Name____

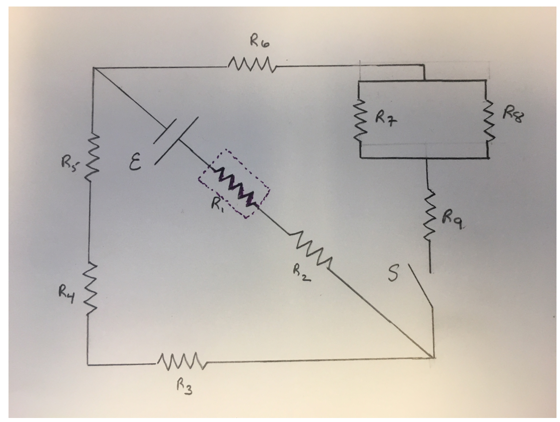
Physics 121 Quiz #5, February 16, 2018

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

Consider the circuit shown below in which all of the resistors have resistance R and are connected to an ideal battery of potential difference $\varepsilon = V$.

1. What is the expression for the equivalent resistance of the circuit in terms of R ? Assume that the switch S is closed for this question.



R_1 and R_2 are in series and $R_{12} = R_1 + R_2 = R + R = 2R$

R_3, R_4 and R_5 are in series

$R_{345} = R_3 + R_4 + R_5 = R + R + R = 3R$.

R_7 and R_8 are in parallel and

$$\frac{1}{R_{78}} = \frac{1}{R_7} + \frac{1}{R_8} = \frac{1}{R} + \frac{1}{R} \rightarrow R_{78} = \frac{R}{2}$$

R_6, R_{78} and R_9 are in series $R_{6789} = R_6 + R_{78} + R_9 = R + \frac{R}{2} + R = \frac{5}{2}R$.

R_{345} and R_{6789} are in parallel $\frac{1}{R_{3456789}} = \frac{1}{R_{345}} + \frac{1}{R_{6789}} = \frac{1}{3R} + \frac{2}{5R} = \frac{11}{15R} \rightarrow R_{3456789} = \frac{15}{11}R$.

$R_{3456789}$ and R_{12} are in series and $R_{123456789} = R_{eq} = R_{12} + R_{3456789} = 2R + \frac{15}{11}R = \frac{37}{11}R$.

2. If $R = 100\Omega$ and $\varepsilon = 10V$, what total current was produced by the battery? Assume that the switch S is closed for this question.

$$I = \frac{\varepsilon}{R_{eq}} = \frac{V}{R_{eq}} = \frac{10V}{\frac{37}{11} \times 100\Omega} = 0.0297A = 29.7mA$$

3. Suppose that you were to leave the switch S open. In this case, the total current and the the total energy dissapated per unit time would be given by which of the following?

   a. $I_{total} \uparrow$ and $P \uparrow$.

   b. $I_{total} \uparrow$ and $P \downarrow$.

   c. $I_{total} \downarrow$ and $P \uparrow$.

   d. $I_{total} \downarrow$ and $P \downarrow$.

   e. None of the above are true.

4. What energy is dissipated across the purple colored resistor inside of the dashed box, if the circuit is powered for one minute? Assume that the switch S is closed.

$$P = \frac{\Delta E}{\Delta t} \rightarrow \Delta E = P\Delta t = I_T^2 R\Delta t = \left(0.029A\right)^2 \times 100\Omega \times 60s = 5.1J$$

5. Due to the fact that there is a current flowing through resistors R_4 and R_5, a magnetic field $B = 5.4\times10^{-7}T$ is produced at a distance of $5mm$ to the left of the resistors R_4 and R_5 that points perpendicular to the plane of the page and is directed out of the page at you. Suppose that a $+q = e$ charge were directed down the page (from the top of the page to the bottom of the page parallel to the line formed by resistors R_4 and R_5 with a speed $2.5\times10^7 \frac{m}{s}$. When the charge enters the magnetic field it feels a force. What are the magnitude and direction of the magnetic force on the charge?

$\left|\vec{F}\right| = qvB\sin\theta = 1.6\times10^{-19}C \times 2.5\times10^7 \frac{m}{s} \times 5.4\times10^{-7}T \times \sin 90 = 2.2\times10^{-18}N$ directed to the left by the right-hand rule, away from the circuit.

