

Physics 120

Exam #3

May 27, 2011

Name _____

Multiple Choice	/16
Problem #1	/28
Problem #2	/28
Problem #3	/28
Total	/100

Part II: Free Response Problems: *The three problems below are worth 84 points total and each subpart is worth 7 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.*

1. Two identical springs of stiffness $k = 100 \text{ N/m}$ lie horizontally on a frictionless track and are attached to either side of a 2kg mass. The mass is initially pulled to the right and the right spring is compressed by an amount $A = 0.25m$ from the right spring's equilibrium length while the left spring is stretched by the same amount $A = 0.25m$ from that spring's equilibrium length. The mass is released from rest at a time $t_i = 0\text{s}$.

- a. Using energy methods, what is the expression for and the speed of the mass as it passes through the equilibrium point?

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(\frac{1}{2}kx_{f,L}^2 - \frac{1}{2}kx_{i,L}^2\right) + \left(\frac{1}{2}kx_{f,R}^2 - \frac{1}{2}kx_{i,R}^2\right)$$

$$0 = \frac{1}{2}mv_f^2 - \frac{1}{2}kx_{i,L}^2 - \frac{1}{2}kx_{i,R}^2 = \frac{1}{2}mv_f^2 - \frac{1}{2}k(A)^2 - \frac{1}{2}k(-A)^2$$

$$v_f = \sqrt{\frac{2kA^2}{m}} = \sqrt{\frac{k_{\text{eff}}}{m}}A = \omega A = \sqrt{\frac{2 \times 100 \frac{\text{N}}{\text{m}}}{2\text{kg}}} \times 0.25m = 2.5 \frac{\text{m}}{\text{s}}$$

- b. Using energy methods, what is the expression for and the speed of the mass when it is at a location $x_f = \frac{A}{2}$ moving to the left?

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(\frac{1}{2}kx_{f,L}^2 - \frac{1}{2}kx_{i,L}^2\right) + \left(\frac{1}{2}kx_{f,R}^2 - \frac{1}{2}kx_{i,R}^2\right)$$

$$0 = \frac{1}{2}mv_f^2 + \left(\frac{1}{2}k\left(\frac{A}{2}\right)^2 - \frac{1}{2}k(A)^2\right) + \left(\frac{1}{2}k\left(\frac{-A}{2}\right)^2 - \frac{1}{2}k(-A)^2\right) = \frac{1}{2}mv_f^2 - \frac{3kA^2}{4}$$

$$v_f = \sqrt{\frac{6kA^2}{4m}} = \sqrt{\frac{6 \times 100 \frac{\text{N}}{\text{m}}}{4 \times 2\text{kg}}} \times 0.25m = 2.17 \frac{\text{m}}{\text{s}}$$

- c. What is the initial acceleration of the mass?

$$\vec{F}_{\text{net}} = \langle -kA - kA, 0, 0 \rangle = \frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = m\vec{a}$$

$$\vec{a} = \frac{\langle -2kA, 0, 0 \rangle}{m} = \left\langle \frac{-2 \times 100 \frac{\text{N}}{\text{m}} \times 0.25m}{2\text{kg}}, 0, 0 \right\rangle = \langle -25, 0, 0 \rangle \frac{\text{m}}{\text{s}^2}$$

- d. What is the period of the masses oscillation?

$$T = 2\pi \sqrt{\frac{m}{k_{\text{eff}}}} = 2\pi \sqrt{\frac{m}{2k}} = 2\pi \sqrt{\frac{2\text{kg}}{2 \times 100 \frac{\text{N}}{\text{m}}}} = 0.1\text{s}$$

2. A karate expert strikes downward with her hand of mass $m_{fist} = 0.7kg$ breaking a $0.14kg$ board. The stiffness of the board is $4.1 \times 10^4 N/m$ and the board breaks at a deflection $y_f = 16mm$ from a starting position of $y_i = 0mm$. You should note that this is not exactly the same situation as you encountered in lab. Here we are going to model the collision between your hand and the board as an inelastic collision and immediately after the collision your hand and the board ($v_{hand + board}$) will be moving with the same speed. The board will do work on your hand bringing it to rest after the board has been deflected by the distance of $16mm$.

