

Physics 111 Homework Solutions Collected on Wednesday 9/24

Friday, September 19, 2014

Chapter 15

Questions

15.5 $\frac{V}{m} = \frac{J}{Cm} = \frac{Nm}{Cm} = \frac{N}{C}$

15.8 They are always perpendicular since if there were a component of the electric field parallel to the equipotential surface then work would be done moving a charge around the equipotential surface. This cannot be the case since the work done in moving a charge around the equipotential surface has to be zero. The only way for this to happen is to have no component of the electric field parallel to the equipotential surface, but rather only perpendicular.

15.11 Suppose that the electric field points along the positive x-axis. If the dipole is oriented so that the negative charge is to the left of the positive charge then the dipole is in a stable equilibrium, for if the charge is displaced slightly off of the axis then the dipole will experience a torque that will return to the dipole to its original configuration. However if the dipole is oriented so that the negative charge is on the right of the positive charge then the dipole is in an unstable equilibrium. Here if the dipole is displaced slightly it will experience a net torque and this torque will rotate the dipole so that the negative charge is on the left of the positive charge.

Multiple-Choice

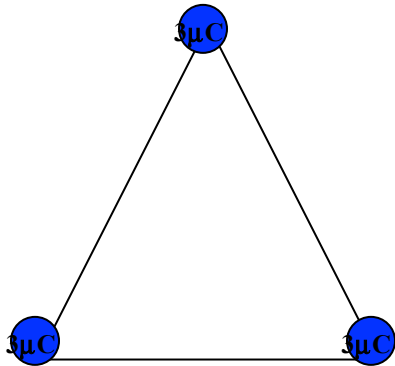
- 15.2 B
- 15.4 A
- 15.5 B
- 15.6 A
- 15.7 D
- 15.8 D

Problems

15.1 The equilateral triangle is given as shown. The potential energy is given by the

equation $PE_{1,2} = \frac{kQ_1Q_2}{r}$. Substituting the values given, we find the

$$PE_{total} = 3 \times PE_{1,2} = 3 \times \frac{(9 \times 10^9 \frac{Nm^2}{C^2})(3 \times 10^{-6}C)^2}{0.05m} = 4.86J.$$



15.2 Bringing each charge Q in one at a time from very far away and assuming that $L = 1.0\text{m}$:

i. the first charge is brought in for free since it feels no forces

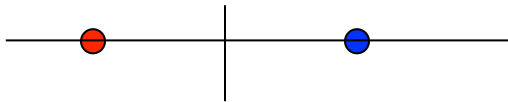
ii. the second charge requires $W = -\frac{kQ^2}{L} = -0.9\text{J}$

iii. the next charge interacts with both of the previous and so the additional work is $W = -\frac{kQ^2}{L} - \frac{kQ^2}{\sqrt{L^2 + L^2}} = -1.54\text{ J}$, where the denominator in the second term is the diagonal distance between charges 1 and 3

iv. the last charge costs an additional $W = -\frac{kQ^2}{L} - \frac{kQ^2}{\sqrt{L^2 + L^2}} - \frac{kQ^2}{L} = -2.44\text{ J}$

The total work to assemble all 4 charges is then -4.88 J

15.3



a. $V = \sum \frac{kQ}{r}$ and since equal and opposite charges are equally distant from the observation point at the origin, the two terms add up to zero – remember these are just + and – numbers, not vectors

b. The electric fields from each charge do not cancel, but both point in the same direction (to the left) since the force from both charges on a positive test charge at the origin is to the left. Adding these up gives $E = 2\frac{kQ}{x^2}$, where $Q =$

$10\ \mu\text{C}$ and $x = 0.1\text{ m}$, so $E = 1.8 \times 10^7\text{ N/C}$, pointing to the left.

c. Since the potential at the origin is $V = 0$, as it is at infinity (very far away), then there is no change in V for the third charge and therefore no net work is required.

Repeating for two positive charges:

- $V = \frac{2kQ}{r} = 1.8 \times 10^6 V$;
- In this case $E = 0$ since the E fields from each charge point in opposite directions and now cancel;
- The net work required is $Q\Delta V = -18 J$ or $18J$ by an external force.

