

## Physics 111 Homework Solutions Week #2 – Wednesday

Friday, January 10, 2014

### Chapter 14

#### Questions

- 14.2 Since objects are charged each will exert equal and opposite forces on each other. If the test charge is massive then its acceleration will be small and both charges will move around in the field of the other. If on the other hand the test charge is small, its acceleration is very large and the test charge will experience the largest change in motion and can be used to map out the electric field of the other charges.

#### Multiple-Choice

- 14.8 C

#### Problems

- 14.7 Assuming a standard Cartesian coordinate system and applying Newton's 2<sup>nd</sup> law we have for the vertical forces ,

$$\sum F_y : F_T \cos 30 - F_W = 0 \rightarrow F_T = \frac{mg}{\cos 30} = \frac{0.02 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2}}{\cos 30} = 0.226 \text{ N}$$
 while for the horizontal forces

$$\sum F_x : -F_T \sin 30 + F_E = 0 \rightarrow F_E = k \frac{Q^2}{r^2} = F_T \sin 30$$

$$\rightarrow Q = \sqrt{\frac{F_T r^2 \sin 30}{k}} = \sqrt{\frac{0.226 \text{ N} \times (0.5 \text{ m})^2 \times \sin 30}{9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}}} = 1.77 \times 10^{-6} \text{ C} = 1.77 \mu\text{C}$$
 Thus

the total charge on the electroscope is twice this value or  $3.6 \mu\text{C}$ .

- 14.8 To determine the amount of charge on either the moon or the Earth, equate the gravitational force law to the electric force law. This produces

$$F_G = F_E \rightarrow \frac{GM_E M_M}{r^2} = \frac{kQ_E Q_M}{r^2} = \frac{kQ^2}{r^2} \rightarrow Q = \sqrt{\frac{G}{k} M_E M_M} =$$

$$Q = \sqrt{\left( \frac{6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}}{9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}} \right) \times 7.35 \times 10^{22} \text{ kg} \times 5.98 \times 10^{24} \text{ kg}} = 5.7 \times 10^{13} \text{ C}$$

Monday, January 13, 2014

### Chapter 14

#### Questions

- None

#### Multiple-Choice

- None

## Problems

14.12 The electric force on a charge

- a. The electric field at the point (1mm,0mm) is the vector sum of the fields due to the positive charge located at (0mm,0.5mm) and the negative charge located at (0mm,-0.5mm). At point A, we have the magnitude of the electric field

$$E_A = E_{upper,y} + E_{lower,y} = -\frac{kQ}{r_{upper}^2} \sin \theta - \frac{kQ}{r_{lower}^2} \sin \theta = -2 \frac{kQ}{r_{upper}^2} \sin \theta = -2 \frac{kQa}{r_{upper}^{3/2}}$$

$$E_A = \frac{-2 \times 9 \times 10^9 \frac{Nm^2}{C^2} \times 5 \times 10^{-6} C \times 0.5 \times 10^{-3} m}{(1.1 \times 10^{-3} m)^{3/2}} = 1.2 \times 10^6 \frac{N}{C}$$

and the direction of the field is vertically down, where we have used the fact that the horizontal components of the electric field vanish due to the symmetry in the problem. We have calculated  $r_{upper}$  and  $r_{lower}$  based on the geometry of the system using the Pythagorean theorem. We find

$$r_{upper} = r_{lower} = \sqrt{(0.5 \times 10^{-3} m)^2 + (1.0 \times 10^{-3} m)^2} = 1.1 \times 10^{-3} m$$

Therefore the electric force is  $F_A = qE_A = 2 \times 10^{-6} C \times 1.2 \times 10^6 \frac{N}{C} = 2.5 N$  vertically down.

- b. The electric field at the point (0mm,1mm) is the vector sum of the fields due to the positive charge located at (0mm,0.5mm) and the negative charge located at (0mm,-0.5mm). At point B, the net electric field points in the positive y-direction and the magnitude is

$$E_B = E_{upper,y} + E_{lower,y} = +\frac{kQ}{r_{upper}^2} - \frac{kQ}{r_{lower}^2} = kQ \left( \frac{1}{r_{upper}^2} - \frac{1}{r_{lower}^2} \right)$$

$$E_B = \left( 9 \times 10^9 \frac{Nm^2}{C^2} \times 5 \times 10^{-6} C \right) \times \left( \frac{1}{(0.5 \times 10^{-3} m)^2} - \frac{1}{(1.5 \times 10^{-3} m)^2} \right) = 1.6 \times 10^{11} \frac{N}{C}$$

Therefore the electric force is  $F_A = qE_A = 2 \times 10^{-6} C \times 1.6 \times 10^{11} \frac{N}{C} = 3.2 \times 10^5 N$  vertically up.

