Physics 111 Homework Solutions Week #8 - Friday

Tuesday, February 25, 2014

Chapter 23

Questions

- None

Multiple-Choice

23.14 C

Problems

23.9 The absorption coefficient is given by

$$\ln\frac{S}{S_0} = \mu x \rightarrow \mu = \frac{1}{x}\ln\frac{S}{S_0} = \frac{1}{0.015m}\ln\frac{0.98S_0}{S_0} = -1.35m^{-1}.$$

23.10 We are given that with the same incident intensity sample 1 has a 95% transmittance, while sample 2 has an 85% transmittance. Thus for sample 1 we find that $\ln(0.95) = \mu_1 x$ and from sample 2, $\ln(0.85) = \mu_2 x$. Taking the ratio we find that $\frac{\mu_1}{\mu_2} = \frac{x\ln(0.95)}{x\ln(0.85)} = 0.32$.

23.11 The CT number is given as $1000\left[\frac{\mu - \mu_w}{\mu_w}\right] = 1000\left[\frac{-7\times10^{-3}x^{-1} + 5\times10^{-3}x^{-1}}{-5\times10^{-3}x^{-1}}\right] = 400.$

ε and ε_w are obtained from

$$\mu = \frac{1}{x}\ln\frac{I}{I_0} = \frac{1}{x}\ln\frac{0.993S_0}{S_0} = -7\times10^{-3}x^{-1}$$

$$\mu_w = \frac{1}{x}\ln\frac{I}{I_0} = \frac{1}{x}\ln\frac{0.995S_0}{S_0} = -5\times10^{-3}x^{-1}.$$

Wednesday, February 26, 2014

Chapter 24

Questions

24.1 Based on special relativity we know that as a particle with mass travels near the speed of light its mass increases. In order to accelerate this object from rest to a speed near that of light would require an ever increasing force (one that rapidly becomes larger by a factor of γ.) There are no known forces that could accelerate a particle with mass to the speed of light in a finite amount of time and with a finite amount of energy. So for objects with no rest mass, as they travel at the speed of light, there mass does not increase with increasing speed and we avoid these problems of accelerating the massless particles.

Multiple-Choice

- None

Problems

24.1 Relativistic energy and momentum for an object of mass m.

For an object with a $m = 1kg$ rest mass, it has a rest energy of $E = mc^2 = 9x10^{16}\ J$.

The Lorentz factor is given by: $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$, with the relativistic momentum $p = \gamma mv$, and the relativistic energy $E^2 = p^2c^2 + m^2c^4$.

a. For a velocity of 0.8c, $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.64}} = 1.67$. The relativistic momentum and energy are therefore,

$p = \gamma mv = 1.67 \times 1kg \times 0.8 \times 3 \times 10^8\ \frac{m}{s} = 4.0 \times 10^8\ \frac{kgm}{s}$ and

$E = \sqrt{p^2c^2 + m^2c^4} = \sqrt{\left(4.0\times 10^8\ \frac{kgm}{s} \times 3 \times 10^8\ \frac{m}{s}\right)^2 + \left(9\times 10^{16}\ J\right)^2} = 1.5 \times 10^{17}\ J$.

b. Following the procedure in part a, for a velocity of 0.9c,

$\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.81}} = 2.29$, the relativistic momentum is $6.19x10^8\ kgm/s$, and the relativistic energy is $2.07x10^{17}\ J$.

c. For a velocity of 0.95c, γ = 3.20, the relativistic momentum is $9.13x10^8\ kgm/s$, and the relativistic energy is $2.88x10^{17}\ J$.

d. For a velocity of 0.99c, γ = 7.09, the relativistic momentum is $2.11x10^9\ kgm/s$, and the relativistic energy is $6.387x10^{17}\ J$.

e. For a velocity of 0.999c, γ = 22.4, the relativistic momentum is $6.70x10^9\ kgm/s$, and the relativistic energy is $2.0x10^{18}\ J$.

