Physics 111 Homework Solutions Collected on Wednesday 11/12

Friday, November 7, 2014

Chapters 24 & 26

Questions

26.1 The atomic number Z is the number of protons in the nucleus. It distinguishes the different types of atoms. N is the number of neutrons in the atom. If we sum the number of neutrons (N) and the number of protons (Z) we get the mass of the nucleus (and the atom if we assume the mass of the electrons are negligible).

α decay is characterized by the following reaction: therefore the mass number of the nucleus decreases by 4 and the atomic number decreases by 2.

β^- decay (for a high speed electron) is characterized by the following reaction: ${}^{A}_{Z}X \rightarrow {}^{0}_{-1}e + {}^{A}_{Z+1}Y$ therefore the mass number of the nucleus is unaffected and the atomic number increases by 1.

β decay (for a high speed positron) is characterized by the following reaction: ${}^{A}_{Z}X \rightarrow {}^{0}_{+1}e + {}^{A}_{Z-1}Y$ therefore the mass number of the nucleus is unaffected and the atomic number decreases by 1.

γ decay is characterized by the following reaction: ${}^{A}_{Z}X^* \rightarrow {}^{0}_{0}\gamma + {}^{A}_{Z}X$ therefore the mass number of the nucleus and the atomic number remain unchanged.

26.4 The three requirements for stability are as follows. 1) The number of neutrons in the nucleus. As more protons are packed into the nucleus those nuclides with significantly more neutrons than protons will tend to produce a stable nucleus. The extra neutrons tend to shield the individual protons from one another. 2) The binding energy of the nucleus. 3) The nuclear energy levels (like those of the electron) should be closed shells. That is the most stable nuclei have equal numbers of protons and neutrons (these numbers are called the magic numbers).

Multiple-Choice

26.1 D
26.2 A
26.4 A

Problems

24.11 A Compton Effect experiment

a. The wavelength and momentum of the incident gamma ray are given as

$$E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E} = \frac{\left(6.63\times10^{-34}\,Js\right)\left(3\times10^{8}\,\frac{m}{s}\right)}{1.6\times10^{6}\,eV \times \frac{1.6\times10^{-19}\,J}{1eV}} = 7.77\times10^{-13}\,m \text{ and}$$

$$p = \frac{h}{\lambda} = \frac{E}{c} = \frac{6.63\times10^{-34}\,Js}{7.77\times10^{-13}\,m} = 8.53\times10^{-22}\,\frac{kgm}{s} \text{ respectively.}$$

b. Using the Compton formula for the wavelength of the scattered photon and the fact that the energy is inversely proportional to the wavelength we can write $\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\phi) \to \frac{\lambda'}{hc} = \frac{\lambda}{hc} + \frac{1}{m_e c^2}(1-\cos\phi) \to \frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{m_e c^2}$.

c. The energy of the scattered gamma ray photon is $\frac{1}{E'} = \frac{1}{1.6MeV} + \frac{(1-\cos 50)}{0.511MeV} = 1.324MeV^{-1} \to E' = 0.755MeV$ where the rest mass of the electron is given as

$$m_e c^2 = (9.11\times10^{-31}kg)(3\times10^8 \tfrac{m}{s})^2 \times \frac{1eV}{1.6\times10^{-19}J} = 0.511MeV.$$

d. The kinetic energy of the recoiling electron is found using conservation of energy where when the incident gamma ray photon interacts with electrons in the sample, the gamma ray photon loses some energy to the electron as it scatters. Thus the kinetic energy of the recoiling electron is

$$E_{incident} = E_{scattered} + KE_{e^-}$$

$$\therefore KE_{e^-} = E_{incident} - E_{scattered} = 1.6MeV - 0.755MeV = 0.845MeV$$

e. The speed of the recoiling electron is given by using the expression for the relativistic kinetic energy. We have

$$KE = 0.845MeV = (\gamma-1)m_e c^2 = (\gamma-1)0.511MeV$$

$$\gamma = 1 + \frac{0.845MeV}{0.511MeV} = 1 + 1.654 = 2.654 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \to v^2 = 0.858c^2 \to v = 0.926c$$

