

# Physics 111 Homework Solutions Week #10 - Wednesday

Wednesday, March 5, 2014

## Chapter 26

### Questions

26.1 The atomic number  $Z$  is the number of protons in the nucleus. It distinguishes the different types of atoms.  $N$  is the number of neutrons in the atom. If we sum the number of neutrons ( $N$ ) and the number of protons ( $Z$ ) we get the mass of the nucleus (and the atom if we assume the mass of the electrons are negligible).

$\alpha$  decay is characterized by the following reaction:  ${}^A_ZX \rightarrow {}^4_2\text{He} + {}^{A-4}_{Z-2}Y$  therefore the mass number of the nucleus decreases by 4 and the atomic number decreases by 2.

$\beta^-$  decay (for a high speed electron) is characterized by the following reaction:  ${}^A_ZX \rightarrow {}^0_{-1}e + {}^A_{Z+1}Y$  therefore the mass number of the nucleus is unaffected and the atomic number increases by 1.

$\beta^+$  decay (for a high speed positron) is characterized by the following reaction:  ${}^A_ZX \rightarrow {}^0_{+1}e + {}^A_{Z-1}Y$  therefore the mass number of the nucleus is unaffected and the atomic number decreases by 1.

$\gamma$  decay is characterized by the following reaction:  ${}^A_ZX^* \rightarrow {}^0_0\gamma + {}^A_ZX$  therefore the mass number of the nucleus and the atomic number remain unchanged.

26.4 The three requirements for stability are as follows. 1) The number of neutrons in the nucleus. As more protons are packed into the nucleus those nuclides with significantly more neutrons than protons will tend to produce a stable nucleus. The extra neutrons tend to shield the individual protons from one another. 2) The binding energy of the nucleus. 3) The nuclear energy levels (like those of the electron) should be closed shells. That is the most stable nuclei have equal numbers of protons and neutrons (these numbers are called the magic numbers).

### Multiple-Choice

- 26.1 D
- 26.2 A
- 26.4 A

## Problems

### 26.2 A neutron star

- a. To calculate the mass of the neutron star we need to know the density of nuclear material and the volume of the star. We model the star as being spherical so it has a volume given by  $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1 \times 10^4 \text{ m})^3 = 4.19 \times 10^{12} \text{ m}^3$ . Given that the nuclear density is  $2 \times 10^{17} \text{ kg/m}^3$  (assuming the nuclear radius is  $1.2 \times 10^{-15} \text{ m}$ ) we have the mass  $M = \rho V = 8.4 \times 10^{29} \text{ kg}$ . Since the mass of the sun is  $2 \times 10^{30} \text{ kg}$  this gives the mass of the neutron star as 0.41 solar masses. As an aside, this is almost  $\frac{1}{2}$  the mass of the sun packed in to a sphere of radius 10 km  $\sim$  6 miles!!
- b. Assuming that the mass of a nucleon (either a proton or neutron) is  $1.67 \times 10^{-27} \text{ kg}$ , this gives the number of nucleons as  $(8.4 \times 10^{29} \text{ kg}) / (1.67 \times 10^{-27} \text{ kg/nucleon}) = 5.03 \times 10^{56}$  particles.

26.3 The density is given by  $\rho = \frac{M}{V} \rightarrow V = \frac{M}{\rho} = \frac{4}{3}\pi r^3$  and therefore the radius is given

$$\text{as } r = \sqrt[3]{\frac{M}{4\pi\rho}} = \sqrt[3]{\frac{2 \times 10^{30} \text{ kg}}{4\pi(2 \times 10^{17} \frac{\text{kg}}{\text{m}^3})}} = 11,675 \text{ m} = 11.7 \text{ km}.$$

**Friday, March 7, 2104**

None due to exam #3

**Monday, March 10, 2014**

## Chapter 26

### Questions

- 26.7  $\alpha$  particles are low energy so they do not penetrate very far into tissue. They are stopped by the skin producing burns to the exposed patch.

$\beta$  particles are higher energy and will penetrate farther into tissue than  $\alpha$  particles. Beta particles travel about 100 times farther than alpha particles. Whereas alpha particles seldom pass beyond the outer, dead layer of the skin, the free, fast-moving electrons and positrons that constitute beta radiation penetrate for about a quarter of an inch into living matter.

$\gamma$  rays are very high-energy photons and are not stopped by the skin. They pass through almost undisturbed. However they will ionize atoms as they pass by. Gamma rays and X rays will pass readily through a large organism; they reach the innermost recesses of the body and injure highly sensitive tissues, but they produce only about one twentieth of the damage inflicted on cells by alpha particles.

- 26.8 To determine the unknown product we use both conservation of mass and charge.
- a) For the following reaction  ${}_{27}^{60}\text{Co} \rightarrow {}_{28}^{60}\text{Ni} + {}_{Z'}^{A'}\text{X}$ , conservation of charge gives me  $27 = 28 + Z'$  which gives  $Z' = -1$ . Conservation of mass gives me  $60 = 60 + A'$  which gives  $A' = 0$ . Thus the unknown particle is a beta particle,  ${}_{Z'}^{A'}\text{X} = {}_{-1}^0\text{e}$ .
- b) For the following reaction  ${}_{Z'}^{A'}\text{X} \rightarrow {}_{91}^{234}\text{Pa} + {}_{-1}^0\text{e}$ , conservation of charge gives me  $Z' = 91 - 1$  which gives  $Z' = 90$ . Conservation of mass gives me  $A' = 234 + 0$  which gives  $A' = 234$ . Thus the unknown particle is a thorium nucleus,  ${}_{Z'}^{A'}\text{X} = {}_{90}^{234}\text{Th}$ .
- c) For the following reaction  ${}_{Z'}^{A'}\text{X} \rightarrow {}_{90}^{234}\text{Th} + {}_2^4\text{He}$ , conservation of charge gives me  $Z' = 90 + 2$  which gives  $Z' = 92$ . Conservation of mass gives me  $A' = 230 + 4$  which gives  $A' = 234$ . Thus the unknown particle is a uranium nucleus,  ${}_{Z'}^{A'}\text{X} = {}_{92}^{234}\text{U}$ .

