1. A Boeing 777 has a mass of 2.43×10^5 kg and each wing has an area of $189 m^2$. During level flight, the pressure on the lower wing surface is 700×10^4 Pa.
   a. What is the pressure on one of the upper wings?
   b. What is the upward acceleration of the aircraft if the pressure on the lower surface were to increase to 702×10^4 Pa? (This increase in pressure is due to the aircraft increasing its forward velocity and assumes that P_{upper} remains constant.)

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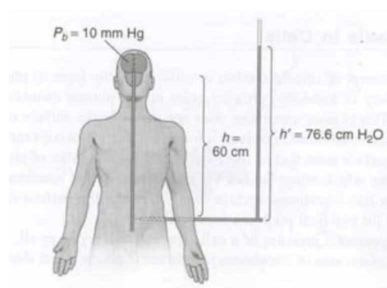
a. If the plane is not accelerating up or down then the difference in pressures above and below the wing gives the lifting force (when multiplied by the wing area.)

$$P_{lower} - P_{upper} = \frac{F_{lift}}{A} = \frac{m_{plane} g}{A_{wing}} \rightarrow P_{upper} = P_{lower} - \frac{m_{plane} g}{A_{wing}} = 6.987 \times 10^6 \tfrac{N}{m^2}$$

b. $$\left(P_{lower} - P_{upper}\right) A_{wing} = F_{unbalanced} \rightarrow F_{unbalanced} = 6.73 \times 10^6 N$$

$$\therefore F_{unbalanced} - m_{plane} g = m_{plane} a \rightarrow a = 15.8 \tfrac{m}{s^2}.$$

2. Suppose that the pressure in the cerebrospinal fluid (CSF) is measured as shown below, using a spinal tap with the patient sitting erect. The pressure due to the weight of the CSF (given that its density is 1050 kg/m^3) in the spinal column increases the pressure?
   a. What is the pressure measured in Pascal and in centimeters of water if the pressure around the brain is 10mm Hg and the tap is at a point 60cm lower than the brain?
   b. What is the pressure measured in Pascal and in centimeters of water if the person is lying down?



a. $$P = P_{brain} + \rho g h = 10 mmHg \times \frac{133.3 \frac{N}{m^2}}{1 mmHg} + \left(1050 \tfrac{kg}{m^3} \times 9.8 \tfrac{m}{s^2} \times 0.6 m\right) = 7507 \tfrac{N}{m^2} = 56.3 mmHg$$

height of water: $$P = \rho g_{water} h_{water} \rightarrow h_{water} = \frac{P}{\rho g_{water}} = 0.766 m = 76.6 cm$$

b. $$P_{lying\ down} = 1333 \tfrac{N}{m^2} = 10 mmHg$$

$$h_{water} = \frac{P}{\rho g_{water}} = 0.136 m = 13.6 cm$$

3. The total flow rate of the adult circulatory system is about 10^{-4} m^3/s. This is the flow that passes through either the right or left side of the heart. The aorta, which carries the blood from the left side of the heart, has a diameter of 2.5cm. What is the flow velocity of the aorta?

$$flow\ rate = Av \rightarrow v = \frac{flow\,rate}{A} = \frac{1\times10^{-4}\ \frac{m^3}{s}}{\pi(0.0125m)^2} = 0.204\,\tfrac{m}{s} = 20.4\,\tfrac{cm}{s}$$

4. Suppose that the aorta has a radius of about 1.25cm and that the typical blood velocity is around 30 cm/s and that it has an average density of 1060 kg/m^3.
   c. What is the average blood velocity in the major arteries if the total cross sectional area of the major arteries is $20cm^2$?
   d. What is the total flow rate?
   e. On the assumption that all the blood in the circulatory system goes through the capillaries, what is the total cross sectional area of the capillaries if the average velocity of the blood in the capillaries is 0.03cm/s?
   f. If a typical capillary has a cross sectional area of $3x10^{-11}$ m^2, about how many capillaries are there in the human body?
   g. What are the kinetic energy per unit volume for blood in the aorta, the major arteries, and the capillaries?
   h. If a capillary has an average length of 0.75mm what is the average time that a red blood cell remains in a capillary?