15.4 The relation between electric field and electric potential

- To calculate the potential at any position x, knowing the other position and potential we use the general relation

$$E = -\frac{\Delta V}{\Delta x} \rightarrow 10 \frac{V}{m} = -\frac{(V_f - (-15V))}{10m - 0m} \rightarrow V_f = -115V.$$

- The potential is zero at infinity and also at a distance x given by

$$E = -\frac{\Delta V}{\Delta x} \rightarrow 10 \frac{V}{m} = -\frac{(0 - 15)V}{x_f - 0m} \rightarrow x = -1.5m.$$

15.11 The energy is related to the charge transferred and the potential difference. We

have $W = q\Delta V \rightarrow \Delta V = \frac{W}{q} = \frac{30 \times 10^6 J}{5C} = 6 \times 10^6 V = 6MV.$

15.24 Rutherford Backscattering Spectrometry

- The electrostatic repulsion due to the two positive nuclei did work bringing the alpha particle to rest. The alpha particle loses its kinetic energy in favor of potential energy as it comes to rest at the quoted distance. Thus we have

$$\Delta KE + \Delta PE = (KE_f - KE_i) + (PE_f - PE_i) = 0 \rightarrow KE_i = PE_f$$

$$KE_i = \frac{kQ_{Au}Q_{\alpha}}{r} = \frac{9 \times 10^9 \frac{Nm^2}{C^2} \left(79e \times \frac{1.6 \times 10^{-19} C}{1e} \right) \left(2e \times \frac{1.6 \times 10^{-19} C}{1e} \right)}{9.23 \times 10^{-15} m} = 3.9 \times 10^{-12} J$$

- The work done accelerating the alpha particle changed its kinetic energy. Thus

$$W = \Delta KE = q\Delta V \rightarrow \Delta V = \frac{W}{q} = \frac{3.9 \times 10^{-12} J}{2e \times \frac{1.6 \times 10^{-19} C}{1e}} = 1.2 \times 10^7 V = 12MV$$

- The work done is $3.9 \times 10^{-12} J$.
- The electrostatic repulsion due to the two positive nuclei did work bringing the alpha particle to rest. The alpha particle loses its kinetic energy in favor of potential energy as it comes to rest at the quoted distance. Thus we have

$$\Delta KE + \Delta PE = (KE_f - KE_i) + (PE_f - PE_i) = 0 \rightarrow KE_i = PE_f$$

$$KE_i = 3.3 \times 10^6 eV \times \frac{1.6 \times 10^{-19} C}{1e} = 5.28 \times 10^{-13} J$$

$$\rightarrow 5.28 \times 10^{-13} J = \frac{kQ_{Au}Q_{\alpha}}{r} = \frac{9 \times 10^9 \frac{Nm^2}{C^2} \left(79e \times \frac{1.6 \times 10^{-19} C}{1e} \right) \left(2e \times \frac{1.6 \times 10^{-19} C}{1e} \right)}{r}$$

$$\therefore r = 6.9 \times 10^{-14} m$$

Monday, September 22, 2014

Chapter 15

Questions

- None

Multiple-Choice

- None

Problems

15.12 The capacitance is given by

$$C = \frac{\kappa \epsilon_0 A}{d} = \frac{1 \times 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \times \left(25mi^2 \times \left(\frac{1600m}{1mi} \right)^2 \right)}{1mi \times \frac{1600m}{1mi}} = 3.54 \times 10^{-7} F$$

The charge stored is given by $Q = CV = 3.54 \times 10^{-7} F \times 50 \times 10^6 V = 17.7 C$. Finally, the energy stored is given as

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 3.54 \times 10^{-7} F \times (50 \times 10^6 V)^2 = 4.43 \times 10^8 J = 443 MJ$$

15.17 An air-spaced parallel plate capacitor

a. $C = Q/V = 36 \mu C / 12 V = 3 \mu F$

b. The dielectric constant of Pyrex glass is 4.7 (from Table 16.1), so $C = \kappa C_0 = 14.1 \mu F$

c. Since the capacitance has increased by a factor of 4.7 and V is the same, then Q increases by the same factor to $Q = 169 \mu C$.

