Name____

Physics 121 Quiz #2, January 19, 2018

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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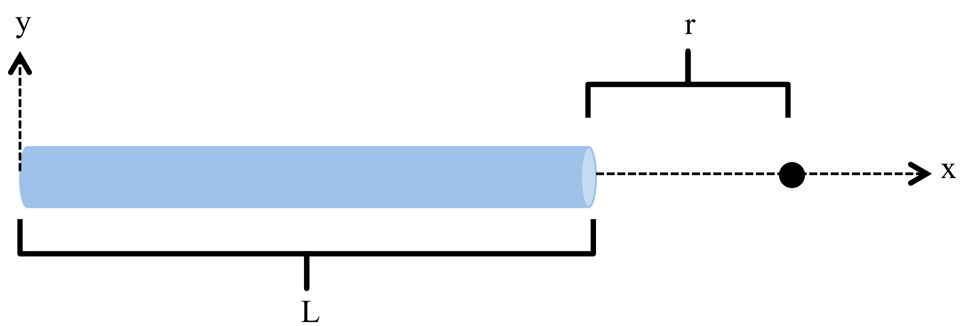
Suppose that you have a line of charge of length L where the linear charge density is distributed along the x-axis according to $\lambda(x) = \alpha x$ where α is a positive constant. The rod and associated geometry are shown below.



1. What is the total charge Q? Hint: The linear charge density is a function of position. "Add up" all of the contributions to the total charge from along the length of the rod.

$$dq = \lambda dx \rightarrow Q = \int dq = \int_0^L \alpha x\, dx = \left(\frac{\alpha x^2}{2}\right)\Bigg|_0^L = \frac{\alpha L^2}{2}$$

2. Set up, *but do not solve*, the integral for the electric field at a point $\langle x,y,z\rangle = \langle L+r,0,0\rangle$.

$$\vec{E} = \int d\vec{E}$$
$$d\vec{E} = \frac{kdq}{p^2}\hat{p}$$
$$dq = \alpha x dx$$
$$p^2 = (L+r-x)^2$$
$$\hat{p} = \frac{\langle L+r,0,0\rangle - \langle x,0,0\rangle}{L+r-x} = \langle 1,0,0\rangle$$
$$\vec{E} = \int d\vec{E} = \int \frac{kdq}{p^2}\hat{p} = k\alpha\left\langle \int_0^L \frac{x}{(L+r-x)^2}dx, 0, 0\right\rangle$$

This was not part of the question, but using Mathematica, the integral evaluates to:

$$E_x = k\alpha\left[\frac{L}{r} + \ln\left(\frac{L+r}{r}\right)\right]$$

3. Suppose that we have a parallel plate capacitor where the plates have area A and are separated by an amount s. The plate on the left has a charge $-Q$ while the plate on the right has a charge $+Q$. Take to the right as the positive x-direction and place an electric dipole (consisting of two equal and opposite charges q separated by a distance d) between the plates of the capacitor with the axis of the dipole perpendicular to the axis of the capacitor plates. In other words, the axis of the electric dipole is along the y-axis. If $d <<< s$, the electric dipole will experience which of the following?
   a. A net force along the x-axis and a torque orienting the dipole along the x-axis.
   b. A net force along the x-axis and zero torque.
   c. A zero net force and a torque orienting the dipole along the x-axis.
   d. A zero net force and a zero net torque leaving the dipole at rest.
   e. unable to be determined from the information given.

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F}=k\frac{Q_1Q_2}{r^2}\hat{r};\ \ \hat{r}=\frac{\vec{r}_o-\vec{r}_s}{\left|\vec{r}_o-\vec{r}_s\right|}$$

$$\vec{E}=\frac{\vec{F}}{q}=\int d\vec{E}=\int dE\hat{r}=\int\frac{kdq}{r^2}\hat{r}$$

$$\vec{E}_Q=k\frac{Q}{r^2}\hat{r}$$

$$\left|\vec{E}_{\parallel}\right|=\frac{2kqs}{r^3};\ \text{dipole } r>>s$$

$$\left|\vec{E}_{\perp}\right|=\frac{kqs}{r^3};\ \text{dipole } r>>s$$

$$\left|\vec{E}_{rod}\right|_{\perp}=\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2+\left(L/2\right)^2}}\right]$$

