Name____

Physics 121 Quiz #3, February 2, 2018

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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A parallel plate capacitor is constructed out of two square metal plates of length $L = 25cm$ separated by $6mm$ of paper ($\kappa = 2.3$).

1. What is the capacitance of the system?

$$C = \frac{\kappa\varepsilon_0 A}{s} = \frac{2.3\times 8.85\times 10^{-12}\,\frac{C^2}{Nm^2}\times(0.25m)^2}{6\times 10^{-6}\,m} = 2.12\times 10^{-10}\,F$$

2. Suppose that two of these capacitors were wired in series and the combination was wired to a $50\,M\Omega$ resistor. What is the time constant of the circuit?

$$\frac{1}{C_{eq}} = \frac{1}{C_2} + \frac{1}{C_1} = \frac{2}{C} \rightarrow C_{eq} = \frac{C}{2} = \frac{2.12\times 10^{-10}\,F}{2} = 1.06\times 10^{-10}\,F$$

$$\tau = RC = 50\times 10^6\,\Omega\times 1.06\times 10^{-10}\,F = 5.3\times 10^{-3}\,s$$

3. The two capacitors and resistor are wired to a battery with constant potential difference V. In multiples or fractions of the time constant, how long will it take to charge this two capacitor system to 83.7% of the rated battery potential?

$$V(t)=0.837V=V\left(1-e^{-\frac{t}{RC}}\right)\rightarrow t=-RC\ln\left(1-\frac{V(t)}{V}\right)=-RC\ln\left(1-\frac{0.837V}{V}\right)=1.8RC$$

4. How much energy is stored in each of the capacitors when fully charged? Assume that the two capacitors and resistor are connected to a $10V$ battery.

$$U_1=U_2=\tfrac{1}{2}CV_1^2=\tfrac{1}{2}\times 2.12\times 10^{-10}F\times(5V)^2=2.7\times 10^{-9}J$$

$$U_T=\tfrac{1}{2}C_{eq}V^2=\tfrac{1}{2}\times 1.06\times 10^{-10}F\times(10V)^2=5.3\times 10^{-9}J=U_1+U_2=2U_1\rightarrow U_1=\frac{U_T}{2}=2.7\times 10^{-9}J$$

5. Suppose instead that you have a single resistor R connected to a capacitor C. This resistor-capacitor combination is connected to a switch S and a battery with constant potential V. The capacitor is initially uncharged and the switch is open. At time $t=0$ the switch is closed and the capacitor begins to charge. As time increases, which of the following is true?
   a. $V_C\uparrow$ and $V_R\uparrow$.
   b. $V_C\uparrow$ and $V_R\downarrow$.
   c. $V_C\downarrow$ and $V_R\uparrow$.
   d. $V_C\downarrow$ and $V_R\downarrow$.
   e. Both V_C and V_R remain at a constant potential because the battery has a constant potential.

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1 Q_2}{r^2}\hat{r};\ \ \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{\left|\vec{r}_o - \vec{r}_s\right|}$$

$$\vec{E} = \frac{\vec{F}}{q} = \int d\vec{E} = \int dE\hat{r} = \int \frac{kdq}{r^2}\hat{r}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$\left|\vec{E}_{\parallel}\right| = \frac{2kqs}{r^3};\ \text{dipole } r >> s$$

$$\left|\vec{E}_{\perp}\right| = \frac{kqs}{r^3};\ \text{dipole } r >> s$$

$$\left|\vec{E}_{rod}\right|_{\perp} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(L/2\right)^2}}\right]$$

$$\left|\vec{E}_{rod}\right|_{\parallel} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\left(L+r\right)}\right];\left|\vec{E}_{rod}\right|_{\parallel} \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{rL}\right)\ \ L >> r$$

$$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{\left(R^2 + z^2\right)^{\frac{3}{2}}}\right]$$

$$\left|\vec{E}_{disk}\right| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right];\ \ \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right]\ \ z << R;\ \ \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A}\ \ z << R$$

$$\left|\vec{E}_{capacitor}\right| \sim \frac{Q}{\varepsilon_0 A};\ \ \left|\vec{E}_{fringe}\right| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$$

$$W = -q\Delta V = -\Delta U = \Delta K;\ U = \sum_{i \neq j}\frac{kQ_i Q_j}{r_{ij}};$$

$$V_Q = \frac{kQ}{r};\ V_{Q's} = \sum_i \frac{kQ_i}{r_i}$$

$$\Delta V = -\int \vec{E}\cdot d\vec{r}$$

$$E_x = -\frac{\Delta V}{\Delta x};\ E_y = -\frac{\Delta V}{\Delta y};\ E_z = -\frac{\Delta V}{\Delta z};\ \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right\rangle$$

$$Q = \left(\frac{\kappa\varepsilon_0 A}{s}\right)\Delta V$$

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right);\ Q = Q_{\max}e^{-\frac{t}{RC}}$$

$$I = \frac{\Delta Q}{\Delta t} = i\left|e\right| = n\left|e\right|Av_d;\ \ \vec{v}_d = \mu\vec{E}$$

$$\vec{B} = \frac{\mu_o}{4\pi}\left(\frac{q\vec{v}\times\hat{r}}{r^2}\right)$$

$$d\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{d\vec{l}\times\hat{r}}{r^2}\right)$$

$$\left|\vec{B}_{wire}\right| = \frac{\mu_o LI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2 + r^2}};\ \ \left|\vec{B}_{wire}\right| \approx \frac{\mu_o I}{2\pi r}\ \ L >> r$$

$$\left|\vec{B}_{ring}\right| = \frac{\mu_o IR^2}{2\left(z^2 + R^2\right)^{\frac{3}{2}}};\left|\vec{B}_{ring}\right| \approx \frac{\mu_o IR^2}{2z^3}\ \ z << R$$

Constants

$$g = 9.8\,\tfrac{m}{s^2}$$

$$1e = 1.6\times10^{-19}C$$

$$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\tfrac{Nm^2}{C^2}$$

$$\varepsilon_o = 8.85\times10^{-12}\,\tfrac{C^2}{Nm^2}$$

$$1eV = 1.6\times10^{-19}J$$

$$\mu_o = 4\pi\times10^{-7}\,\tfrac{Tm}{A}$$

$$c = 3\times10^8\,\tfrac{m}{s}$$

$$h = 6.63\times10^{-34}Js$$

$$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$$

$$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$$

$$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$$

$$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$$

$$N_A = 6.02\times10^{23}$$

$$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2 - 4AC}}{2A}$$