*Name*____________________________
PIXE Homework #3 - Physics 100
Union College Fall 2014

1. If a spectrograph (a graph of the energy or wavelength spectrum) had a wavelength resolution of $\Delta\lambda = 10^{-12}$ m, would it be able to separate the K_α lines for platinum(Z=78) and gold(Z=79)? (Resolution means that anything smaller than this value, and I won't be able to distinguish the lines from each other.)

$$\Delta\lambda = \frac{hc}{\Delta E} = \frac{6.63\times10^{-34}\,Js\times\frac{1eV}{1.6\times10^{-19}\,J}\times3\times10^{8}\,\frac{m}{s}}{62056.8eV-60475.8eV} = 7.86\times10^{-10}\,m.$$

Since this is larger than 10^{-12}m I'll be able to see them as separate. The K_α energies of Pt and Au are given from

$$E_\alpha = -13.6eV\times(Z-1)^2\left[\frac{1}{n_{upper}^2}-\frac{1}{n_{lower}^2}\right]$$

$$E_{\alpha,Pt} = -13.6eV\times(78-1)^2\left[\frac{1}{2^2}-\frac{1}{1^2}\right] = 60475.8eV$$

$$E_{\alpha,Au} = -13.6eV\times(79-1)^2\left[\frac{1}{2^2}-\frac{1}{1^2}\right] = 62056.8eV$$

2. What corresponding energy resolution for $\Delta\lambda$ given in problem 1? (In other words, given $\Delta\lambda$ above, what is the difference in energies, ΔE that two elements must have so I can tell them apart?)

$$\Delta E = \frac{hc}{\Delta\lambda} = \frac{6.63\times10^{-34}\,Js\times\frac{1eV}{1.6\times10^{-19}\,J}\times3\times10^{8}\,\frac{m}{s}}{1\times10^{-12}\,m} = 1.24MeV$$

3. What are the two shortest wavelengths for a molybdenum(Z=42) atom?

The two shortest wavelengths are given from the energies. The energies are calculated using

$$\Delta E_\alpha = E_2 - E_1 = -13.6eV(Z-1)^2\left[\frac{1}{4}-1\right] = 17.15keV = 2.74\times10^{-15}J$$

$$\Delta E_\beta = E_3 - E_1 = -13.6eV(Z-1)^2\left[\frac{1}{9}-1\right] = 20.32keV = 3.25\times10^{-15}J$$

and the wavelengths are given respectively as:

$$\lambda = \frac{hc}{E}$$

$$\lambda_\alpha = \frac{6.63\times10^{-34}Js\times3\times10^{8}\,\tfrac{m}{s}}{2.74\times10^{-15}J} = 7.26\times10^{-11}m$$

$$\lambda_\beta = \frac{6.63\times10^{-34}Js\times3\times10^{8}\,\tfrac{m}{s}}{3.25\times10^{-15}J} = 6.11\times10^{-11}m$$

4. An unknown single element target is used in a PIXE experiment and characteristic x-rays are produced with wavelengths of 1.55x10^{-10}m and 1.31x10^{-10}m. What is the elemental make up of the target?

Using $E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}Js\times\frac{1eV}{1.6\times10^{-19}J}\times3\times10^{8}\,\frac{m}{s}}{1.55\times10^{-10}m} = 8.02keV$ *and* $E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}Js\times\frac{1eV}{1.6\times10^{-19}J}\times3\times10^{8}\,\frac{m}{s}}{1.31\times10^{-10}m} = 9.49keV$*, the element could be Copper using the table of x-ray energies, or it could be Zinc. The actual element is copper because the* K_α *energy is exact.*

5. Show that the Moseley's law for K_α radiation may be expressed as $\sqrt{f} = \sqrt{\frac{3}{4}\left(\frac{13.6eV}{h}\right)}(Z-1)$ where f is the x-ray frequency.

$$E_n = -13.6eV(Z-1)^2\left[\frac{1}{n_{upper}^2}-\frac{1}{n_{lower}^2}\right] = hf \rightarrow f = -\frac{13.6eV(Z-1)^2}{h}\left[\frac{1}{n_{upper}^2}-\frac{1}{n_{lower}^2}\right] \rightarrow$$

$$f = \frac{-13.6eV(Z-1)^2}{h}\times\left(\frac{1}{2^2}-\frac{1}{1^2}\right) = \frac{3}{4}\times\frac{13.6eV(Z-1)^2}{h}$$

$$\sqrt{f} = \sqrt{\frac{3}{4}\times\frac{13.6eV}{h}}(Z-1)$$