Or by evaluating the cross product:

$$\vec{F} = q\vec{v}\times\vec{B} = q\begin{vmatrix} i & j & k \\ 0 & -v & 0 \\ 0 & 0 & B \end{vmatrix} = \left\langle -qvB,0,0\right\rangle = \left\langle -2.2\times10^{-18},0,0\right\rangle N$$

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r};\ \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|}$$

$$\vec{E} = \frac{\vec{F}}{q} = \int d\vec{E} = \int dE\hat{r} = \int \frac{kdq}{r^2}\hat{r}$$

$$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$$

$$\left|\vec{E}_{\parallel}\right| = \frac{2kqs}{r^3};\ \text{dipole } r >> s$$

$$\left|\vec{E}_{\perp}\right| = \frac{kqs}{r^3};\ \text{dipole } r >> s$$

$$\left|\vec{E}_{rod}\right|_{\perp} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(L/2\right)^2}}\right]$$

$$\left|\vec{E}_{rod}\right|_{\parallel} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r(L+r)}\right];\ \left|\vec{E}_{rod}\right|_{\parallel} \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{rL}\right)\quad L >> r$$

$$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{\left(R^2 + z^2\right)^{\frac{3}{2}}}\right]$$

$$\left|\vec{E}_{disk}\right| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right];\quad \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right]\quad z << R;\quad \left|\vec{E}_{disk}\right| \sim \frac{Q}{2\varepsilon_0 A}\quad z << R$$

$$\left|\vec{E}_{capacitor}\right| \sim \frac{Q}{\varepsilon_0 A};\quad \left|\vec{E}_{fringe}\right| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$$

$$W = -q\Delta V = -\Delta U = \Delta K;\ U = \sum_{i \neq j} \frac{kQ_iQ_j}{r_{ij}};$$

$$V_Q = \frac{kQ}{r};\ V_{Q's} = \sum_i \frac{kQ_i}{r_i}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$E_x = -\frac{\Delta V}{\Delta x};\ E_y = -\frac{\Delta V}{\Delta y};\ E_z = -\frac{\Delta V}{\Delta z};\ \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz} \right\rangle$$

$$Q = \left(\frac{\kappa\varepsilon_0 A}{s}\right)\Delta V$$

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

$$Q = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right);\ Q = Q_{\max}e^{-\frac{t}{RC}}$$

$$I = \frac{dQ}{dt} = n|e|Av_d = \int \vec{J} \cdot d\vec{A}$$

$$n = \frac{\rho}{M}N_A$$

$$\vec{J} = n|e|\vec{v}_d = \sigma\vec{E} = \tfrac{1}{\rho}\vec{E} \to V = IR;\ R = \frac{\rho L}{A}$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$\rho = \rho_0(1 + \alpha\Delta T)$$

$$\vec{F} = q\vec{v} \times \vec{B};\ \left|\vec{F}\right| = qvB\sin\theta$$

$$\vec{B} = \frac{\mu_o}{4\pi}\left(\frac{q\vec{v} \times \hat{r}}{r^2}\right)$$

$$d\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{d\vec{l} \times \hat{r}}{r^2}\right)$$

$$\left|\vec{B}_{wire}\right| = \frac{\mu_o LI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2 + r^2}};\ \left|\vec{B}_{wire}\right| \approx \frac{\mu_o I}{2\pi r}\quad L >> r$$

$$\left|\vec{B}_{ring}\right| = \frac{\mu_o IR^2}{2\left(z^2 + R^2\right)^{\frac{3}{2}}};\ \left|\vec{B}_{ring}\right| \approx \frac{\mu_o IR^2}{2z^3}\quad z << R$$

Constants

$$g = 9.8\tfrac{m}{s^2}$$

$$1e = 1.6 \times 10^{-19}C$$

$$k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9\tfrac{Nm^2}{C^2}$$

$$\varepsilon_o = 8.85 \times 10^{-12}\tfrac{C^2}{Nm^2}$$

$$1eV = 1.6 \times 10^{-19}J$$

$$\mu_o = 4\pi \times 10^{-7}\tfrac{Tm}{A}$$

$$c = 3 \times 10^8\tfrac{m}{s}$$

$$h = 6.63 \times 10^{-34}Js$$

$$m_e = 9.11 \times 10^{-31}kg = \frac{0.511MeV}{c^2}$$

$$m_p = 1.67 \times 10^{-27}kg = \frac{937.1MeV}{c^2}$$

$$m_n = 1.69 \times 10^{-27}kg = \frac{948.3MeV}{c^2}$$

$$1amu = 1.66 \times 10^{-27}kg = \frac{931.5MeV}{c^2}$$

$$N_A = 6.02 \times 10^{23}$$

$$Ax^2 + Bx + C = 0 \to x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$