24.2 For an electron with rest mass $9.11x10^{-31}\ kg$, it has a rest energy of $E = mc^2 = 8.199x10^{-14}\ J = 0.511\ MeV$. The Lorentz factor is given by: $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$, and the relativistic momentum and energy are given respectively as $p = \gamma mv$ and $E = \sqrt{p^2c^2 + m^2c^4}$.

a. For the electron with a velocity of $0.8c$, $\gamma = 1.67$. The relativistic momentum and energy are therefore,

$p = \gamma mv = 1.67 \times 9.11 \times 10^{-31} kg \times 0.8 \times 3 \times 10^8\ \frac{m}{s} = 3.65 \times 10^{-22}\ \frac{kgm}{s}$ and

$E = \sqrt{p^2c^2 + m^2c^4} = \sqrt{\left(3.65\times 10^{-22}\ \frac{kgm}{s} \times 3 \times 10^8\ \frac{m}{s}\right)^2 + \left(8.199\times 10 - 14J\right)^2} = 1.37 \times 10^{-13}\ J$

Using the fact that $1.6x10^{-19}\ J = 1\ eV$, the relativistic energy is given as

$E = 1.37 \times 10^{-13}\ J \times \dfrac{1eV}{1.6\times 10^{-19}\ J} = 0.855 \times 10^6\ eV = 0.855 MeV$. Further it can be shown that the total relativistic energy is given as $E = \gamma mc^2 = \gamma E_{rest}$. Thus the relativistic energy can be computed in a more efficient method.

$E = \gamma E_{rest} = 1.67 \times 0.511 MeV = 0.855 MeV$.

b. Following the method outlined in part a, for a velocity of *0.9c*, $\gamma = 2.29$. The relativistic momentum is therefore,

$p = \gamma mv = 2.29 \times 9.11 \times 10^{-31} kg \times 0.9 \times 3 \times 10^{8} \frac{m}{s} = 5.63 \times 10^{-22} \frac{kgm}{s}$ and

$E = \gamma E_{rest} = 2.29 \times 0.511 MeV = 1.17 MeV$.

c. For a velocity of *0.95c*, $\gamma = 3.2$. The relativistic momentum is therefore,

$p = \gamma mv = 3.2 \times 9.11 \times 10^{-31} kg \times 0.95 \times 3 \times 10^{8} \frac{m}{s} = 8.31 \times 10^{-22} \frac{kgm}{s}$ and

$E = \gamma E_{rest} = 3.2 \times 0.511 MeV = 1.63 MeV$.

d. For a velocity of *0.99c*, $\gamma = 7.09$. The relativistic momentum is therefore,

$p = \gamma mv = 7.09 \times 9.11 \times 10^{-31} kg \times 0.99 \times 3 \times 10^{8} \frac{m}{s} = 1.92 \times 10^{-21} \frac{kgm}{s}$ and

$E = \gamma E_{rest} = 7.09 \times 0.511 MeV = 3.62 MeV$.

e. For a velocity of *0.999c*, $\gamma = 22.4$. The relativistic momentum is therefore,

$p = \gamma mv = 22.4 \times 9.11 \times 10^{-31} kg \times 0.999 \times 3 \times 10^{8} \frac{m}{s} = 6.12 \times 10^{-21} \frac{kgm}{s}$ and

$E = \gamma E_{rest} = 22.4 \times 0.511 MeV = 11.45 MeV$.

24.4 The relativistic momentum and relativistic energy are given as $p = \gamma mv$ and $E = \gamma mc^2$ respectively. To show the relation between energy and momentum, equation (24.8), we start by squaring the relativistic energy. This gives us $E^2 = \gamma^2 m^2 c^4$. Next, we use a mathematical "trick." We add and subtract the same quantity from the right hand side of the equation we just developed. The quantity we want to add and subtract is v^2. This produces factoring out a factor of c^2, $E^2 = \gamma^2 m^2 c^2 \left(c^2 + v^2 - v^2\right)$. Expanding this result we get $E^2 = \gamma^2 m^2 c^2 v^2 + \gamma^2 m^2 c^2 \left(c^2 - v^2\right)$. Recognizing that the first term is nothing more than p^2c^2 allows us to write $E^2 = p^2 c^2 + \gamma^2 m^2 c^2 \left(c^2 - v^2\right)$. Factoring out a c^2 from the 2nd term on the right hand side give us $E^2 = p^2 c^2 + \gamma^2 m^2 c^4 \left(1 - \frac{v^2}{c^2}\right)$.

The quantity $\left(1 - \frac{v^2}{c^2}\right)$ is simply $\frac{1}{\gamma^2}$. Therefore we arrive at the desired result, $E^2 = p^2 c^2 + m^2 c^4$.