Physics 123 Homework Solutions

Week #4 Unit T Thermodynamics

T3B.2

If the volume change is sufficiently small then the gas pressure change will be constant and is given as P = 95kPa. A volume change of 1% corresponds to $3cm^3$ and $\Delta V = V_f - V_i = 297cm^3 - 300cm^3 = -3cm^3$. Thus the work done in an isobaric compression is given by $W = -P\Delta V = (-95kPa)*(-3cm^3) = 0.29J$. This result is consistent with the idea that a positive amount of work done means energy flowed into the system.

T3B.6

If the volume of the cylinder is $600cm^3$ initially and $450cm^3$ finally, while the initial pressure $P_i = 80kPa$, we have for an isothermal process $P_f V_f = Nk_B T = P_i V_i \rightarrow P_f = \frac{P_i V_i}{V_f} = 80kPa\left(\frac{600cm^3}{450cm^3}\right) = 107kPa$

T3B.7

Let the initial volume and pressure of the cylinder be $600cm^3$ and 60kPa respectively, while the final volume is $450cm^3$. For monatomic gas, the adiabatic index $\gamma = 1.67$, so the final pressure is given $P_f V_f^\gamma = P_i V_i^\gamma \rightarrow P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = 60kPa\left(\frac{600cm^3}{450cm^3}\right)^{1.67} = 97kPa$. The pressure has increased as we would have expected in a compression.

T3S.8

Let the initial volume be $800m^3$ and the initial pressure is that of the atmosphere P_i = 1atm. The final pressure is given as $P_f = 0.45P_i$. If the monatomic helium expands adiabatically the final volume is given by $P_f V_f^\gamma = P_i V_i^\gamma \rightarrow V_f = V_i\left(\frac{P_i}{P_f}\right)^{\frac{1}{\gamma}} = 800m^3\left(\frac{P_1}{0.45P_1}\right)^{\frac{1}{1.67}} = 5100m^3$, and the final temperature is given as (starting at say 295K)

$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \rightarrow T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = T_i\left(\frac{P_f}{P_i}\right)^{\frac{\gamma-1}{\lambda}} = 295K\left(\frac{0.45P_1}{P_1}\right)^{\frac{0.67}{1.67}} = 82K$. This seems pretty cold (Nitrogen liquefies at 77K!) The problem is that the expansion is probably not adiabatic over the entire trip the balloon takes to its final altitude. The balloon probably does heat up some.

T3S.10

a. If the pressure and volume are functions of temperature, then applying the product rule to the ideal gas law we find $\frac{dP}{dT}V + P\frac{dV}{dT} = \frac{d}{dT}\left(Nk_BT\right) = Nk_B \rightarrow VdP + PdV = Nk_BdT$.

b. For any infinitesimal process, the first law of thermodynamics implies that $dE_{thermal}$ = dQ + dW. For an adiabatic process, dQ = 0, so $dE_{thermal}$ = dW. For an ideal gas, $E_{thermal} = \frac{f}{2}Nk_BT \rightarrow dE_{thermal} = \frac{f}{2}Nk_BdT = dW$. For an infinitesimal quasistatic expansion or compression dW = -PdV. Substituting this into the above gives $dW = -PdV = -\frac{f}{2}Nk_BdT$.

c. So, using the result in part b, we have that $-\frac{2}{f}PdV = Nk_BdT$, which when plugged into part a becomes $VdP + PdV = Nk_BdT = -\frac{2}{f}PdV \rightarrow 0 = VdP + \gamma PdV$. Dividing by VdV gives $0 = \frac{dP}{dV} + \gamma\frac{P}{V}$.

d. Now, multiply by V^γ we get $0 = \frac{dP}{dV}V^\gamma + \gamma V^\gamma\frac{P}{V} = \frac{dP}{dV}V^\gamma + \gamma V^{\gamma-1}P = \frac{dP}{dV}V^\gamma + P\frac{d\left(V^\gamma\right)}{dV} = \frac{d}{dV}\left(PV^\gamma\right)$ or $\frac{d}{dV}\left(PV^\gamma\right) = 0 \rightarrow PV^\gamma = \text{constant}$.

e. Using the ideal gas law we get $0 = \frac{d}{dV}\left(PVV^{\gamma-1}\right) = \frac{d}{dV}\left(Nk_BTV^{\gamma-1}\right) = Nk_B\frac{d}{dV}\left(TV^{\gamma-1}\right)$ which will be true only if $TV^{\gamma-1}$ = constant.