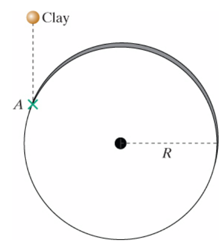
Name________________________________

Physics 120 Quiz #7, March 9, 2012

*Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.*

A rotating uniform-density disk of mass *5kg* and radius *0.6m* is mounted in the vertical plane as shown in the figure. The axle is help up by supports that are not shown and the disk is free to rotate on a nearly frictionless axle. A lump of clay with mass *0.4kg* falls and stick to the outer edge of the wheel at the location x on the diagram with coordinates $\langle -0.36, 0.48, 0\rangle m$, relative to an origin at the center of the axle. Just before impact, the clay has speed *8m/s* and the disk is rotating clockwise with angular speed *0.51rad/s*.




1. Just before the impact what is the angular momentum of the combined system of wheel plus clay about the axle? (As usual, *x is to the right*, *y is up*, and *z is out of the page toward you* and the moment of inertia of a solid cylinder spun about its center is $\frac{1}{2}MR^2$.)

$$\vec{L}_i = \vec{r} \times \vec{p}_{ic} + I_w \vec{\omega}_w = \langle 0,0,+Rp\sin\theta \rangle + \left(\tfrac{1}{2}MR^2\right)\langle 0,0,-\omega_i \rangle$$

$$\vec{L}_i = \left\langle 0,0,(0.6m) \times (3.2\tfrac{kgm}{s}) \times \left(\frac{0.36m}{0.6m}\right)\right\rangle + \left(\tfrac{1}{2}(5kg)(0.6m)^2\right)\langle 0,0,-0.51\tfrac{rad}{s}\rangle$$

$$\vec{L}_i = \langle 0,0,+0.691\rangle \tfrac{kgm^2}{s}$$

2. Just after the impact, what is the angular momentum of the combined system of the clay and wheel about the axle?

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net} = 0 \rightarrow d\vec{L} = 0 \Rightarrow \vec{L}_i = \vec{L}_f = \langle 0,0,+0.691\rangle \tfrac{kgm^2}{s}$$

3. Just after impact, what is the angular velocity of the wheel?

$$\vec{L}_f = I\vec{\omega}_f \rightarrow \vec{\omega}_f = \frac{\vec{L}_f}{I} = \frac{\langle 0,0,0.691\rangle \tfrac{kgm^2}{s}}{\left(m_c R^2 + \tfrac{1}{2}MR^2\right)} = \frac{\langle 0,0,0.691\rangle \tfrac{kgm^2}{s}}{\left((2.9kg)(0.6m)^2\right)} = \langle 0,0,0.662\rangle \tfrac{rad}{s}$$

4. Qualitatively, what happens to the linear momentum of the combined system?
   a. There is no change because linear momentum is always conserved.
   b. Some of the linear momentum is changed into angular momentum.
   c. Some of the linear momentum is changed into energy.
   d. The downward linear momentum decreases because the axle exerts an upward force.

## *Useful formulas:*

$\vec{p} = \gamma m\vec{v}$ $\quad k_{eff,parallel} = n_{parallel}k_{individual};$ $\quad stress = Ystrain \rightarrow \frac{F}{A} = Y\frac{\Delta L}{L}$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ $\quad k_{eff,series} = \frac{k_{individual}}{n_{series}}$

$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} \Rightarrow v_{avg} = R\omega_{avg};$ special case : constant acceleration $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2; \quad \vec{v}_f = \vec{v}_i + \vec{a}t$

$\vec{F}_g = m\vec{g}; \quad F_C = m\frac{v^2}{R}$

$\vec{F}_{gravity} = \frac{GM_1M_2}{r_{12}^2}\hat{r}_{12}$

$\vec{F}_{spring} = -k\vec{s}; \quad \vec{s} = (L - L_o)\hat{s}; \quad T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k_{eff}}}$

$W = \int \vec{F}\cdot d\vec{r} = \Delta KE = -\Delta U$

$U_g = -\frac{GM_1M_2}{r}$

$U_g = mgy$

$U_s = \frac{1}{2}ks^2$

$KE_T = \frac{1}{2}mv^2$

$KE_R = \frac{1}{2}I\omega^2$

$KE = (\gamma - 1)mc^2$

$E_{rest} = mc^2$

## *Constants*

$g = 9.8\,\frac{m}{s^2}$

$G = 6.67\times10^{-11}\,\frac{Nm^2}{kg^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{C^2}{Nm^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{Nm^2}{C^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

## *Momentum Principle:*

$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$

$\frac{d\vec{p}}{dt} = \vec{F}_{net}$

## *Angular Momentum Principle:*

$\vec{L}_f = \vec{L}_i + \vec{\tau}_{net}\Delta t; \quad \vec{\tau} = \vec{r}\times\vec{F}$

$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}; \quad \vec{L} = \vec{r}\times\vec{p} = I\vec{\omega}; \quad v = R\omega; \quad a = \frac{dv}{dt} = R\frac{d\omega}{dt} = R\alpha$

## *Position-update:*

$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t = \vec{r}_i + \frac{\vec{p}}{m\sqrt{1+\frac{p^2}{m^2c^2}}}\Delta t$

## *Energy Principle:*

$\Delta E = \Delta W + \Delta Q; \quad Q = mc\Delta T$

$\Delta E = 0 = \Delta U_g + \Delta U_s + \Delta KE_T + \Delta KE_R$

$W = \int \vec{F}\cdot d\vec{r} + \int \vec{\tau}\cdot d\vec{\theta}$

## *Vectors*

magnitude of a vector : $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

writing a vector : $\vec{a} = \langle a_x, a_y, a_z\rangle = |\vec{a}|\hat{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

dot product : $\vec{A}\cdot\vec{B} = AB\cos\theta$

cross product : $C = A\times B = \langle (A_yB_z - A_zB_y), (A_zB_x - A_xB_z), (A_xB_y - A_yB_x)\rangle$

magnitude of the cross product : $\vec{A}\times\vec{B} = AB\sin\theta$

## *Geometry*

*Circles* : $C = 2\pi r = \pi D \quad A = \pi r^2$

*Triangles* : $A = \frac{1}{2}bh$

*Spheres* : $A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3$