Physics 111 Homework Solutions Collected on Monday 9/22

Wednesday, September 16, 2014

Chapter 14

Questions

- None

Multiple-Choice

-None

Problems

14.12 The electric force on a charge

a. The electric field at the point (1mm,0mm) is the vector sum of the fields due to the positive charge located at (0mm,0.5mm) and the negative charge located at (0mm,-0.5mm). At point A, we have the magnitude of the electric field

$$E_A = E_{upper,y} + E_{lower,y} = -\frac{kQ}{r_{upper}^2}\sin\theta - \frac{kQ}{r_{lower}^2}\sin\theta = -2\frac{kQ}{r_{upper}^2}\sin\theta = -2\frac{kQa}{r_{upper}^{3/2}}$$

$$E_A = \frac{-2\times 9\times 10^9\,\tfrac{Nm^2}{C^2}\times 5\times 10^{-6}C\times 0.5\times 10^{-3}m}{\left(1.1\times 10^{-3}m\right)^{3/2}} = 1.2\times 10^6\,\tfrac{N}{C}$$

and the direction of the field is vertically down, where we have used the fact that the horizontal components of the electric field vanish due to the symmetry in the problem. We have calculated r_{upper} and r_{lower} based on the geometry of the system using the Pythagorean theorem. We find

$$r_{upper} = r_{lower} = \sqrt{\left(0.5\times 10^{-3}m\right)^2 + \left(1.0\times 10^{-3}m\right)^2} = 1.1\times 10^{-3}m$$

Therefore the electric force is $F_A = qE_A = 2\times 10^{-6}C\times 1.2\times 10^6\,\tfrac{N}{C} = 2.5N$ vertically down.

b. The electric field is at the point (0mm,1mm) is the vector sum of the fields due to the positive charge located at (0mm,0.5mm) and the negative charge located at (0mm,-0.5mm). At point B, the net electric field points in the positive y-direction and the magnitude is

$$E_B = E_{upper,y} + E_{lower,y} = +\frac{kQ}{r_{upper}^2} - \frac{kQ}{r_{lower}^2} = kQ\left(\frac{1}{r_{upper}^2} - \frac{1}{r_{lower}^2}\right)$$

$$E_B = \left(9\times 10^9\,\tfrac{Nm^2}{C^2}\times 5\times 10^{-6}C\right)\times\left(\frac{1}{\left(0.5\times 10^{-3}m\right)^2} - \frac{1}{\left(1.5\times 10^{-3}m\right)^2}\right) = 1.6\times 10^{11}\,\tfrac{N}{C}.$$

Therefore the electric force is $F_A = qE_A = 2\times 10^{-6}C\times 1.6\times 10^{11}\,\tfrac{N}{C} = 3.2\times 10^5 N$ vertically up.

c. The electric field is at the point (0mm,-1mm) is the vector sum of the fields due to the positive charge located at (0mm,0.5mm) and the negative charge located at (0mm,-0.5mm). At point C, the net electric field

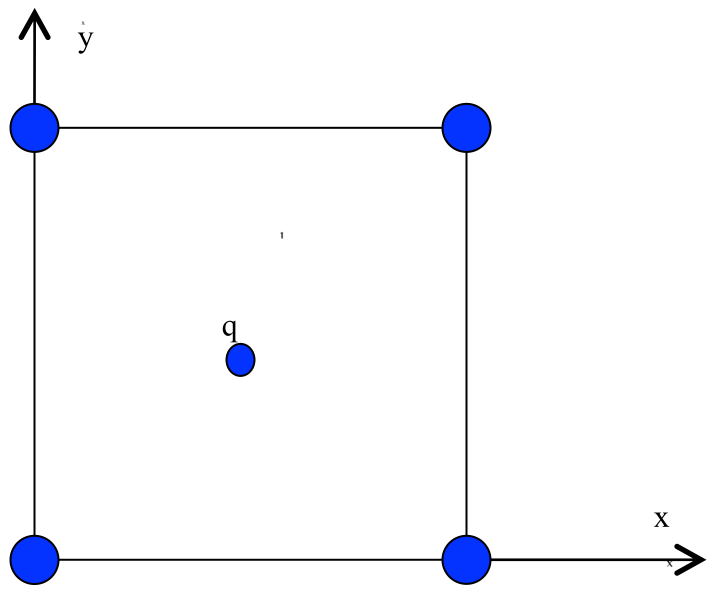
$$E_C = E_{upper,y} + E_{lower,y} = -\frac{kQ}{r_{upper}^2} + \frac{kQ}{r_{lower}^2} = kQ\left(-\frac{1}{r_{upper}^2} + \frac{1}{r_{lower}^2}\right)$$

