Name $\qquad$
Physics 120 Quiz \#6, May 16, 2014
Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

Suppose you have the arrangement of masses connected to a spring as shown below where $m_{1}=1 \mathrm{~kg}$, $m_{2}=4 \mathrm{~kg}, k=100 \frac{N}{m}$, and the angles are $\theta_{1}=28^{\circ}$ and $\theta_{2}=57^{\circ}$.

a. Taking the system to be the masses, the spring, and the earth, if the masses are released from rest when the spring is at its equilibrium position, what is the maximum extension of the spring?

Applying the energy principle we have
$\Delta E=\Delta U_{g}+\Delta U_{s}+\Delta K E+\Delta E_{\text {rest }}=0$
$0=\left(m_{2} g y_{2 f}-m_{2} g y_{2 i}\right)+\left(m_{1} g y_{1 f}-m_{1} g y_{1 i}\right)+\left(\frac{1}{2} k y_{f}^{2}-\frac{1}{2} k y_{i}^{2}\right)$
$0=-m_{2} g\left(d \sin \theta_{2}\right)+m_{1} g\left(d \sin \theta_{1}\right)+\frac{1}{2} k d^{2}$
$d_{\text {min }}=0$;
$d_{\text {max }}=\frac{2\left[m_{2} g \sin \theta_{2}-m_{1} g \sin \theta_{1}\right]}{k}=\frac{2 \times 9.8 \frac{\mathrm{~m}}{s^{2}}[4 \mathrm{~kg} \sin 57-1 \mathrm{~kg} \sin 28]}{100 \frac{N}{m}}=0.57 \mathrm{~m}$
b. When the spring has been displaced from its equilibrium position by an amount $d=0.4 m$, what is the speed of mass $m_{2}$ ?

Applying the energy principle we have
$\Delta E=\Delta U_{g}+\Delta U_{s}+\Delta K E+\Delta E_{\text {rest }}=0$
$0=\left(m_{2} g y_{2 f}-m_{2} g y_{2 i}\right)+\left(m_{1} g y_{1 f}-m_{1} g y_{1 i}\right)+\left(\frac{1}{2} m y_{2 f}^{2}-\frac{1}{2} m y_{2 i}^{2}\right)+\left(\frac{1}{2} m y_{1 f}^{2}-\frac{1}{2} m v_{1 i}^{2}\right)+\left(\frac{1}{2} k y_{f}^{2}-\frac{1}{2} k y_{i}^{2}\right)$
$v_{f}=\sqrt{\frac{2\left[m_{2} g d \sin \theta_{2}-m_{1} g d \sin \theta_{1}-\frac{1}{2} k d^{2}\right]}{\left(m_{1}+m_{2}\right)}}$
$v_{f}=\sqrt{\frac{2\left[\left(4 \mathrm{~kg} \times 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \times 0.4 \mathrm{~m} \sin 57\right)-\left(1 \mathrm{~kg} \times 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \times 0.4 \mathrm{~m} \sin 28\right)-\frac{1}{2} \times 100 \frac{\mathrm{~N}}{\mathrm{~m}}(0.4 \mathrm{~m})^{2}\right]}{(1 \mathrm{~kg}+4 \mathrm{~kg})}}=0.81 \frac{\mathrm{~m}}{\mathrm{~s}}$
c. From the point the spring reaches its maximum displacement from equilibrium and for all subsequent times after this point, the system of masses

1. moves back in the direction opposite their motion up to the point of maximum displacement of the spring from equilibrium. For example, $m_{2}$ moves back up the incline.
2. oscillates up and down the incline.
3. remains at rest on the incline.
4. moves but in a way that is not easily predicted.

## Physics 120 Equation Sheet

$$
\begin{aligned}
& \vec{r}=\left\langle r_{x}, r_{y}, r_{z}\right\rangle=|\vec{r}| \cdot \hat{r} \\
& \text { magnitude of a vector: } r=|\vec{r}|=\sqrt{r_{x}^{2}+r_{y}^{2}+r_{z}^{2}} \\
& \text { unit vector }: \hat{r}=\frac{\vec{r}}{|\vec{r}|} \\
& \vec{v}=\frac{\Delta \vec{r}}{\Delta t} ; \vec{v}_{\text {avg }}=\frac{\vec{v}_{i}+\vec{v}_{f}}{2} \\
& \vec{r}_{f}=\vec{r}_{i}+\vec{v}_{\text {avg }} \Delta t \\
& \vec{F}_{n e t}=\frac{\Delta \vec{p}}{\Delta t} \\
& \vec{p}=\gamma m \vec{v} ; \lim _{v<c c}(\vec{p}) \sim m \vec{v} \\
& \vec{p}_{f}=\vec{p}_{i}+\vec{F}_{n e t} \Delta t \\
& \vec{r}_{f}=\vec{r}_{i}+\vec{v}_{i} \Delta t+\frac{\vec{F}_{\text {net }}}{2 m}(\Delta t)^{2} \\
& \vec{F}_{G}=-\frac{G M_{1} M_{2}}{r^{2}} \hat{r} \\
& \vec{F}_{g} \sim m \vec{g} \\
& \text { Constants: } \\
& g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \\
& G=6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}} \\
& m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=0.51 \frac{\mathrm{MeV}}{\mathrm{c}^{2}} \\
& m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=938.5 \frac{\mathrm{MeV}}{\mathrm{c}^{2}} \\
& m_{E}=6 \times 10^{24} \mathrm{~kg} \\
& R_{E}=6.4 \times 10^{6} \mathrm{~m} \\
& N_{A}=6.02 \times 10^{23} \\
& c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}} \\
& 1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J} \\
& \frac{d \vec{p}}{d t}=\vec{F}_{n e t} \\
& \vec{F}_{s}=-k \vec{s} \\
& F_{f r}=\mu F_{N} \\
& k_{\text {parallel }}=\sum_{i=1}^{N} k_{i} \\
& \frac{1}{k_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{k_{i}} \\
& \rho=\frac{m}{V} \\
& Y=\frac{k_{I A B}}{d} \\
& \text { Stress }=Y \times \text { Strain; Stress }=\frac{F}{A} ; \text { Strain }=\frac{\Delta l}{l} \\
& \vec{F}_{\|}=\frac{d p}{d t} \hat{p} \\
& \vec{F}_{\perp}=p \frac{d \hat{p}}{d t} \rightarrow\left|\vec{F}_{\perp}\right|=\frac{m v^{2}}{R} ; \\
& \vec{F}_{\text {Net }}=\vec{F}_{\|}+\vec{F}_{\perp}=\frac{d \vec{p}}{d t}=\frac{d p}{d t} \hat{p}+p \frac{d \hat{p}}{d t} \\
& \Delta E=\Delta Q+\Delta W=\Delta K E+\Delta U_{g}+\Delta U_{s}+\Delta E_{\text {rest }} \\
& \Delta W=\Delta K E=\int \vec{F} \cdot d \vec{r} ; \quad \vec{F} \cdot d \vec{r}=F_{x} d x+F_{y} d y+F_{z} d z \\
& K E=(\gamma-1) m c^{2} ; \lim _{v<c c}(K E) \sim \frac{1}{2} m v^{2} \\
& E_{T}=\gamma m c^{2} ; E_{T}^{2}=p^{2} c^{2}+m^{2} c^{4} ; E_{\text {rest }}=m c^{2} \\
& U_{g}=m g y \\
& U_{s}=\frac{1}{2} k x^{2}
\end{aligned}
$$