$a.$ $$v_{arteries} = \frac{flow\,rate}{A_{arteries}} = \frac{1.47\times10^{-4}\ \frac{m^3}{s}}{0.002m^2} = 0.0735\,\tfrac{m}{s} = 7.35\,\tfrac{cm}{s}$$

$b.$ $$flow\,rate = 1.47\times10^{-4}\ \tfrac{m^3}{s}$$

$c.$ $$flow\,rate = A_{cap}v_{cap} \rightarrow A_{cap} = \frac{1.47\times10^{-4}\ \frac{m^3}{s}}{3\times10^{-4}\ \frac{m}{s}} = 0.49m^2$$

$d.$ $$\#capillaries = \frac{0.49m^2}{3\times10^{-11}m^2} = 1.6\times10^{10} = 16billion$$

$e.$ $$KE_{aorta} = \tfrac{1}{2}\rho_{blood}v_{aorta}^2 = 47.7\,\tfrac{J}{m^3}$$

$$KE_{arteries} = \tfrac{1}{2}\rho_{blood}v_{arteries}^2 = 2.9\,\tfrac{J}{m^3}$$

$$KE_{capillaries} = \tfrac{1}{2}\rho_{blood}v_{capillaires}^2 = 4.8\times10^{-5}\ \tfrac{J}{m^3}$$

$f.$ $$v = \frac{d}{t} \rightarrow t = \frac{d}{v} = \frac{0.75\times10^{-3}m}{3.0\times10^{-4}\ \frac{m}{s}} = 2.5s$$

5. An air duct is of rectangular cross section 300mm wide by 450mm deep. Determine the mean velocity in the duct when the rate of flow is $0.42m^3/s$. If the duct tapers to a cross-section 150mm wide by 400mm deep, what will the mean velocity in the reduced section be assuming that the density remains constant?

   a. *Constant cross-section:* The velocity v = Q / A, where $Q = 0.42m^3s^{-1}$, A = 0.3m x 0.45m = $0.135m^2$, the velocity v = 3.11 m/s.

   b. *Tapering cross-section:* The velocity v = Q / A, where $Q = 0.42m^3s^{-1}$, A = 0.15m x 0.40m = $0.06m^2$, the velocity v = 7.00 m/s.

6. Water flows through a pipe (AB) 1.2m in diameter at 3m/s and then passes into another pipe (CD) which is 1.5m in diameter. At C the pipe forks and branch CD is 0.8m in diameter and carries one-third of the flow in AB. The velocity in branch CE is 2.5m/s.
   a. What is the volume rate of flow in AB?
   b. What is the velocity in BC?
   c. What is the velocity in CD?
   d. What is the diameter CE?

$$a.\ \text{Discharge in } AB:\quad Q_{AB} = v_{AB}A_{AB} = 3\tfrac{m}{s}\times 1.131m^2 = 3.393\tfrac{m^3}{s}$$

$$b.\ \text{Velocity in } BC:\quad v_{BC} = \frac{Q_{BC}}{A_{BC}} = \frac{Q_{AB}}{A_{BC}} = \frac{3.393\frac{m^3}{s}}{1.767m^2} = 1.92\tfrac{m}{s}$$

$$c.\ \text{Velocity in } CD:\quad v_{CD} = \frac{Q_{CD}}{A_{CD}} = \frac{Q_{AB}}{3A_{CD}} = \frac{1.131\frac{m^3}{s}}{0.503m^2} = 2.25\tfrac{m}{s}$$

$$d.\ \text{Diameter of } CE:\quad A_{CE} = \frac{\pi D_{CE}^2}{4} \rightarrow$$

$$D_{CE} = \sqrt{\frac{4}{\pi}\frac{Q_{CE}}{v_{CE}}} = \sqrt{\frac{4}{\pi}\times\frac{2}{3}\times\frac{Q_{AB}}{v_{CE}}} = \sqrt{\frac{4}{\pi}\times\frac{2}{3}\times\frac{3.393\frac{m^3}{s}}{2.5\frac{m}{s}}} = 1.073m$$

7. A ram-jet engine consumes 20kg of air per second an 0.6kg of fuel per second. The exit velocity of the gases is 520m/s relative to the engine and the flight velocity is 200m/s. What power is developed by the engine?

We know that for a jet engine, the force exerted on the fluid is given by:

$F = \dot{m}_2 v_2 - \dot{m}_1 v_1$ where $\dot{m}_1 = \dot{m}_{air} = 20\frac{kg}{s}$ and

$\dot{m}_2 = \dot{m}_{air} + \dot{m}_{fuel} = 20\frac{kg}{s} + 0.6\frac{kg}{s} = 20.6\frac{kg}{s}$.

Also, $v_2 = 520\frac{m}{s}$ and $v_1 = 200\frac{m}{s}$ therefore:

$F = \dot{m}_2 v_2 - \dot{m}_1 v_1 = \left(20.6\frac{kg}{s}\times 520\frac{m}{s}\right) - \left(20\frac{kg}{s}\times 200\frac{m}{s}\right) = 6712\text{ N}$

We also know that Power $= F\times v_1 = 6712N\times 200\frac{m}{s} = 1340$ kW