$$E_B = \left(9\times10^9\,\tfrac{Nm^2}{C^2}\times5\times10^{-6}C\right)\times\left(-\frac{1}{\left(1.5\times10^{-3}m\right)^2} + \frac{1}{\left(0.5\times10^{-3}m\right)^2}\right) = 1.6\times10^{11}\,\tfrac{N}{C}.$$

Therefore the electric force is $F_A = qE_A = 2\times10^{-6}C\times1.6\times10^{11}\,\tfrac{N}{C} = 3.2\times10^5 N$ vertically up.

14.14 The electric field at the center of a square.

a.



Easy way: By symmetry Arguments: $\vec{E}_{net} = 0\,\tfrac{N}{C}$

Harder way: $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

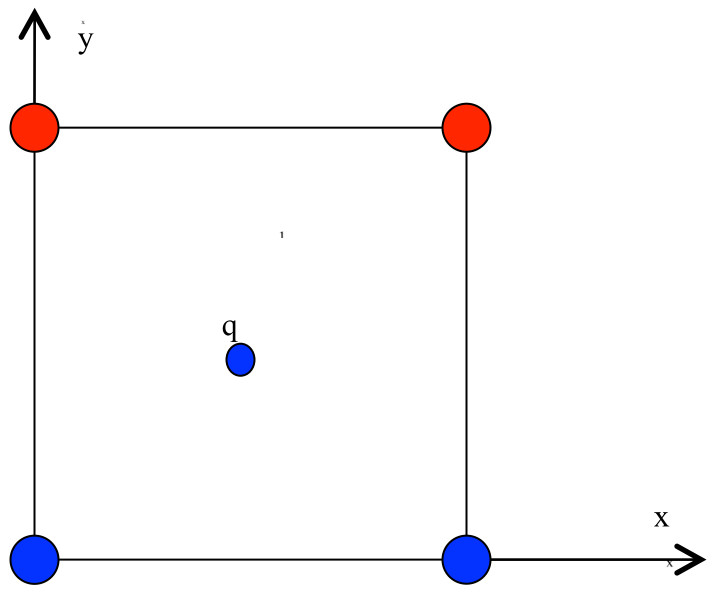
$$E_{net,x} = -\frac{kQ}{r^2}\cos 45 + \frac{kQ}{r^2}\cos 45 + \frac{kQ}{r^2}\cos 45 - \frac{kQ}{r^2}\cos 45 = 0\,\tfrac{N}{C}$$

$$E_{net,y} = +\frac{kQ}{r^2}\sin 45 + \frac{kQ}{r^2}\sin 45 - \frac{kQ}{r^2}\sin 45 - \frac{kQ}{r^2}\sin 45 = 0\,\tfrac{N}{C}$$

for $r = \frac{a}{\sqrt{2}}$, where a is the length of the side of the square.

Therefore, $\vec{E}_{net} = 0\,\tfrac{N}{C}$

b.



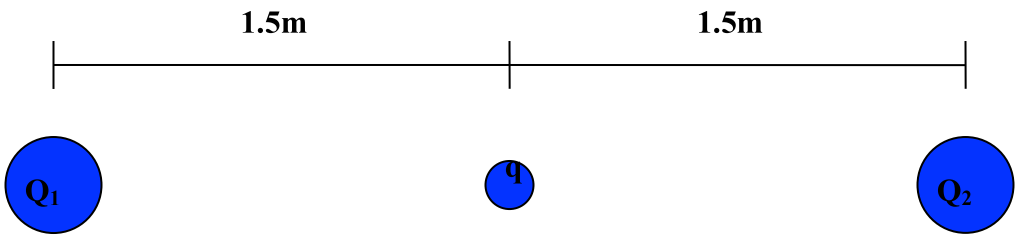
$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$E_{net,x} = -\frac{kQ}{r^2}\cos 45 - \frac{kQ}{r^2}\cos 45 + \frac{kQ}{r^2}\cos 45 + \frac{kQ}{r^2}\cos 45 = 0\,\tfrac{N}{C}$$

$$E_{net,y} = +\frac{kQ}{r^2}\sin 45 + \frac{kQ}{r^2}\sin 45 + \frac{kQ}{r^2}\sin 45 + \frac{kQ}{r^2}\sin 45 = \frac{4\sqrt{2}kQ}{a^2}\,\tfrac{N}{C}$$

for $r = \frac{a}{\sqrt{2}}$, where a is the length of the side of the square.