- c. The electric field at the point (0mm,-1mm) is the vector sum of the fields due to the positive charge located at (0mm,0.5mm) and the negative charge located at (0mm,-0.5mm). At point C, the net electric field

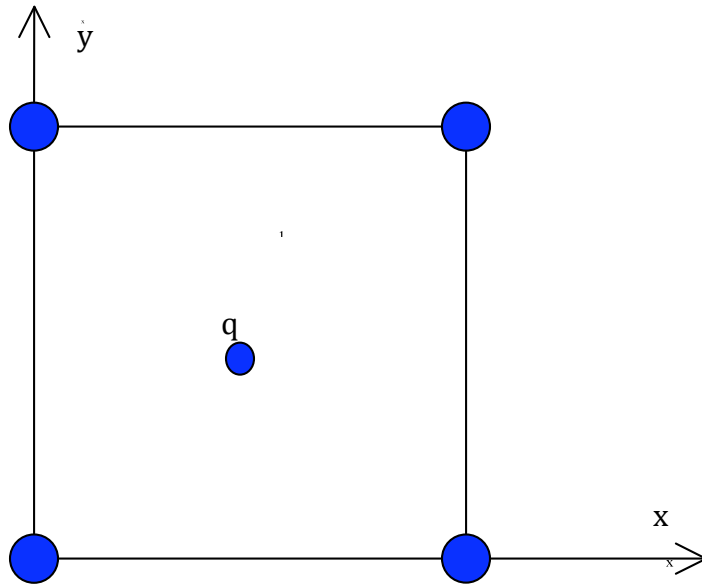
$$E_C = E_{upper,y} + E_{lower,y} = -\frac{kQ}{r_{upper}^2} + \frac{kQ}{r_{lower}^2} = kQ \left( -\frac{1}{r_{upper}^2} + \frac{1}{r_{lower}^2} \right)$$

$$E_B = \left( 9 \times 10^9 \frac{Nm^2}{C^2} \times 5 \times 10^{-6} C \right) \times \left( -\frac{1}{(1.5 \times 10^{-3} m)^2} + \frac{1}{(0.5 \times 10^{-3} m)^2} \right) = 1.6 \times 10^{11} \frac{N}{C}$$

Therefore the electric force is  $F_A = qE_A = 2 \times 10^{-6} C \times 1.6 \times 10^{11} \frac{N}{C} = 3.2 \times 10^5 N$  vertically up.

14.14 The electric field at the center of a square.

a.



Easy way: By symmetry Arguments:  $\vec{E}_{net} = 0 \frac{N}{C}$

Harder way:  $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

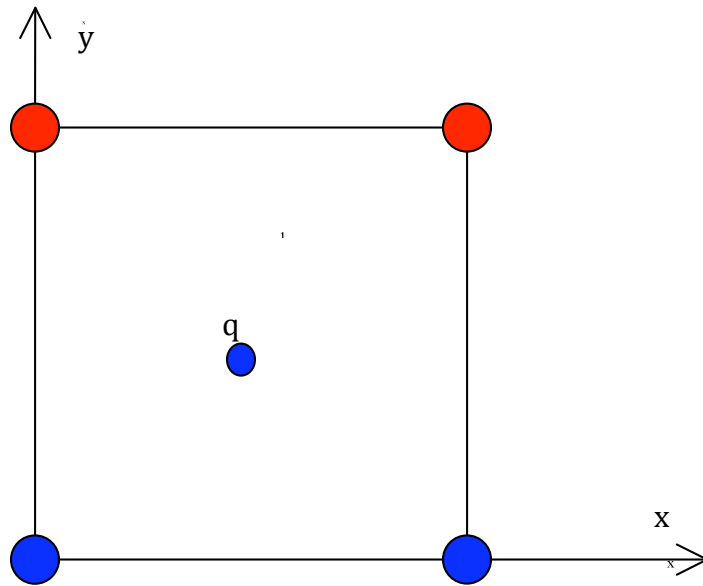
$$E_{net,x} = -\frac{kQ}{r^2} \cos 45 + \frac{kQ}{r^2} \cos 45 + \frac{kQ}{r^2} \cos 45 - \frac{kQ}{r^2} \cos 45 = 0 \frac{N}{C}$$

$$E_{net,y} = +\frac{kQ}{r^2} \sin 45 + \frac{kQ}{r^2} \sin 45 - \frac{kQ}{r^2} \sin 45 - \frac{kQ}{r^2} \sin 45 = 0 \frac{N}{C}$$

for  $r = \frac{a}{\sqrt{2}}$ , where a is the length of the side of the square.

Therefore,  $\vec{E}_{net} = 0 \frac{N}{C}$

b.