### Multiple-Choice

None

### Problems

26.4 The nuclear binding energy is given through:

$$NBE = Zm_p c^2 + Nm_N c^2 - m_{atom} c^2; \text{ where } m_p = 1.00727u \text{ and } m_n = 1.00867u.$$

For radium-226:

$$NBE = [88(1.00727u) + 138(1.00867u) - 225.97709u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1731.8\text{MeV}.$$

$$\text{The } \frac{NBE}{\text{nucleon}} = \frac{1731.8\text{MeV}}{226} = 7.66 \frac{\text{MeV}}{\text{nucleon}}.$$

For radium-228:

$$NBE = [88(1.00727u) + 140(1.00867u) - 227.98275u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1742.7\text{MeV}.$$

$$\text{The } \frac{NBE}{\text{nucleon}} = \frac{1742.7\text{MeV}}{228} = 7.64 \frac{\text{MeV}}{\text{nucleon}}.$$

For thorium-232:

$$NBE = [90(1.00727u) + 142(1.00867u) - 231.98864u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1766.9\text{MeV}.$$

$$\text{The } \frac{NBE}{\text{nucleon}} = \frac{1766.9\text{MeV}}{232} = 7.62 \frac{\text{MeV}}{\text{nucleon}}.$$

26.6 For the  $\beta$ -decay reaction of  ${}^{24}\text{Na}$ ,

$$Q = (M_{{}_{11}^{24}\text{Na}} - M_{{}_{12}^{24}\text{Mg}} - M_{{}_{-1}^0\text{e}})c^2$$

$$\therefore Q = (23.98492 - 23.97845 - 5.49 \times 10^{-4})uc^2 \times \frac{931.5\text{MeV}}{1uc^2} = 5.52\text{MeV}.$$

26.7 For alpha decay:  $Q = (M_{parent} - M_{daughter} - M_{He})c^2$ . If the parent is at rest when it decays, then from conservation of momentum, the daughter gets a recoil velocity in the direction opposite direction to the velocity of the alpha particle. Conservation of

momentum gives:  $0 = -m_{daughter}v_{daughter} + m_{\alpha}v_{\alpha} \rightarrow v_{daughter} = \frac{m_{\alpha}}{m_{daughter}}v_{\alpha}$ . Next we

apply conservation of energy and we find:

$m_{parent}c^2 = \frac{1}{2}m_{daughter}v_{daughter}^2 + \frac{1}{2}m_{He}v_{He}^2 + m_{daughter}c^2 + m_{He}c^2$ . Then we can replace the

velocity of the recoiling daughter atom in terms of the velocity of the alpha particle, which can be measured. We have

$m_{parent}c^2 = \frac{1}{2}m_{daughter}\left(\frac{m_{\alpha}}{m_{daughter}}v_{\alpha}\right)^2 + \frac{1}{2}m_{He}v_{He}^2 + m_{daughter}c^2 + m_{He}c^2$ . Bringing all of the

rest of the energy terms to one side and combining the terms involving the velocity of the alpha particle we find that

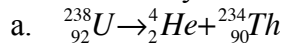
$(m_{parent}c^2 - m_{daughter}c^2 - m_{He}c^2) = Q = \left[\frac{1}{2}\frac{m_{\alpha}^2}{m_{daughter}} + \frac{1}{2}m_{\alpha}\right]v_{\alpha}^2$ . Factoring out the mass of

the alpha particle on the left hand side we have  $Q = \frac{1}{2}m_{\alpha}v_{\alpha}^2\left[1 + \frac{m_{\alpha}}{m_{daughter}}\right]$  which is the

desired result.

From example 27.2,  $Q = 4.28 \text{ MeV}$ ,  $m_{\alpha} = 4.0015\text{u}$ ,  $m_{daughter} = m_{thorium} = 233.99409\text{u}$ , and we find the velocity of the alpha particle to be  $1.42 \times 10^7 \text{ m/s}$ .

### 26.16 Uranium Decay



b. The daughter nucleus is thorium.

c. The kinetic energy of the emitted alpha particle is given by (ignoring the recoil of the thorium nucleus):

$$KE = [M_U - M_\alpha - M_{Th}]c^2$$

$$KE = \left\{ [238.000187u - 4.00150u - 233.99409u] \times \frac{931.5 \text{ MeV}/c^2}{1u} \right\} c^2 = 4.282 \text{ MeV}$$

d. To determine the speed of the emitted alpha particle we use the relativistic form of the kinetic energy since we don't know if the alpha particle is relativistic or not. Thus

$$KE = 4.282 \text{ MeV} = (\gamma - 1)m_\alpha c^2 = (\gamma - 1) \left[ 4.00150u \times \frac{931.5 \text{ MeV}/c^2}{1u} \right] c^2 \rightarrow \gamma = 1.0015$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow v = \sqrt{1 - \frac{1}{\gamma^2}} c = \sqrt{1 - \frac{1}{(1.0015)^2}} c = 0.055c$$

e. Based on part d, the alpha particle is almost at the relativistic cutoff. However, we have actually ignored the recoil of the thorium nucleus, so if we were to include the recoil of the thorium atom, the alpha particle is probably not actually relativistic.