Name $\qquad$
Physics 111 Quiz \#4, February 7, 2014
Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

Below are two independent questions involving magnetic fields and forces.
a. A mass spectrometer is an analytical instrument used to identify various atoms or molecules in a sample. The samples to be analyzed consist of singly ionized atoms or singly ionized molecules and these atoms or molecules are accelerated through a potential difference $\Delta V$ and then enter a region of uniform magnetic field as shown below. The interaction of the atom or molecule with the magnetic field bends the species into a circle where they either strike either the detector or the wall. As the potential difference is changed different ions can be selected out based on their mass. Typical design values have the magnetic field strength $B=0.2 T$ and the spacing between the entrance and exit holes $d=8 \mathrm{~cm}$. If $\Delta V=15.4 M V$ what atom or molecule might have been detected? (Hints: ${ }^{1)} \mathrm{A}$ table of selected atoms is below and you may have to combine elements from the table to form a molecule. ${ }^{2)}$
$1 u=1.66 \times 10^{-27} \mathrm{~kg}$ )

| Ion | Mass |
| :--- | :--- |
| ${ }^{12} \mathrm{C}$ | $12.0000 u$ |
| ${ }^{14} N$ | $14.0031 u$ |
| ${ }^{16} O$ | $15.9949 u$ |


$F_{b}=q v B=\frac{m v^{2}}{R} \rightarrow m=\frac{q R B}{v} \rightarrow m^{2}=\frac{q^{2} R^{2} B^{2}}{v^{2}}=\frac{q^{2} R^{2} B^{2} m}{2 q V} ;$ where $W=q V=\frac{1}{2} m v^{2} \rightarrow v^{2}=\frac{2 q V}{m}$
$\Rightarrow m=\frac{q R^{2} B^{2}}{2 V}=\frac{q(d / 2)^{2} B^{2}}{2 V}=\frac{e d^{2} B^{2}}{8 V}=\left[\frac{1.6 \times 10^{-19} \mathrm{C} \times(0.08 \mathrm{~m})^{2} \times(0.2 \mathrm{~T})^{2}}{8 \times 15.4 \times 10^{6} \mathrm{~V}}\right] \times \frac{1 \mathrm{u}}{1.66 \times 10^{-27} \mathrm{~kg}}=2 \times 10^{-4} \mathrm{u}$
Now, you may notice that this mass doesn't correspond to anything in the chart. Clearly I'm not immune to math errors myself. In my original calculation I wanted $O_{2}$ and I used this mass to determine the accelerating potential difference I would need to see $O_{2}$ bend and hit the detector. However, in the $R$ term above, I wrote in my original solution when I worked out the problem I had $R=2 d$, which will give $V=15.4 M V$. Clearly $R \neq 2 d$ but $R=d / 2$ which produces no value for any mass combination in the table. However, if you wanted to
detect any of those elements or molecules (say ${ }^{12} \mathrm{C},{ }^{14} \mathrm{~N},{ }^{16} \mathrm{O}, \mathrm{CO}, \mathrm{NO}, \mathrm{N}_{2}$, or $\mathrm{O}_{2}$ ) then you would need the accelerating potentials given by
$V=\frac{q R^{2} B^{2}}{2 m}=\frac{q(d / 2)^{2} B^{2}}{2 m}=\frac{e d^{2} B^{2}}{8 m}$ as shown in the chart below.

| Ion | Mass $(u)$ | Accelerating $\mathrm{V}(V)$ |
| :--- | :--- | :--- |
| ${ }^{12} \mathrm{C}$ | 12.0000 | 257.03 |
| ${ }^{14} \mathrm{~N}$ | 14.0031 | 220.26 |
| ${ }^{16} \mathrm{O}$ | 15.9949 | 192.83 |
| CO | 27.9949 | 110.17 |
| NO | 29.9980 | 102.82 |
| $\mathrm{O}_{2}$ | 21.9898 | 140.26 |
| $\mathrm{~N}_{2}$ | 28.0062 | 110.13 |

b. A 10-turn loop of wire, shown below, lies in a horizontal plane, parallel to a uniform horizontal magnetic field $B$. The wire has a current $I=2.0 \mathrm{~A}$ running through it supplied by a battery that is not shown and a 50 g mass hangs from one edge of the loop. The loop is free to rotate about a non-magnetic axle through the center of the loop. What magnetic field strength $B$ and what direction for the current flow will prevent the loop from rotating about the axle? (Hint: In order to define the direction of the current flow look at the loop as if it were on a table and you are above it looking down.)

