Physics 120

Spring 2007

Exam #3

June 1, 2007

Name____________________

Part	
Multiple Choice	/ 10
Problem #1	/ 30
Problem #2	/ 30
Problem #3	/ 30
Total	/ 100

In keeping with the Union College policy on academic honesty, it is assumed that you will neither accept nor provide unauthorized assistance in the completion of this work.

Multiple choice questions are worth 2 points each. Please circle the best answer to each question.

1. What is the magnitude in $\mathrm{rad}/\mathrm{s}^2$ of the angular acceleration of the second hand of a clock?

   a. 0 b. $\frac{\pi}{1200}$ c. $\frac{\pi}{1800}$ d. $\frac{\pi}{2400}$

2. Stars originate as large bodies of slowly rotating gas. Because of gravity, these clumps of gas slowly decrease in size. The angular velocity of a star increases as it shrinks because of
   a. conservation of angular momentum.
   b. conservation of linear momentum.
   c. conservation of energy.
   d. the law of universal gravitation.

3. A person facing a road sees a car come down the road headed to the person's left. When the driver of the car steps on the accelerator, the front of the car moves upwards because
   a. the road exerts a backwards force on the tires.
   b. of a counterclockwise torque on the car.
   c. of a clockwise torque on the car.
   d. a clockwise torque and a counterclockwise torque cancel each other out.

4. A one-kilogram mass suspended from a heavy spring is replaced by a nine-kilogram mass. By what factor is the frequency of vibration changed?

   a. 1/9 b. 1/3 c. 3.0 d. 9.0

5. A simple pendulum is moving with simple harmonic motion and is at its maximum displacement from equilibrium. Which of the following is also at its maximum?

   a. Speed b. Acceleration c. Period d. Kinetic energy

Part II: Free Response Problems

Please show all work in order to receive partial credit. If your solutions are illegible no credit will be given. Please use the back of the page if necessary, but number the problem you are working on. The numbers in parentheses following the question correspond to the point values for each part.

1. A tall, cylinder-shaped chimney falls over when its base is ruptured. Suppose that we can treat the chimney as a thin, uniform rod of height $H = 30.8m$ and letting θ be the angle that the chimney makes with respect to the vertical.

   a. Derive an expression for the angular speed of the chimney about the pivot point on the ground. Evaluate your result at θ = 30°. (Hints: The moment of inertia for a thin rod, pivoted about one end is $I_{rod} = \frac{1}{3}MR^2$ and you can consider all of the mass of the chimney to be located at its geometric center. Also, you may want to use energy methods to solve this problem.) (10 points)

$$E_i = E_f \rightarrow mgh_{com} = mgh' + \tfrac{1}{2}I\omega^2$$

$$mg\frac{H}{2} = mg\frac{H}{2}\cos\theta + \tfrac{1}{2}\left(\tfrac{1}{3}mH^2\right)\omega^2$$

$$\therefore \omega = \sqrt{\frac{3g(1-\cos\theta)}{H}} = \sqrt{\frac{3\times 9.8\,\tfrac{m}{s^2}\times(1-\cos 30)}{30.8m}} = 0.36\,\tfrac{rad}{s}$$

   b. Derive an expression and then evaluate it (at θ = 30°) for the tangential acceleration of the chimney's top. (10 points)

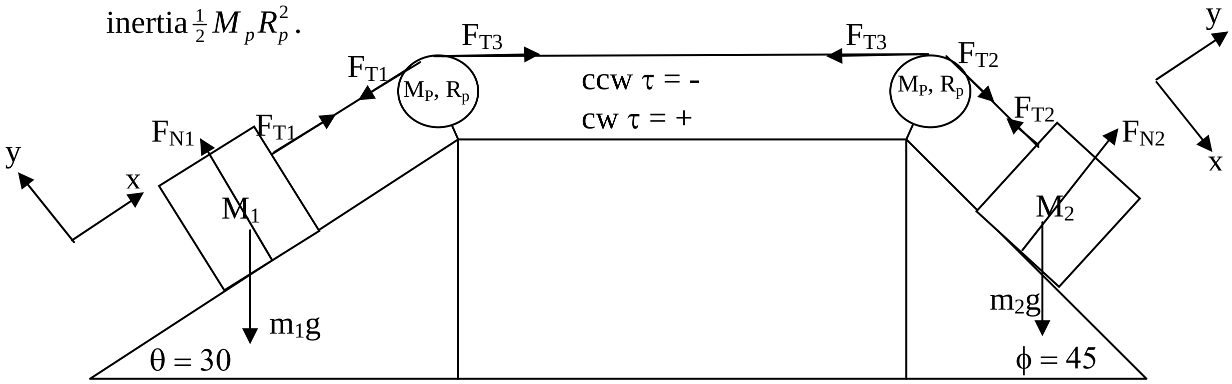
$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\sqrt{\frac{3g(1-\cos\theta)}{H}}\right) = \left[\tfrac{1}{2}\left(\frac{1}{\sqrt{\frac{3g(1-\cos\theta)}{H}}}\right)\frac{3g}{H}\sin\theta\right]\frac{d\theta}{dt} = \left[\tfrac{1}{2}\left(\frac{1}{\sqrt{\frac{3g(1-\cos\theta)}{H}}}\right)\frac{3g}{H}\sin\theta\right]\omega$$