- a. Apply conservation of energy to the situation of your hand striking the board and determine how fast are *the board and your hand* ($v_{hand + board}$) are moving just after you strike the board.

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\frac{1}{2} m_{h+b} v_{h+b,f}^2 - \frac{1}{2} m_{h+b} v_{h+b,i}^2 \right) + (m_{h+b} g y_f - m_{h+b} g y_i) + \left(\frac{1}{2} k y_f^2 - \frac{1}{2} k y_i^2 \right)$$

$$0 = -\frac{1}{2} m_{h+b} v_{h+b,i}^2 + m_{h+b} g y_f + \frac{1}{2} k y_f^2 = -\frac{1}{2} m_{h+b} v_{h+b,i}^2 - m_{h+b} g d + \frac{1}{2} k (-d)^2$$

$$v_{h+b} = \sqrt{\frac{2 \left(-m_{h+b} g d + \frac{1}{2} k (-d)^2 \right)}{m_{h+b}}} = \sqrt{\frac{2 \left((-0.84kg \times 9.8 \frac{m}{s^2} \times 16 \times 10^{-3} m) + \left(\frac{1}{2} \times 4.1 \times 10^4 \frac{N}{m} (16 \times 10^{-3} m)^2 \right) \right)}{0.84kg}}$$

$$v_{h+b} = 3.55 \frac{m}{s}$$

- b. What is the minimum speed that your hand (v_{hand}) must be moving before it collides with the board so that you can break this karate board?

$$\frac{d\vec{p}}{dt} = \vec{F}_{net,ext} = 0 \rightarrow \Delta \vec{p} = 0 \rightarrow \vec{p}_i = \vec{p}_f$$

$$\langle 0, -m_h v_h, 0 \rangle = \langle 0, -m_{h+b} v_{h+b}, 0 \rangle$$

$$v_h = \frac{m_{h+b} v_{h+b}}{m_h} = \frac{0.84kg \times 3.55 \frac{m}{s}}{0.7kg} = 4.26 \frac{m}{s}$$

- c. If instead of a breaking board, you wanted to break a cement block of mass 3.2kg . What minimum speed would your hand, v_{hand} , need to be moving to break a cement block? Take $k_{block} = 2.6 \times 10^6 \text{N/m}$ and suppose that a deflection of 1mm will break the block.

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\frac{1}{2} m_{h+b} v_{h+b,f}^2 - \frac{1}{2} m_{h+b} v_{h+b,i}^2 \right) + (m_{h+b} g y_f - m_{h+b} g y_i) + \left(\frac{1}{2} k y_f^2 - \frac{1}{2} k y_i^2 \right)$$

$$0 = -\frac{1}{2} m_{h+b} v_{h+b,i}^2 + m_{h+b} g y_f + \frac{1}{2} k y_f^2 = -\frac{1}{2} m_{h+b} v_{h+b,i}^2 - m_{h+b} g d + \frac{1}{2} k (-d)^2$$

$$v_{h+b} = \sqrt{\frac{2 \left(-m_{h+b} g d + \frac{1}{2} k (-d)^2 \right)}{m_{h+b}}} = \sqrt{\frac{2 \left((-3.9\text{kg} \times 9.8 \frac{\text{m}}{\text{s}^2} \times 1 \times 10^{-3} \text{m}) + \left(\frac{1}{2} \times 2.6 \times 10^6 \frac{\text{N}}{\text{m}} (1 \times 10^{-3} \text{m})^2 \right) \right)}{3.9\text{kg}}}$$

$$v_{h+b} = 0.80 \frac{\text{m}}{\text{s}}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net,ext} = 0 \rightarrow \Delta\vec{p} = 0 \rightarrow \vec{p}_i = \vec{p}_f$$

$$\langle 0, -m_h v_h, 0 \rangle = \langle 0, -m_{h+b} v_{h+b}, 0 \rangle$$

$$v_h = \frac{m_{h+b} v_{h+b}}{m_h} = \frac{3.9\text{kg} \times 0.80 \frac{\text{m}}{\text{s}}}{0.7\text{kg}} = 4.48 \frac{\text{m}}{\text{s}}$$

Which coincidentally is not much faster than you'd need to break the board.

