Name_____

Physics 120 Quiz #2, April 11, 2014

*Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.*

*I affirm that I have carried out my academic endeavors with full academic honesty.*

_____

A field goal kicker in football is attempting to kick a $50\,yd\,(\sim 45.7m)$ field goal. The kicker kicks the ball always at a $53^o$ angle but is capable of varying the initial speed of the kicked football. The bottom bar of the goal post uprights is at a location $\langle 45.7, 3.1, 0 \rangle m$ assumes that the football is kicked from the ground at a location $\langle 0,0,0 \rangle m$. (For a $50\,yd$ attempt, the football would actually be placed on the $40\,yd$ line, the end zone is $10\,yd$ deep. This information is not needed for the actual problem.)

1. What velocity would the kicker have to kick the football with so that it just barely clears the bottom bar of the goal post uprights?

The initial velocity vector is given by:
$$\vec{v}_i = \langle v_{ix}, v_{ix}, v_{ix} \rangle = v_i \langle \cos\alpha, \cos\beta, \cos\gamma \rangle = v_i \langle \cos 53, \cos 37, \cos 90 \rangle = v_i \langle 0.602, 0.799, 0 \rangle .$$

We know the initial and final locations of the ball, so the position update will be used to calculate the time of flight and the initial velocity.

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \vec{r}_i + \left( \frac{\vec{v}_i + \vec{v}_f}{2} \right)\Delta t$$

$$\langle 45.7, 3.1, 0 \rangle = \langle 0,0,0 \rangle_i + \left( \frac{\langle 0.602 v_i, 0.799 v_i, 0 \rangle + \left[ \langle 0.602 v_i, 0.799 v_i, 0 \rangle + \langle 0, -g, 0 \rangle \Delta t \right]}{2} \right)\Delta t$$

where the final velocity of the projectile is written from the momentum principle ($\vec{v}_f = \vec{v}_i + \dfrac{\vec{F}_{net}}{m}\Delta t$ ). Above, there are 2 equations and 2 unknowns, the initial speed of launch and the time of flight. From the horizontal motion we have: $45.7 = 0.602 v_i \Delta t \rightarrow \Delta t = \dfrac{45.7}{0.602 v_i}$ and inserting this into the vertical motion solves for the initial velocity. We have

$$3.1 = 0.799 v_i \left( \frac{45.7}{0.602 v_i} \right) - \frac{9.8}{2}\left( \frac{45.7}{0.602 v_i} \right)^2 \rightarrow v_i = 22.2 \tfrac{m}{s}.$$ The initial velocity is therefore,

$$\vec{v}_i = 22.2 \langle 0.602, 0.799, 0 \rangle \tfrac{m}{s}.$$

2.  What is the time of flight of the football from its kickoff point until it just barely clears the bottom bar of the uprights?

    From either the x- or y-motion we can calculate the time of flight.  To avoid solving a quadratic equation, from say the x-motion, $\Delta t = \dfrac{x_f}{0.602 v_i} = \dfrac{45.7m}{0.602 \times 22.2 \frac{m}{s}} = 3.4s$ .

3.  What is the velocity of the football just as it passes the bottom bar of the uprights?

    From the momentum principle, the final velocity is
    $$\vec{v}_f = \vec{v}_i + \dfrac{\vec{F}_{net}}{m}\Delta t = \langle (22.2 \times 0.602),(22.2 \times 0.799 - 9.8 \times 3.4),0 \rangle = \langle 13.4, -31.6, 0 \rangle \frac{m}{s}\,.$$

4.  Suppose that the kicker were to kick the ball with an initial velocity that is about $10\%$ greater than the value you found in *part 1*.  Assuming that the distances involved do not change and that the kicker still kicks the ball at the $53^o$ angle, the ball

    a. ⃝ will clear the uprights by passing above the bottom bar and a goal will be scored.
    b.    will clear the uprights by passing below the bottom bar and a goal will not be scored.
    c.    will never make it to the uprights and actually land in front of the uprights and a goal will not be scored.
    d.    and it's motion cannot be determined since there is not enough information given.

    So what we want to calculate is the final vertical height. If it is greater than 3.1m then it clears above the goal post, less than 3.1m under the goal post, and if the final vertical height is negative then it never made it to the goal post landing in front and short.

    To see this solution, consider the fact that the distances remain the same as does the launch angle. Since the initial velocity increases, from the horizontal motion, the time needed to reach the goalpost must decrease. Therefore in the vertical trajectory equation, the first term involving the velocity and the time, this piece remains constant. However, in the last piece this term decreases. Therefore the final height is a constant minus a smaller number than in part 1. Therefore the ball clears the goal posts by passing above the bar.

# *Physics 120 Equation Sheet*

$$\vec{r} = <r_x, r_y, r_z> = |\vec{r}| \cdot \hat{r}$$

magnitude of a vector : $r = |\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$

unit vector : $\hat{r} = \dfrac{\vec{r}}{|\vec{r}|}$

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}; \ \vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2}$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg} \Delta t$$

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \frac{\vec{F}_{net}}{2m} (\Delta t)^2$$