$$\alpha = \frac{3g}{2H}\sin\theta = \frac{3\times 9.8\,\tfrac{m}{s^2}}{2\times 30.8m}\sin 30 = 0.24\,\tfrac{rad}{s^2}$$

   c. At what angle, θ, does the tangential acceleration equal g? (10 points)

$$a_t = R\alpha \rightarrow g = H\times\frac{3g}{2H}\sin\theta = \frac{3g}{2}\sin\theta \rightarrow \sin\theta = \frac{2}{3} \rightarrow \theta = 41.2^o$$

2. Consider the arrangement of masses shown below. M_1 has a mass of 2kg, is on a ramp inclined at 30^o above the horizontal, while mass M_2 has a mass of 5kg and is on a ramp inclined at 45^o. The blocks are connected by pullies and all of the surfaces that the blocks ride along are frictionless. The pullies are identical with each having a mass M_p of 1kg and a radius of R_p of 3cm so that each has a moment of inertia $\frac{1}{2}M_pR_p^2$.



a. From carefully labeled free body diagrams, what is the acceleration of the blocks? (12 points)

$$m_1: \quad \sum F_x: \quad F_{T1} - m_1 g \sin\theta = m_1 a \rightarrow F_{T1} = m_1 a + m_1 g \sin\theta$$

$$m_2: \quad \sum F_y: \quad -F_{T2} + m_2 g \sin\phi = m_2 a \rightarrow F_{T2} = m_2 g \sin\phi - m_2 a$$

$$Left\ Pully: \quad \sum \tau: \quad -F_{T1}R_p + F_{T3}R_p = I\alpha \rightarrow -F_{T1} + F_{T3} = I\frac{a}{R_p^2}$$

$$Right\ Pully: \quad \sum \tau: \quad -F_{T3}R_p + F_{T2}R_p = I\alpha \rightarrow -F_{T3} + F_{T2} = I\frac{a}{R_p^2}$$

$$\therefore F_{T3} = F_{T2} - I\frac{a}{R_p^2} = I\frac{a}{R_p^2} + F_{T1} \Rightarrow F_{T2} - F_{T1} = 2I\frac{a}{R_p^2} = 2\left(\tfrac{1}{2}M_pR_p^2\right)\frac{a}{R_p^2} = M_p a$$

$$Thus, \quad m_2 g \sin\phi - m_2 a - (m_1 a + m_1 g \sin\theta) = M_p a$$

$$a = \frac{m_2 g \sin\phi - m_1 g \sin\theta}{m_1 + m_2 + M_p} = \frac{(5kg \sin 45 - 2kg \sin 30) \times 9.8\frac{m}{s^2}}{(2+5+1)kg} = 3.11\tfrac{m}{s^2}$$

b. What are the tensions in the cords that connect masses M_1 and M_2 to their respective pullies? (Hint: The tension force between the two pullies along the horizontal portion of the system is the same.) (12 points)

$$F_{T1} = m_1(a + g\sin\theta) = 2kg \times \left(3.11\tfrac{m}{s^2} + 9.8\tfrac{m}{s^2}\sin 30\right) = 16N$$

$$F_{T2} = m_2(g\sin\phi - a) = 5kg \times \left(9.8\tfrac{m}{s^2}\sin 45 - 3.11\tfrac{m}{s^2}\right) = 19.1N$$

c. What is the angular acceleration of the one of the pullies? (6 points)

$$a = R_p\alpha \rightarrow \alpha = \frac{a}{R_p} = \frac{3.11\frac{m}{s^2}}{0.03m} = 103.7\tfrac{rad}{s^2} \text{ clockwise}$$

3. In Edgar Allen Poe's masterpiece of terror, "*The Pit and the Pendulum,*" a prisoner who is strapped flat on a floor spies a seemingly motionless pendulum 12m above him. Then, to his horror, he realizes that the pendulum consists of a "*crescent of glittering steel, … the under edge as keen as that of a razor*" and that it is gradually descending. As hours go by, the pendulum's motion becomes mesmerizing, with the left-right sweep and the speed at the lowest point of each swing both increasing. The pendulum's intent becomes clear: to seep directly across the prisoner's heart. "*Down – steadily it crept. I took a frenzied pleasure in contrasting its downward with its lateral velocity. To the right to the left – far and wide – with the shriek of a damned spirit!...Down – certainly, relentlessly down!*"