- d. How much work was done on your hand by the board in performing this “feat of strength?” by the block?

$$W_{board} = \Delta KE_{hand} = \frac{1}{2} m_{h+bo} v_{f,after}^2 - \frac{1}{2} m_{h+bo} v_{i,after}^2 = -\frac{1}{2} m_{h+bo} v_{i,after}^2 = -\frac{1}{2} (0.84\text{kg}) \left(3.55 \frac{\text{m}}{\text{s}} \right)^2 = -5.29\text{J}$$

$$W_{board} = \Delta KE_{hand} = \frac{1}{2} m_{h+bl} v_{f,after}^2 - \frac{1}{2} m_{h+bl} v_{i,after}^2 = -\frac{1}{2} m_{h+bl} v_{i,after}^2 = -\frac{1}{2} (3.9\text{kg}) \left(0.8 \frac{\text{m}}{\text{s}} \right)^2 = -1.25\text{J}$$

3. You are driving your car ($m_1 = 1000\text{kg}$) down a straight road in the middle of winter and the road you are driving on is icy (with a coefficient of kinetic friction between your tires and the ice of $\mu_k = 0.4$.) In front of you is another car (of mass $m_2 = 1400\text{kg}$). The traffic light ahead on the road that both your car and the one in front of you are traveling on turns from green to red and both of you apply the brakes in your respective cars. You both slide along the ice and the car in front of you stops at the light, but you do not quite make it and unfortunately hit the stopped car in front of you.

- a. After the collision at the light you stop a distance of 7.3m from the point of impact with the car in front of you. How fast were you going after the collision?

$$W = -F_{fr}d = \Delta KE = \frac{1}{2}mv_{f,after}^2 - \frac{1}{2}mv_{i,after}^2$$

$$v_{i,after} = \sqrt{\frac{2F_{fr}d}{m}} = \sqrt{\frac{2\mu_k mgd}{m}} = \sqrt{2\mu_k gd} = \sqrt{2 \times 0.4 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 7.3\text{m}}$$

$$v_{i,after} = 7.6 \frac{\text{m}}{\text{s}}$$

- b. If the car you hit ended up at a distance of 10.4m ahead of the point of impact, with what initial speed did you hit the car in front of you with? Compare this speed with the posted speed on the road ($30\text{mph} \sim 13\text{m/s}$.) Would a cop have issued you a ticket? Justify your answer with calculations.

$$\frac{d\vec{p}}{dt} = \vec{F}_{net,ext} = 0 \rightarrow \Delta\vec{p} = 0 \rightarrow \vec{p}_i = \vec{p}_f$$

$$\langle m_y v_{y,before}, 0, 0 \rangle = \langle m_y v_{y,after} + m_o v_{o,after}, 0, 0 \rangle$$

$$v_{y,before} = \frac{m_y v_{y,after} + m_o v_{o,after}}{m_y} = \frac{1100\text{kg} \times 7.6 \frac{\text{m}}{\text{s}} + 1400\text{kg} \times 9.0 \frac{\text{m}}{\text{s}}}{1100\text{kg}} = 19.1 \frac{\text{m}}{\text{s}}, \text{where}$$

$$v_{o,after} = \sqrt{\frac{2F_{fr}d}{m}} = \sqrt{\frac{2\mu_k mgd}{m}} = \sqrt{2\mu_k gd} = \sqrt{2 \times 0.4 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 10.4\text{m}}$$

$$v_{o,after} = 9.0 \frac{\text{m}}{\text{s}}$$

Not only would you have probably got a ticket for speeding ($19.1\text{m/s} \sim 43\text{mph}$) but you would also probably have been issued one for failure to drive according to the road conditions, which means that you should have probably been doing less than 30mph on the ice anyway.

