Physics 123 Homework Solutions

Week #9 Unit T Thermodynamics

T8B.1

For an ideal monatomic gas, $E_{\text{int}ernal} = \frac{3}{2} N k_B T$. Substituting this into equation T8.8 we find $S(T,V,N) = \frac{3}{2} N k_B \ln\left(\frac{8mbV^{\frac{2}{3}} \frac{3}{2} N k_B T}{3h^2 N}\right) - k_B \ln(N!) = \frac{3}{2} N k_B \ln\left(\frac{8mbk_B V^{\frac{2}{3}} T}{2h^2}\right) - k_B \ln(N!)$.

T8B.2

Since the internal energy is proportional to T, doubling the gas's temperature means doubling its internal energy. Therefore the change in the gas's entropy will

$$\Delta S = \frac{3}{2} N k_B \ln\left(\frac{8mb\left(\frac{V}{2}\right)^{\frac{2}{3}} 2E_{\text{int}}}{3h^2 N}\right) - k_B \ln(N!) - \left[\frac{3}{2} N k_B \ln\left(\frac{8mb(V)^{\frac{2}{3}} E_{\text{int}}}{3h^2 N}\right) - k_B \ln(N!)\right]$$

be

$$= \frac{3}{2} N k_B \ln\left(\frac{8mb\left(\frac{V}{2}\right)^{\frac{2}{3}} 2E \Big/ 3h^2 N}{8mb(V)^{\frac{2}{3}} E \Big/ 3h^2 N}\right) = \tfrac{3}{2} N k_B \ln\left(2^{\frac{1}{3}}\right) = \tfrac{1}{2} N k_B \ln(2)$$

T8B.5

According to equation T8.30, the change in entropy of an object that gains or loses heat when no work is involved is given by $\Delta S = mc \ln\left(\frac{T_f}{T_i}\right) = 0.220kg \times 900 \tfrac{J}{kgK} \times \ln\left(\frac{299K}{291K}\right) = 5.4 \tfrac{J}{K}$. This assumes that the block's specific heat doesn't change during the temperature change, a pretty good assumption for a small temperature change.

T8S.7

Imagine that we use a piston to gradually allow the gas to expand to its final volume. As the gas expands against the piston, it will do work, decreasing the thermal energy of the system. We want the final energy of the gas to be the same as it originally was, so we need to add heat energy to replace the work energy lost. We can do this in an easy way by purring the gas in thermal contact with a reservoir at 304K. Since the thermal energy depends on T and N, but not on V, keeping the gas's temperature fixed will automatically add whatever heat is needed to keep its internal energy fixed. Thus the heat added in this process is equal to the work that the gas does as it expands isothermally. This is given as $\Delta Q = -W = -\int PdV = Nk_B T \ln\left(\frac{V_f}{V_i}\right)$. Therefore ΔS is given

as $\Delta S = \frac{\Delta Q}{T} = \frac{Nk_B T \ln\left(\frac{V_f}{V_i}\right)}{T} = Nk_B \ln\left(\frac{V_f}{V_i}\right) = nR\ln = 0.4mol \times 8.31\,\tfrac{J}{Kmol}\ln 3 = 3.7\,\tfrac{J}{K}$

T8S.10

Let us assume that the bullet's entire kinetic energy is transferred to the aluminum block as thermal energy. Let's also assume that the block is initially at room temperature (295K). Sine the process described in the problem is irreversible; we know that the entropy of the block must have increased. One replacement process that takes us to the same final macrostate is the following. Imagine that we drill a hole in the block and place the bullet in the hole, adjust the temperature to 295K and then simply add an amount of heat to the block that is equal in magnitude to the kinetic energy of the bullet in this situation. No work is done in the process. Since the block is so massive relative to the bullet, it is a credible assumption that the final temperature of the block will not be very different from its initial temperature. Thus $T_i \sim T_f$. If this is so, then we can use equation T8.19 to calculate the entropy change of the block. The thermal energy added to the block in this process is the kinetic energy of the bullet just before it embeds itself in the block. Thus $\Delta S = \frac{\Delta Q_{blick}}{T} = \frac{KE_{bullet}}{T} = \frac{\frac{1}{2}m_{bullet}v_{bullet}^2}{T} = \frac{\frac{1}{2}\times 0.003kg \times 420\,\frac{m}{s}}{295K} = 0.90\,\tfrac{J}{K}$.