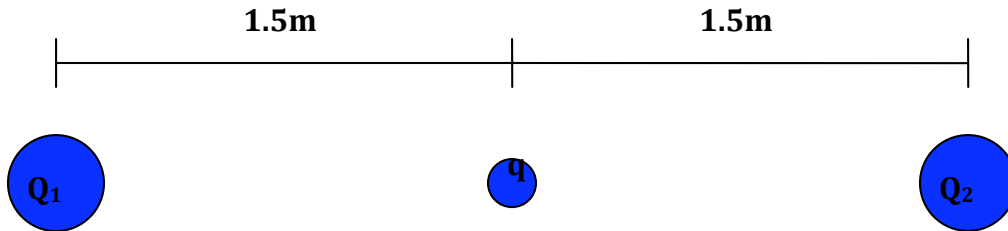
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$E_{net,x} = -\frac{kQ}{r^2} \cos 45 - \frac{kQ}{r^2} \cos 45 + \frac{kQ}{r^2} \cos 45 + \frac{kQ}{r^2} \cos 45 = 0 \frac{N}{C}$$

$$E_{net,y} = +\frac{kQ}{r^2} \sin 45 + \frac{kQ}{r^2} \sin 45 + \frac{kQ}{r^2} \sin 45 + \frac{kQ}{r^2} \sin 45 = \frac{4\sqrt{2}kQ}{a^2} \frac{N}{C}$$

for  $r = \frac{a}{\sqrt{2}}$ , where a is the length of the side of the square.

- 14.16 We place a positive test charge  $q$  at the midpoint between the two point charges  $Q$  and calculate the electric field. The magnitude of the electric field is that of a point charge and the directions are found from Coulomb's law applied to the test charge  $q$ .



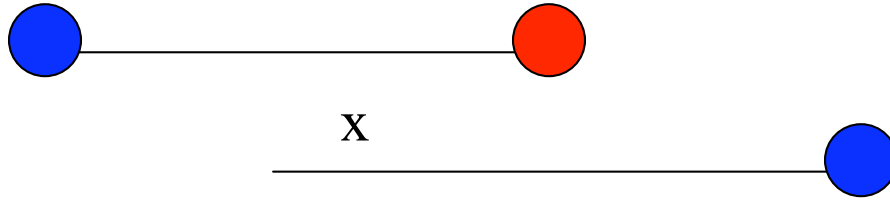
$$\begin{aligned}\vec{E}_{\text{midpoint}} &= \vec{E}_{Q_1} + \vec{E}_{Q_2} = \left( \frac{kQ_1}{r_1^2} - \frac{kQ_2}{r_2^2} \right) \hat{i} \\ &= \left( 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \right) \left( 5 \times 10^{-6} \text{C} \right) \left( \frac{1}{(1.5\text{m})^2} - \frac{1}{(1.5\text{m})^2} \right) \hat{i} \\ &= 0 \frac{\text{N}}{\text{C}}.\end{aligned}$$

- 14.17 Assume that the  $5\mu\text{C}$  point charge is at the upper vertex of the triangle and that the two  $-10\mu\text{C}$  charges lie along the x-axis. Due to the symmetry of the problem, the horizontal components of the electric field will vanish and we will only have a vertically downward component to the electric field. We have for the net vertically downward electric field, a magnitude of

$$\begin{aligned}E_{\text{net},y} &= \frac{kQ_5}{r_5^2} + \frac{kQ_{10}}{r_{10}^2} \sin \theta + \frac{kQ_{10}}{r_{10}^2} \sin \theta \\ E_{\text{net},y} &= 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \left[ \frac{5 \times 10^{-6} \text{C}}{(0.33\text{m})^2} + \frac{10 \times 10^{-6} \text{C}}{(0.33\text{m})^2} \sin 30 + \frac{10 \times 10^{-6} \text{C}}{(0.33\text{m})^2} \sin 30 \right] = 1.23 \times 10^6 \frac{\text{N}}{\text{C}}\end{aligned}$$

and where  $r_5$  and  $r_{10}$  are calculated using the geometry of the problem and the Pythagorean Theorem.

14.18



Defining the distance from the midpoint between the first two charges to the location of the added charge as  $x$ , we have

$$E_{net@0.6m} = E_5 + E_{-8} - E_{added5} = \frac{k(5 \times 10^{-6} C)}{(0.6m)^2} + \frac{k(8 \times 10^{-6} C)}{(0.6m)^2} - \frac{k(5 \times 10^{-6} C)}{x^2} = 0$$

$$x = 0.37m$$

14.22 In order to calculate the minimum charge, we assume that the electric fields detected by the fish are due to point charges. Therefore,

$$E = 7 \times 10^{-6} \frac{N}{m} \text{ at } r = 1m \text{ and } E = \frac{kQ}{r^2} \rightarrow Q = \frac{Er^2}{k}$$

$$\text{Thus, } Q = \frac{(7 \times 10^{-6} \frac{N}{m})(1m)^2}{9 \times 10^9 \frac{Nm^2}{C^2}} = 7.8 \times 10^{-16} C.$$