15.18 Another air-spaced parallel plate capacitor

a. The energy stored in a parallel plate capacitor is given by $E = \frac{1}{2} CV^2$. Here we need the capacitance which is given by $C = q/V = (0.05 \times 10^{-6} C) / (10V) = 5 \times 10^{-9} F$.

Thus the energy is $E = \frac{1}{2} CV^2 = \frac{1}{2} (5 \times 10^{-9} F) (10V)^2 = 2.5 \times 10^{-7} J$.

b. The electric field is related to the potential difference and the plate separation

through $E = \frac{\Delta V}{\Delta d}$ and thus $E_{before} = \frac{10V}{0.1 \times 10^{-3} m} = 100,000 \frac{V}{m}$

c. The final voltage across the capacitor is the same as the battery, or 10V. The

final energy is given again by $E = \frac{1}{2} CV^2 = \frac{1}{2} (1 \times 10^{-9} \text{ F})(10 \text{ V})^2 = 8.3 \times 10^{-8} \text{ J}$, where the new capacitance is obtained from the old capacitance through

$C_{\text{new}} = \frac{1}{3} C_{\text{old}} = 1.7 \times 10^{-9} \text{ F}$. And the new energy is given by

$E = \frac{Q^2}{2C_{\text{new}}} = \frac{(0.05 \times 10^{-6} \text{ C})^2}{2 \times 1.7 \times 10^{-9} \text{ F}} = 7.5 \times 10^{-7} \text{ J}$. Lastly the potential difference

changes and is related to the old potential difference through $V_{\text{new}} = 3V_{\text{old}} = 30 \text{ V}$

and the new field is $E_{\text{new}} = \frac{\Delta V}{\Delta d} = \frac{30 \text{ V}}{0.3 \times 10^{-3} \text{ m}} = 1 \times 10^5 \frac{\text{V}}{\text{m}} + E_{\text{old}}$

d. The work done in pulling the plates apart is

$$W = \frac{qEd}{2} = \frac{0.05 \times 10^{-6} \text{ C} \times 1 \times 10^5 \frac{\text{V}}{\text{m}} \times 0.2 \times 10^{-3} \text{ m}}{2} = 5 \times 10^{-7} \text{ J}.$$

15.22 The charge that flows due to the sodium ions is

$$Q = \frac{50 \text{ channels}}{1 \mu\text{m}^2} \times 100 \mu\text{m}^2 \times \frac{1000 \text{ Na}^+ \text{ ion}}{1 \text{ channel}} \times \frac{1.6 \times 10^{-19} \text{ C}}{\text{Na}^+ \text{ ion}} = 8.0 \times 10^{-13} \text{ C}.$$
 The

specific capacitance can be related to the capacitance through

$$C = \left(\frac{C}{A} \right) A = \frac{1 \times 10^{-6} \text{ F}}{1 \text{ cm}^2 \times \left(\frac{1 \text{ m}}{100 \text{ cm}} \right)^2} \times 1000 \mu\text{m}^2 \times \left(\frac{1 \text{ m}}{1 \times 10^6 \mu\text{m}} \right)^2 = 1.0 \times 10^{-12} \text{ F}.$$
 Therefore

the voltage change across the membrane due to the ion flow is given by

$$Q = CV \rightarrow \Delta V = \frac{\Delta Q}{C} - 100 \text{ mV} = \frac{8.0 \times 10^{-13} \text{ C}}{1.0 \times 10^{-12} \text{ F}} - 100 \text{ mV} = 0.8 \text{ V} - 100 \text{ mV} = 700 \text{ mV}.$$

15.23 The energy in a charged capacitor is given by

$$E = \frac{1}{2} CV^2 \rightarrow V = \sqrt{\frac{2E}{C}} = \sqrt{\frac{2 \times 300 \text{ J}}{30 \times 10^{-6} \text{ F}}} = 4470 \text{ V}$$