$\tau_{\text {net }}=\tau_{\text {loop }}-\tau_{\text {weight }}=0 \rightarrow \tau_{\text {loop }}=\tau_{\text {weight }} \rightarrow$ NIAB $=r m g$
$B=\frac{r m g}{N I A}=\frac{0.025 \mathrm{~m} \times 0.050 \mathrm{~kg} \times 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}{10 \times 2 \mathrm{~A} \times(0.05 \mathrm{~m} \times 0.1 \mathrm{~m})}=0.1225 \mathrm{~T}=122.5 \mathrm{mT}$
On the side of the wire with the weight, the down ward force due to the weight needs to be balanced by an upward force on the wire due to the interaction of the current flowing with the magnetic field. This, by the right hand rule, produces a current that flows clockwise when viewed from above.

## Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r} \\
& P E=k \frac{Q_{1} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V}{\Delta x} \\
& W_{A, B}=q \Delta V_{A, B}
\end{aligned}
$$

## Magnetic Forces and Fields

$$
\begin{aligned}
& F=q v B \sin \theta \\
& F=I l B \sin \theta \\
& \tau=N I A B \sin \theta=\mu B \sin \theta \\
& P E=-\mu B \cos \theta \\
& B=\frac{\mu_{0} I}{2 \pi r} \\
& \varepsilon_{\text {induced }}=-N \frac{\Delta \phi_{B}}{\Delta t}=-N \frac{\Delta(B A \cos \theta)}{\Delta t}
\end{aligned}
$$

## Constants

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\mathrm{C}^{2}}{\mathrm{Nm}}{ }^{2}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{Nn} n^{2}}{\mathrm{C}^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{\mathrm{Tm}}{4}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{c^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A x^{2}+B x+C=0 \rightarrow x=\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}$

Electric Circuits

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t} \\
& V=I R=I\left(\frac{\rho L}{A}\right) \\
& R_{\text {series }}=\sum_{i=1}^{N} R_{i} \\
& \frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}} \\
& P=I V=I^{2} R=\frac{V^{2}}{R} \\
& Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V=\left(\kappa C_{0}\right) V \\
& P E=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C} \\
& Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right) \\
& Q_{\text {discharge }}(t)=Q_{\max } e^{-\frac{t}{R C}} \\
& C_{\text {parallel }}=\sum_{i=1}^{N} C_{i} \\
& \frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}
\end{aligned}
$$

Light as a Particle \& Relativity

$$
\begin{aligned}
& E=h f=\frac{h c}{\lambda}=p c \\
& K E_{\max }=h f-\phi=e V_{\text {stop }} \\
& \Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi) \\
& \gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& p=\gamma m v \\
& E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2} \\
& E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4} \\
& E_{\text {rest }}=m c^{2} \\
& K E=(\gamma-1) m c^{2}
\end{aligned}
$$

Geometry
Circles: $C=2 \pi r=\pi D$

Triangles: $\quad A=\frac{1}{2} b h$
Spheres: $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$

Light as a Wave

$$
\begin{aligned}
& c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}} \\
& S(t)=\frac{\text { energy }}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}} \\
& I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}} \\
& P=\frac{S}{c}=\frac{\text { Force }}{\text { Area }} \\
& S=S_{o} \cos ^{2} \theta \\
& v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n} \\
& \theta_{\text {inc }}=\theta_{\text {ref } l} \\
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \\
& \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}} \\
& M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}} \\
& M_{\text {total }}=\prod_{i=1}^{N} M_{i} \\
& d \sin \theta=m \lambda \text { or }\left(m+\frac{1}{2}\right) \lambda \\
& a \sin \phi=m^{\prime} \lambda \\
& \text { Nuclear Physics } \\
& E_{\text {binding }}=\left(Z m_{p}+N m_{n}-m_{\text {rest }}\right) \varepsilon^{2} \\
& \frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t} \\
& A(t)=A_{o} e^{-\lambda t} \\
& m(t)=m_{o} e^{-\lambda t} \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{aligned}
$$

Misc. Physics 110 Formulae

$$
\begin{aligned}
& \vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a} \\
& \vec{F}=-k \vec{y} \\
& \vec{F}_{C}=m \frac{v^{2}}{R} \hat{r} \\
& W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E \\
& P E_{\text {gravity }}=m g y \\
& P E_{\text {spring }}=\frac{1}{2} k y^{2}
\end{aligned}
$$

$$
x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}
$$

$v_{f x}=v_{i x}+a_{x} t$
$v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