14.16 We place a positive test charge q at the midpoint between the two point charges Q and calculate the electric field. The magnitude of the electric field is that of a point charge and the directions are found from Coulomb's law applied to the test charge q.



$$\vec{E}_{\text{midpoint}} = \vec{E}_{Q_1} + \vec{E}_{Q_2} = \left(\frac{kQ_1}{r_1^2} - \frac{kQ_2}{r_2^2}\right)\hat{i}$$
$$= \left(9\times10^9 \tfrac{\text{Nm}^2}{\text{C}^2}\right)\left(5\times10^{-6}\,\text{C}\right)^2\left(\frac{1}{(1.5\text{m})^2} - \frac{1}{(1.5\text{m})^2}\right)\hat{i}$$
$$= 0\,\tfrac{\text{N}}{\text{C}}.$$

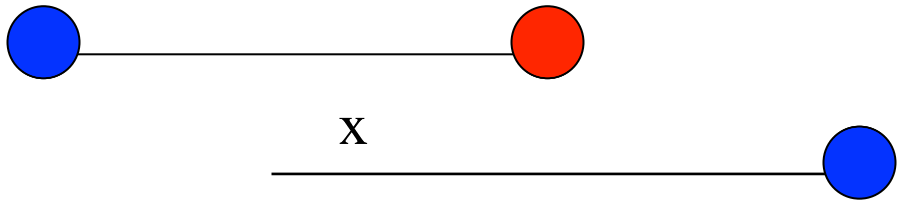
14.17 Assume that the 5μC point charge is at the upper vertex of the triangle and that the two -10μC charges lie along the x-axis. Due to the symmetry of the problem, the horizontal components of the electric field will vanish and we will only have a vertically downward component to the electric field. We have for the net vertically downward electric field, a magnitude of

$$E_{net,y} = \frac{kQ_5}{r_5^2} + \frac{kQ_{10}}{r_{10}^2}\sin\theta + \frac{kQ_{10}}{r_{10}^2}\sin\theta$$

$$E_{nety} = 9\times10^9 \tfrac{Nm^2}{C^2}\left[\frac{5\times10^{-6}C}{(0.33m)^2} + \frac{10\times10^{-6}C}{(0.33m)^2}\sin 30 + \frac{10\times10^{-6}C}{(0.33m)^2}\sin 30\right] = 1.23\times10^6 \tfrac{N}{C}$$

and where r_5 and r_{10} are calculated using the geometry of the problem and the Pythagorean Theorem.

14.18



Defining the distance from the midpoint between the first two charges to the location of the added charge as x, we have

$$E_{net@0.6m} = E_5 + E_{-8} - E_{added\,5} = \frac{k(5\times10^{-6}C)}{(0.6m)^2} + \frac{k(8\times10^{-6}C)}{(0.6m)^2} - \frac{k(5\times10^{-6}C)}{x^2} = 0$$

$$x = 0.37m$$

14.22 In order to calculate the minimum charge, we assume that the electric fields detected by the fish are due to point charges. Therefore,

$$E = 7\times10^{-6}\,\tfrac{\mathrm{N}}{\mathrm{m}} \text{ at } \mathrm{r} = 1\mathrm{m} \text{ and } E = \frac{kQ}{r^2} \rightarrow Q = \frac{Er^2}{k}.$$

Thus, $$Q = \frac{\left(7\times10^{-6}\,\tfrac{\mathrm{N}}{\mathrm{m}}\right)(1\mathrm{m})^2}{9\times10^{9}\,\tfrac{\mathrm{Nm}^2}{\mathrm{C}^2}} = 7.8\times10^{-16}\,\mathrm{C}.$$

Thursday, September 17, 2014

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Questions

- None

Multiple-Choice

- None

Problems

- None