- c. How much work did friction do in bringing you to rest after you hit the car in front of you?

$$W_{fr} = -F_{fr}d = -\mu_k mgd = -0.4 \times 1100kg \times 9.8 \frac{m}{s^2} \times 7.3m = -3.2 \times 10^4 J$$

$$W_{fr} = \Delta KE = \frac{1}{2} m_y v_{y,f,after}^2 - \frac{1}{2} m_y v_{y,i,after}^2 = -\frac{1}{2} \times 1100kg \times \left(7.6 \frac{m}{s}\right)^2 = -3.2 \times 10^4 J$$

- d. If the collision lasted for a time $\Delta t = 0.8s$, what was the impulse force on your car due to the collision with the car in front of you?

$$\frac{d\vec{p}_y}{dt} = \frac{\langle m_y v_{y,i,after} - m_y v_{y,before}, 0, 0 \rangle}{dt} = \vec{F}_{net,ext,y}$$

$$\vec{F}_{net,ext,y} = \frac{1100kg \times \langle 7.6 \frac{m}{s} - 19.1 \frac{m}{s}, 0, 0 \rangle}{0.8s} = \langle -1.6 \times 10^4, 0, 0 \rangle N$$

Useful formulas:

$$\vec{p} = \gamma m \vec{v}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

$$\vec{F}_g = m\vec{g}$$

$$\vec{F}_{gravity} = -\frac{GM_1M_2}{r_{12}^2} \hat{r}_{12} \quad T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k_{eff}}}$$

$$\vec{F}_{spring} = -k\vec{s}; \quad \vec{s} = (L - L_o)\hat{s}$$

$$W = \int \vec{F} \cdot d\vec{r} = \Delta KE = -\Delta U$$

$$U_g = -\frac{GM_1M_2}{r}$$

$$U_g = mgy$$

$$U_s = \frac{1}{2}ks^2$$

$$KE = \frac{1}{2}mv^2$$

$$KE = (\gamma - 1)mc^2$$

$$E_{rest} = mc^2$$

Momentum Principle:

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

Position-update:

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \vec{r}_i + \frac{\vec{p}}{m\sqrt{1 + \frac{p^2}{m^2c^2}}}\Delta t$$

Energy Principle:

$$\Delta E = W = \Delta U_g + \Delta U_s + \Delta KE$$

$$W = \int \vec{F} \cdot d\vec{r}$$

Vectors

magnitude of a vector: $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

writing a vector: $\vec{a} = \langle a_x, a_y, a_z \rangle = |\vec{a}|\hat{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

dot product: $\vec{A} \cdot \vec{B} = AB \cos \theta$

magnitude of the cross product: $|\vec{A} \times \vec{B}| = AB \sin \theta$

Geometry

Circles: $C = 2\pi r = \pi D \quad A = \pi r^2$

Triangles: $A = \frac{1}{2}bh$

Spheres: $A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3$

Constants

$$g = 9.8 \frac{m}{s^2}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

$$1e = 1.6 \times 10^{-19} C$$

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{C^2}{Nm^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{Nm^2}{C^2}$$

$$1eV = 1.6 \times 10^{-19} J$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$$

$$c = 3 \times 10^8 \frac{m}{s}$$

$$h = 6.63 \times 10^{-34} Js$$

$$m_e = 9.11 \times 10^{-31} kg = \frac{0.511 MeV}{c^2}$$

$$m_p = 1.67 \times 10^{-27} kg = \frac{937.1 MeV}{c^2}$$

$$m_n = 1.69 \times 10^{-27} kg = \frac{948.3 MeV}{c^2}$$

$$1amu = 1.66 \times 10^{-27} kg = \frac{931.5 MeV}{c^2}$$

$$N_A = 6.02 \times 10^{23}$$

$$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$