$$\left|\vec{E}_{rod}\right|_{\parallel}=\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r(L+r)}\right];\left|\vec{E}_{rod}\right|_{\parallel}\sim\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{rL}\right)\ \ L>>r$$

$$\left|\vec{E}_{ring}\right|=\frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{\left(R^2+z^2\right)^{\frac{3}{2}}}\right]$$

$$\left|\vec{E}_{disk}\right|=\frac{Q}{2\pi\varepsilon_0R^2}\left[1-\frac{z}{\sqrt{R^2+z^2}}\right];\ \ \left|\vec{E}_{disk}\right|\sim\frac{Q}{2\varepsilon_0A}\left[1-\frac{z}{R}\right]\ \ z<<R;\ \ \left|\vec{E}_{disk}\right|\sim\frac{Q}{2\varepsilon_0A}\ \ z<<R$$

$$\left|\vec{E}_{capacitor}\right|\sim\frac{Q}{\varepsilon_0A};\ \ \ \left|\vec{E}_{fringe}\right|\sim\frac{Q}{2\varepsilon_0A}\left(\frac{s}{R}\right)$$

$$W=-q\Delta V=-\Delta U=\Delta K;\ \ U=\sum_{i\neq j}\frac{kQ_iQ_j}{r_{ij}};$$

$$V_Q=\frac{kQ}{r};\ \ V_{Q's}=\sum_i\frac{kQ_i}{r_i}$$

$$\Delta V=-\int\vec{E}\cdot d\vec{r}$$

$$E_x=-\frac{\Delta V}{\Delta x};\ \ E_y=-\frac{\Delta V}{\Delta y};\ \ E_z=-\frac{\Delta V}{\Delta z};\ \ \vec{E}=-\left\langle\frac{dV}{dx},\frac{dV}{dy},\frac{dV}{dz}\right\rangle$$

$$Q=\left(\frac{\varepsilon_0A}{s}\right)\Delta V$$

$$I=\frac{\Delta Q}{\Delta t}=i\left|e\right|=n\left|e\right|Av_d;\ \ \ \vec{v}_d=\mu\vec{E}$$

$$\vec{B}=\frac{\mu_o}{4\pi}\left(\frac{q\vec{v}\times\hat{r}}{r^2}\right)$$

$$d\vec{B}=\frac{\mu_oI}{4\pi}\left(\frac{d\vec{l}\times\hat{r}}{r^2}\right)$$

$$\left|\vec{B}_{wire}\right|=\frac{\mu_oLI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2+r^2}};\ \ \left|\vec{B}_{wire}\right|\approx\frac{\mu_oI}{2\pi r}\ \ L>>r$$

$$\left|\vec{B}_{ring}\right|=\frac{\mu_oIR^2}{2\left(z^2+R^2\right)^{\frac{3}{2}}};\left|\vec{B}_{ring}\right|\approx\frac{\mu_oIR^2}{2z^3}\ \ z<<R$$

Constants

$$g=9.8\tfrac{m}{s^2}$$

$$1e=1.6\times10^{-19}C$$

$$k=\frac{1}{4\pi\varepsilon_o}=9\times10^9\tfrac{Nm^2}{C^2}$$

$$\varepsilon_o=8.85\times10^{-12}\tfrac{C^2}{Nm^2}$$

$$1eV=1.6\times10^{-19}J$$

$$\mu_o=4\pi\times10^{-7}\tfrac{Tm}{A}$$

$$c=3\times10^8\tfrac{m}{s}$$

$$h=6.63\times10^{-34}Js$$

$$m_e=9.11\times10^{-31}kg=\frac{0.511MeV}{c^2}$$

$$m_p=1.67\times10^{-27}kg=\frac{937.1MeV}{c^2}$$

$$m_n=1.69\times10^{-27}kg=\frac{948.3MeV}{c^2}$$

$$1amu=1.66\times10^{-27}kg=\frac{931.5MeV}{c^2}$$

$$N_A=6.02\times10^{23}$$

$$Ax^2+Bx+C=0\rightarrow x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$