26.2 A neutron star

a) To calculate the mass of the neutron star we need to know the density of nuclear material and the volume of the star. We model the star as being spherical so it has a volume given by $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1\times10^4 m)^3 = 4.19\times10^{12} m^3$. Given that the nuclear density is $2x10^{17}$ kg/m^3 (assuming the nuclear radius is $1.2x10^{-15}$ m) we have the mass M = ρV = $8.4x10^{29}$ kg. Since the mass of the sun is 2x1030kg this gives the mass of the neutron star as 0.41 solar masses. As an aside, this is almost ½ the mass of the sun packed in to a sphere of radius 10 km ~ 6 miles!!

b) Assuming that the mass of a nucleon (either a proton or neutron) is $1.67x10^{-27}$ kg, this gives the number of nucleons as ($8.4x10^{29}$ kg) / ($1.67x10^{-27}$ kg/nucleon) = $5.03x10^{56}$ particles.

26.3 The density is given by $\rho = \frac{M}{V} \to V = 4\pi r^3 = \frac{M}{\rho}$ and therefore the radius is given as

$$r = \sqrt[3]{\frac{M}{4\pi\rho}} = \sqrt[3]{\frac{2\times10^{30}\text{kg}}{4\pi\left(2\times10^{17}\,\frac{\text{kg}}{\text{m}^3}\right)}} = 11{,}675\text{m} = 11.7\text{km}.$$

Monday, November 10, 2014

Chapter 26
Questions

26.7 α particles are low energy so they do not penetrate very far into tissue. They are stopped by the skin producing burns to the exposed patch.

β particles are higher energy and will penetrate farther into tissue than α particles. Beta particles travel about 100 times farther than alpha particles. Whereas alpha particles seldom pass beyond the outer, dead layer of the skin, the free, fast-moving electrons and positrons that constitute beta radiation penetrate for about a quarter of an inch into living matter.

γ rays are very high-energy photons and are not stopped by the skin. They pass through almost undisturbed. However they will ionize atoms as they pass by. Gamma rays and X rays will pass readily through a large organism; they reach the innermost recesses of the body and injure highly sensitive tissues, but they produce only about one twentieth of the damage inflicted on cells by alpha particles.

26.8 To determine the unknown product we use both conservation of mass and charge.
a) For the following reaction $^{60}_{27}Co \rightarrow ^{60}_{28}Ni + ^{A'}_{Z'}X$, conservation of charge gives me 27 = 28 + Z' which gives Z' = -1. Conservation of mass gives me 60 = 60 + A' which gives A' = 0. Thus the unknown particle is a beta particle, $^{A'}_{Z'}X = ^{0}_{-1}e$.
b) For the following reaction $^{A'}_{Z'}X \rightarrow ^{234}_{91}Pa + ^{0}_{-1}e$, conservation of charge gives me Z' = 91-1' which gives Z' = 90. Conservation of mass gives me A' = 234 + 0 which gives A' = 234. Thus the unknown particle is a thorium nucleus, $^{A'}_{Z'}X = ^{234}_{90}Th$.
c) For the following reaction $^{A'}_{Z'}X \rightarrow ^{234}_{90}Th + ^{4}_{2}He$, conservation of charge gives me Z' = 90 + 2 which gives Z' = 92. Conservation of mass gives me A' = 230 + 4 which gives *A' = 234*. Thus the unknown particle is a uranium nucleus, $^{A'}_{Z'}X = ^{234}_{92}U$.

26.16 The electron-positron annihilation occurs when the positron encounters an electron and both are assumed to be at rest. To conserve momentum, the two photons produced are done so at exactly 180°. Thus photons from the annihilation are detected coincidently 180° apart. If the two photons are not detected coincidentally 180° apart, they are from two different annihilation events.

Multiple-Choice

24.2 B
24.3 C
24.4 D
24.6 C

Problems

26.4 The nuclear binding energy is given through:

$$NBE = Zm_p c^2 + Nm_N c^2 - m_{atom}c^2; \text{where } m_p = 1.00727u \text{ and } m_n = 1.00867u.$$

For radium-226:

$$NBE = [88(1.00727u) + 138(1.00867u) - 225.97709u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1731.8\text{MeV}.$$

The $\frac{NBE}{nucleon} = \frac{1731.8MeV}{226} = 7.66\frac{MeV}{nucleon}$.