   Assuming that the pendulum is ideal, and consists of a particle of mass m on the end of a massless cord of length r, and that the pendulum descends by small increments in r, take the initial cord length r_o to be 0.8m and the initial maximum angular speed $\omega_{o,\max}$ (when the pendulum passes through $\theta = 0$) to be 1.30 rad/s. Assume also that the pendulum descends only in small steps and only as it passes through $\theta = 0$, which means that the angular momentum of the pulley does not change during each step of the descent.

   a. In terms of the length of the cord, r, what are the maximum angular speed and kinetic energy of the pendulum. (Hints: Use conservation of angular momentum to relate the initial radius and angular velocity to the final angular velocity. Also, the moment of inertia for a point mass is MR^2.) (8)

   $$L_{\max} = L_i \rightarrow I_{\max}\omega_{\max} = I_i\omega_{o,\max} \rightarrow mr^2\omega_{\max} = mr_0^2\omega_{o,\max}$$

   $$\therefore \omega_{\max} = \left(\frac{r_0}{r}\right)^2 \omega_{o,\max} = \frac{0.832}{r^2}$$

   $$KE_{\max} = \tfrac{1}{2} I_{\max}\omega_{\max}^2 = \tfrac{1}{2}\left(mr^2\right)\left(\frac{r_0}{r}\right)^4 \omega_{o,\max}^2 = \frac{mr_0^4\omega_{o,\max}^2}{2r^2}$$

   b. As r increases, does the kinetic energy change? Justify your answer with a statement or two. (7)

   From above, we see that the kinetic energy depends inversely on r^2, so as r *increases* toward the victim, the kinetic energy *decreases*.

   c. What is the maximum angle through attained by the pendulum during the upward swing? (Hint: Assume that for any given value of r, the total mechanical energy is conserved.) (8)

   $$KE_{\max} = PE_{\max} \rightarrow \frac{mr_0^4\omega_{o,\max}^2}{2r^2} = mgr\left(1 - \cos\theta_{\max}\right)$$

   $$\theta_{\max} = \cos^{-1}\left(1 - \frac{r_0^4\omega_{o,\max}^2}{2gr^3}\right)$$

   d. What is the period of the pendulum? Does it increase or decrease as the length of the cord increases close to the victim? (7)

   $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{r}{g}}$, which increases with increasing r.

Useful formulas:

Motion in the x, y or z-directions

$r_f = r_0 + v_{0r}t + \frac{1}{2}a_r t^2$

$v_{fr} = v_{ir} + a_r t$

$v_{fr}^{\ 2} = v_{ir}^{\ 2} + 2a_r \Delta r$

Uniform Circular Motion

$a_r = \frac{v^2}{r}$

$F_r = ma_r = m\frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation: $ax^2 + bx + c = 0$,

whose solutions are given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$\text{magnitude of a vector} = \sqrt{v_x^{\ 2} + v_y^2}$

$\text{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\,{}^{m}\!/_{s^2}$ $\quad G = 6.67 \times 10^{-11}\,{}^{Nm^2}\!/_{kg^2}$

$N_A = 6.02 \times 10^{23}\,{}^{atoms}\!/_{mole}$ $\quad k_B = 1.38 \times 10^{-23}\,{}^{J}\!/_{K}$

$\sigma = 5.67 \times 10^{-8}\,{}^{W}\!/_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}\!/_{s}$ $\quad P_{air} = 1.013 \times 10^5\, N/m^2$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\,\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = \int \vec{F} \cdot d\vec{r} = \int F dr \cos\theta = \Delta E_T$

$W_R = \int \tau d\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$P = \frac{\Delta E}{\Delta t}$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$

$a_r = r\omega^2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\omega\cos\left(\frac{2\pi t}{T}\right) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\omega^2\sin\left(\frac{2\pi t}{T}\right) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

Other Misc. Formulas

$$\vec{r}_{com} = \frac{\sum_{i=1}^{N} m_i\vec{r}_i}{\sum_{i=1}^{N} m_i} \Rightarrow \lim_{N\to\infty}\left(\frac{\sum_{i=1}^{N} m_i\vec{r}_i}{\sum_{i=1}^{N} m_i}\right) = \int \vec{r}dm = \vec{r}_{com}$$

$$v_{1f} = \frac{2m_2}{m_1 + m_2}v_{2i} + \frac{m_1 - m_2}{m_1 + m_2}v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i}$$