Name____

Physics 121 Quiz #7, March 9, 2018

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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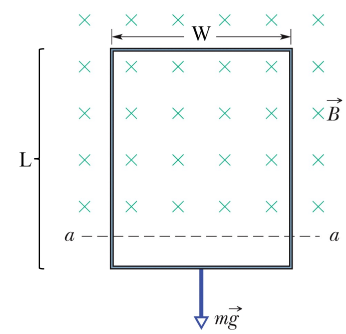
A rectangular loop of conducting wire of length $L = 25cm$ and width $W = 17cm$ has a mass $m = 250g$ and an electrical resistance $R = 0.018\Omega$. The loop is suspended vertically in an $B = 8T$ magnetic field oriented perpendicular to the plane of the conducting loop. The magnetic filed exists only above the line labeled AA. The wire loop is released from rest and is allowed to fall vertically through the magnetic field. As it falls it accelerates and ultimately reaches a terminal speed v_t, where the terminal speed is the speed reached when the net acceleration vanishes.



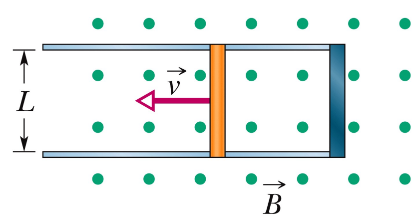
1. What is the terminal speed of the wire loop?

$$F_B - F_W = ma_y = 0$$

$$\rightarrow IWB = \left(\frac{BWv}{R}\right)WB = \frac{B^2W^2v_t}{R} = mg$$

$$v_t = \frac{Rmg}{B^2W^2} = \frac{0.018\Omega \times 0.25kg \times 9.8\frac{m}{s^2}}{(8T \times 0.17m)^2} = 0.024\frac{m}{s} = 2.4\frac{cm}{s}$$

2. Suppose that you have a metal rod forced to move with a constant velocity $\vec{v}$ along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude points $|\vec{B}| = 0.35T$ out of the page. The rails are separated by $L = 25cm$ and the speed of the rod is $|\vec{v}| = 55\frac{cm}{s}$. What is the induced potential difference generated across the loop?



$$\varepsilon = \left|-N\frac{d\phi_B}{dt}\right| = \left|\frac{d(BA\cos\theta)}{dt}\right| = \left|B\frac{dA}{dt}\right| = BLv = 0.35T \times 0.25m \times 0.55\frac{m}{s} = 0.0481V = 48.1mV$$

3. If the rod has a resistance of $R = 18\Omega$ and the rails and connector have negligible resistance, what is the magnitude and direction of the current induced in the rod?

$I = \frac{\varepsilon}{R} = \frac{0.0481V}{18\Omega} = 0.00267A = 2.67mA$ and by the right-hand rule, the current in the circuit flows clockwise to oppose the change in magnetic flux.

4. What is the magnitude and direction of the electric field induced in the rod?

$\left|\vec{E}\right| = \left|-\frac{\Delta V}{\Delta y}\right| = \frac{\varepsilon}{L} = \frac{0.0481V}{0.25m} = 0.193\tfrac{V}{m} = 193\tfrac{mV}{m}$ and the direction of the electric field is down the rod.

5. What is the rate at which energy is being transferred to heat in the rod?

$$P = \frac{V^2}{R} = \frac{(BLv)^2}{R} = \frac{(0.0481V)^2}{18\Omega} = 0.000128W = 0.128mW$$

Physics 121 Equation Sheet

Electric Forces, Fields and Potentials

$\vec{F} = k\frac{Q_1Q_2}{r^2}\hat{r};\ \hat{r} = \frac{\vec{r}_o - \vec{r}_s}{|\vec{r}_o - \vec{r}_s|}$

$\vec{E} = \frac{\vec{F}}{q} = \int d\vec{E} = \int dE\hat{r} = \int \frac{kdq}{r^2}\hat{r}$

$\vec{E}_Q = k\frac{Q}{r^2}\hat{r}$

$|\vec{E}_{\parallel}| = \frac{2kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{\perp}| = \frac{kqs}{r^3}$; dipole $r >> s$

$|\vec{E}_{rod}|_{\perp} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r\sqrt{r^2 + \left(L/2\right)^2}}\right]$

$|\vec{E}_{rod}|_{\parallel} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r(L+r)}\right]; |\vec{E}_{rod}|_{\parallel} \sim \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{rL}\right)\ \ L >> r$

