Physics 123 Homework Solutions

Week #2 Unit Q Wave Optics

Q2B.1

The angles for constructive interference are given by $\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{m(2.5cm)}{12.0cm}\right) = \sin^{-1}(0.21m)$. Thus the angles are 0°, 12°, 25°, 39°, 56° and for m > 4 we get angles larger than 90°, so we stop here.

Q2B.2

The sound waves will interfere and diffract as if they had gone through 2 slits having the size of the speaker spacing. Thus the angles are given by $\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{m(1.1m)}{3.9m}\right) = \sin^{-1}(0.49m)$, where the wavelength is the velocity of sound divided by the frequency, or 343m/s / 320 Hz = 1.1m. Thus the angles are 0°, 16°, 34°, 58°, and for m > 3 we get angles larger than 90°, so we stop here.

Q2B.6

The angle for the first constructive interference above the centerline is given by $\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{570\times10^{-9}m}{3\times10^{-5}m}\right) = 1.1^o$. From the geometry of the setup, the distance between the central and 1st bright spots is given by $y_1 = D\tan\theta_1 = 2.4m\tan(1.1) = 4.6cm$.

Q2B.13

The total angle covered by the central maximum goes from $-\theta_1$ to $+\theta_1$ where $\theta = \sin^{-1}\frac{\lambda}{a}$. Using the small angle approximation, we have $\frac{w}{D} \approx 2\theta_1 \approx 2\frac{\lambda}{a} \rightarrow a = \frac{2D\lambda}{w} = \frac{2\times2m\times441\times10^{-9}m}{0.015m} = 0.12mm$.

Q2S.1

The wavelength of the sound waves with a frequency of 440Hz is given as $\lambda = \frac{v}{f} = \frac{343\frac{m}{s}}{440s^{-1}} = 0.78m$. The speakers will act like two slits that emit circular waves in all directions since the speakers with is smaller than this. Thus we will have constructive and destructive interferences given by $y_m = D\tan\theta_m = D\tan\left(\sin^{-1}\left(\frac{m\lambda}{d}\right)\right)$. From the information given, the distance from the central maximum to the 1st constructive interference on either side is given as 5m. Thus if you walk 2.5m from the central maximum you should hear nothing (the amplitude of the sound waves will decrease to zero) and if you continue for another 2.5m the amplitude should increase to its maximum value.

Q2S.2

The interferences will be constructive when the waves from the lower antennae travel ml farther than waves from the other antenna. For the $m = 1$ constructive interference, the angle above the centerline is given as $\theta_1 = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{c}{fd}\right) = \sin^{-1}\left(\frac{3\times10^8\ \frac{m}{s}}{60m\times100\times10^6\ Hz}\right) = 2.9^o$. Therefore the distance above the central maximum is given by $y_1 = D\tan\theta_1 = 5000m\tan(2.9) = 250m$.

Q2S.8

The eye acts as a circular aperture and can barely resolve two headlights for angles $\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{550\times10^{-9}m}{0.008m}\right) = 8.39\times10^{-5}rad$, where the pupil has a diameter of about 8mm at night and we've assumed that white light has a maximum wavelength of color yellow, to which the eye is most sensitive. Thus if the car is directly in front of you, the minimum distance is given by

$$\theta_{\min} \approx \frac{\text{spacing of lights}}{\text{distance away}} \to \text{distance away} = \frac{\text{spacing of lights}}{\theta_{\min}} = \frac{1.4m}{8.39\times10^{-5}rad} = 16.7km!!$$

Take this result with a grain of salt. This assumes that you have a perfectly clear atmosphere and that your eye is working at close to their diffraction limit. A more reasonable estimate may be 10x smaller, or 1.67km (~1 mi).

Q2S.9

The closest Mars gets to Earth is about 0.5AU (or about 50 million kilometers.) The separation of two objects using your eye is given from the minimum resolvable angle at this distance, which we use from Q2S.8 as, $\theta_{min} = 8.39\text{x}10^{-5}$ rad. At the distance of closest approach we calculate the minimum separation to be $\theta_{\min} \approx \frac{\text{separation}}{\text{distance away}} \to separation = \text{distance away}\times\theta_{\min} = 50\times10^6\,km\times8.39\times10^{-5}\,rad = 4000km\cdot$

The separation of two objects using the Hubble Telescope is given from the minimum resolvable angle at this distance, which we use from Q2S.8 as, $\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{550\times10^{-9}m}{1.0m}\right) = 6.71\times10^{-7}rad$. At the distance of closest approach we calculate the minimum separation to be $\theta_{\min} \approx \frac{\text{separation}}{\text{distance away}} \to separation = \text{distance away}\times\theta_{\min} = 50\times10^6\,km\times6.71\times10^{-7}\,rad = 34km\cdot$

Q2S.10

We wish to distinguish objects separated by d = 0.75mm at a distance of D. Thus since the wavelength of light we are using (575nm) is smaller than the width of the pupil, we have small angles involved. To see the text, we would have to be at a distance of $D = \frac{ad}{1.22\lambda} = \frac{0.003m \times 0.00075mm}{1.22 \times 575 \times 10^{-9} m} = 3.4m$ or less.

Q2S.12

To tell hair color of a set of riders, Legolas had to be able to distinguish riders' heads from other objects. To do this Legolas has to be able to resolve objects that are on the order of magnitude 8 inches (or 20cm) apart at a distance of 5 Leagues. Looking up a "League" I get about 3 miles. Therefore the angular separation of these objects would be about $\theta \approx \frac{d}{D} = \frac{0.20m}{15mi \times \frac{1.6km}{1mi} \times \frac{1000m}{1km}} = 8.33 \times 10^{-6} rad$. At the diffraction limit we can estimate the size of Legolas' pupils to be $a = \frac{1.22\lambda}{\theta} = \frac{1.22 \times 575 \times 10^{-9} m}{8.33 \times 10^{-6}} = 0.084m = 8.4cm$. Or, Legolas' pupils would have to be 8cm in diameter!! That's some great eyes! Either Legolas was using some sort of non-visual perception or he was merely guessing!

Q2R.2

The star witness is sitting about D = 600 feet (183m) from the crime scene. To see the "A" on the defendant's hat, the witness would have to resolve features of the letter that are about ¼ the size of the letter (from Q2S.10). If the letter is about 2.4 inches (6cm) tall, then the witness would have to resolve features about 1.5cm across at this distance. The angular separation of the features would be given as θmin ~ d / D. This corresponds to a width to the pupil of approximately $a = \frac{1.22\lambda D}{d} = \frac{1.22 \times 575 \times 10^{-9} m \times 183m}{0.015m} = 0.0086m = 8.5mm$. Given that this was during the daytime, and at night the diameter of the pupil is about 8mm, I'm sure this is too large, by maybe a factor of 2 or 3. Therefore the witness probably could not tell the letter on the cap.