# Physics 123 Homework Solutions

## Week #3 Unit T Thermodynamics

**T1B.8**

We convert body temperature of 98.6°F to Celsius by $T\left({}^{o}C\right)=\frac{5}{9}\left(T\left({}^{o}F\right)-32^{o}F\right)=\frac{5}{9}\left(98.6^{o}F-32^{o}F\right)=37^{o}C$ .

**T1S.3**

In order for the cylinder of water to remain at rest, the total force acting on it in the vertical direction must be zero. From the diagram, the net force is $-F_{top}+F_{bottom}-m_{water}g=0\rightarrow m_{water}g=F_{bottom}-F_{top}=\left(P_{bottom}-P_{top}\right)A$. The mass of the column of fluid is given by its density ρ, where $m_{water}=\rho V=\rho hA$ for a cylinder of cross-sectional area A and height h. Combining these two expressions, we find $m_{water}g=\rho_{water}hAg=\left(P_{bottom}-P_{top}\right)A\rightarrow\left(P_{bottom}-P_{top}\right)=\rho_{water}hg$ where $h = |z_1 - z_2|$.

**T1S.8**

a. The energy dU an object gains when its temperature increases by dT is given by $dU = McdT$ , where M is the objects mass and c is the specific heat. If the two objects are thermally insulated then conservation of energy requires that the heat lost by one object must have been gained by the other object. Therefore, $dU_A=-dU_B\rightarrow M_Ac_AdT_A=-M_Bc_BdT_B$. Therefore the ratio of the temperatures is given $-\frac{dT_B}{dT_A}=\frac{M_Ac_A}{M_Bc_B}\equiv u$ .

b. Here we have, $dT_A = T_f - T_A$ and $dT_B = T_f - T_B$, therefore $-\frac{dT_B}{dT_A}=-\left(\frac{T_f-T_B}{T_f-T_A}\right)=u\rightarrow T_B-T_f=u\left(T_f-T_A\right)$. Gathering all terms involving $T_f$ to one side we find $T_f=T_B+\frac{u}{u+1}\left(T_A-T_B\right)$.

c. If the cold object *B* is much more massive than the hot object *A*, we would expect the final temperature to be close to the value of $T_B$. If $m_A >> m_B$ then u is small and the ratio $\frac{u}{u+1}\approx 0$ so that $T_f \sim T_B$ as expected.

**T1R.2**

a. From figure T1.7 the gas exerts a total downward force of PA in the left tube. The surrounding atmosphere exerts a total force of $P_aA$ downward on the mercury in the right tube. If these forces are not equal, the mercury will move upward in which ever tube has the smaller pressure (the left tube in the figure) until the net force on the mercury is zero. Here we have two cases for the pressure difference in the two tubes. For the first case given (left tube has higher Hg level) we have for our summation of the forces: $PA + m_{Hg}g = P_a A \rightarrow P = P_a - \rho_{Hg}gh$. For the second case given (right tube has higher Hg level) we have for our summation of the forces: $PA = P_a A + m_{Hg}g \rightarrow P = P_a + \rho_{Hg}gh$. Depending on which tube has the higher level of Hg, we can use the difference in height, h, to determine the pressure in the thermometer.

b. The thermometer shown in the problem has several advantages over the one discussed in the text and in class. First any friction between the piston and cylinder walls is difficult to calculate making the pressure measurement hard for the in-chapter thermometer. (“Frictionless” pistons are difficult to come by.) Second it may be difficult to find a fine enough mass set in order to determine the exact weight needed to calculate the pressure. The mercury thermometer is easy to build in practice and the pressure in the tubes is simply found from the difference in heights of the column of mercury. Both systems suffer from the fact that both volumes will not be constant if the materials that hold the gas and mercury expand with increasing temperatures.

**T1A.1**

Supposing that z = 0 is the earth’s surface, and the density of the atmosphere never goes to zero based on the equation given. Consider the top of the atmosphere to be located at z = infinity. The change in pressure is given by $P_\infty - P_0 = \int_0^\infty - \rho(z)dz = -g\rho_o \int_0^\infty e^{-\frac{z}{a}}dz = +g\rho_o a\left(e^{-\frac{z}{a}}\right)_0^\infty = -g\rho_o a$. Taking the pressure at infinity to be zero and we noting that $P_o$ = 101.3κΠα ανδ $\rho_o$ = 1.2 kg/m$^3$ we find *a* = 8600m, or the height at which the density falls to *1/e* of its sea-level value.