For radium-228:

$$NBE = [88(1.00727u)+140(1.00867u)-227.98275u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1742.7\text{MeV}.$$

The $\frac{NBE}{nucleon} = \frac{1742.7MeV}{228} = 7.64\frac{MeV}{nucleon}$.

For thorium-232:

$$NBE = [90(1.00727u)+142(1.00867u)-231.98864u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1766.9\text{MeV}.$$

The $\frac{NBE}{nucleon} = \frac{1766.9MeV}{232} = 7.62\frac{MeV}{nucleon}$.

26.6 For the β-decay reaction of ^{24}Na,

$$Q = \left(M_{^{24}_{11}Na} - M_{^{24}_{12}Mg} - M_{^{0}_{-1}e}\right)c^2$$

$$\therefore Q = \left(23.98492 - 23.97845 - 5.49\times10^{-4}\right)uc^2 \times \frac{931.5MeV}{uc^2} = 5.52MeV.$$

26.7 For alpha decay: $Q = \left(M_{parent} - M_{daughter} - M_{He}\right)c^2$. If the parent is at rest when it decays, then from conservation of momentum, the daughter gets a recoil velocity in the direction opposite direction to the velocity of the alpha particle. Conservation of momentum gives: $0 = -m_{daughter}v_{daughter} + m_\alpha v_\alpha \rightarrow v_{daughter} = \frac{m_\alpha}{m_{daughter}}v_\alpha$. Next we apply conservation of energy and we find:

$m_{parent}c^2 = \frac{1}{2}m_{daughter}v_{daughter}^2 + \frac{1}{2}m_{He}v_{He}^2 + m_{daughter}c^2 + m_{He}c^2$. Then we can replace the velocity of the recoiling daughter atom in terms of the velocity of the alpha particle, which can be measured. We have

$m_{parent}c^2 = \frac{1}{2}m_{daughter}\left(\frac{m_\alpha}{m_{daughter}}v_\alpha\right)^2 + \frac{1}{2}m_{He}v_{He}^2 + m_{daughter}c^2 + m_{He}c^2$. Bringing all of the rest of the energy terms to one side and combining the terms involving the velocity of the alpha particle we find that

$\left(m_{parent}c^2 - m_{daughter}c^2 - m_{He}c^2\right) = Q = \left[\frac{1}{2}\frac{m_\alpha^2}{m_{daughter}} + \frac{1}{2}m_\alpha\right]v_\alpha$. Factoring out the mass of the alpha particle on the left hand side we have $\text{Q} = \frac{1}{2}m_\alpha v_\alpha^2\left[1 + \frac{m_\alpha}{m_{daughter}}\right]$ which is the desired result.

From example 27.2, Q = 4.28 MeV, $m_\alpha = 4.0015u$, $m_{daughter} = m_{thorium} = 233.99409u$, and we find the velocity of the alpha particle to be 1.42×10^7 m/s.

26.16 Uranium Decay

a. $^{238}_{92}U \rightarrow ^{4}_{2}He + ^{234}_{90}Th$
b. The daughter nucleus is thorium.
c. The kinetic energy of the emitted alpha particle is given by (ignoring the recoil of the thorium nucleus):

$$KE = [M_U - M_\alpha - M_{Th}]c^2$$

$$KE = \left\{[238.000187u - 4.00150u - 233.99409u] \times \frac{931.5 MeV/c^2}{1u}\right\}c^2 = 4.282 MeV$$

d. To determine the speed of the emitted alpha particle we use the relativistic form of the kinetic energy since we don't know if the alpha particle is relativistic or not. Thus

$$KE = 4.282 MeV = (\gamma - 1)m_\alpha c^2 = (\gamma - 1)\left[4.00150u \times \frac{931.5 MeV/c^2}{1u}\right]c^2 \rightarrow \gamma = 1.0015$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow v = \sqrt{1 - \frac{1}{\gamma^2}}c = \sqrt{1 - \frac{1}{(1.0015)^2}}c = 0.055c$$

e. Based on part d, the alpha particle is almost at the relativistic cutoff. However, we have actually ignored the recoil of the thorium nucleus, so if we were to include the recoil of the thorium atom, the alpha particle is probably not actually relativistic.