$|\vec{E}_{ring}| = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qz}{(R^2+z^2)^{\frac{3}{2}}}\right]$

$|\vec{E}_{disk}| = \frac{Q}{2\pi\varepsilon_0 R^2}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right];\ \ |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\left[1 - \frac{z}{R}\right]\ \ z << R;\ \ |\vec{E}_{disk}| \sim \frac{Q}{2\varepsilon_0 A}\ \ z << R$

$|\vec{E}_{capacitor}| \sim \frac{Q}{\varepsilon_0 A};\ \ |\vec{E}_{fringe}| \sim \frac{Q}{2\varepsilon_0 A}\left(\frac{s}{R}\right)$

$W = -q\Delta V = -\Delta U = \Delta K;\ U = \sum_{i \neq j}\frac{kQ_iQ_j}{r_{ij}};$

$V_Q = \frac{kQ}{r};\ V_{Q's} = \sum_i \frac{kQ_i}{r_i}$

$\Delta V = -\int \vec{E}\cdot d\vec{r}$

$E_x = -\frac{\Delta V}{\Delta x};\ E_y = -\frac{\Delta V}{\Delta y};\ E_z = -\frac{\Delta V}{\Delta z};\ \vec{E} = -\left\langle \frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right\rangle$

$Q = \left(\frac{\kappa\varepsilon_0 A}{s}\right)\Delta V$

$U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$

$Q = Q_{\max}\left(1 - e^{-\frac{t}{RC}}\right);\ Q = Q_{\max}e^{-\frac{t}{RC}}$

$I = \frac{dQ}{dt} = n|e|Av_d = \int \vec{J}\cdot d\vec{A}$

$n = \frac{\rho}{M}N_A$

$\vec{J} = n|e|\vec{v}_d = \sigma\vec{E} = \tfrac{1}{\rho}\vec{E} \rightarrow V = IR; R = \frac{\rho L}{A}$

$P = IV = I^2R = \frac{V^2}{R}$

$\rho = \rho_0(1 + \alpha\Delta T)$

$\vec{F} = q\vec{v}\times\vec{B};\ |\vec{F}| = qvB\sin\theta$

$\vec{B} = \frac{\mu_o}{4\pi}\left(\frac{q\vec{v}\times\hat{r}}{r^2}\right)$

$d\vec{B} = \frac{\mu_o I}{4\pi}\left(\frac{d\vec{l}\times\hat{r}}{r^2}\right)$

$|\vec{B}_{wire}| = \frac{\mu_o LI}{4\pi r\sqrt{\left(\frac{L}{2}\right)^2 + r^2}};\ \ |\vec{B}_{wire}| \approx \frac{\mu_o I}{2\pi r}\ \ L >> r$

$|\vec{B}_{ring}| = \frac{\mu_o IR^2}{2(z^2+R^2)^{\frac{3}{2}}};\ |\vec{B}_{ring}| \approx \frac{\mu_o IR^2}{2z^3}\ \ z << R$

$\vec{F} = I\vec{L}\times\vec{B}; |\vec{F}| = ILB\sin\theta$

$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$

$R_{eq} = \sum_i R_i$

$C_{eq} = \sum_i C_i$

$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$

$\phi_B = \int \vec{B}\cdot d\vec{A}$

$\varepsilon = -N\frac{d\phi_B}{dt}$

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$

$\vec{A}\times\vec{B} = \left\langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x\right\rangle$

Constants

$g = 9.8\,\frac{m}{s^2}$

$1e = 1.6\times10^{-19}C$

$k = \frac{1}{4\pi\varepsilon_o} = 9\times10^9\,\frac{Nm^2}{C^2}$

$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{Nm^2}$

$1eV = 1.6\times10^{-19}J$

$\mu_o = 4\pi\times10^{-7}\,\frac{Tm}{A}$

$c = 3\times10^8\,\frac{m}{s}$

$h = 6.63\times10^{-34}Js$

$m_e = 9.11\times10^{-31}kg = \frac{0.511MeV}{c^2}$

$m_p = 1.67\times10^{-27}kg = \frac{937.1MeV}{c^2}$

$m_n = 1.69\times10^{-27}kg = \frac{948.3MeV}{c^2}$

$1amu = 1.66\times10^{-27}kg = \frac{931.5MeV}{c^2}$

$N_A = 6.02\times10^{23}$

$Ax^2 + Bx + C = 0 \rightarrow x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$