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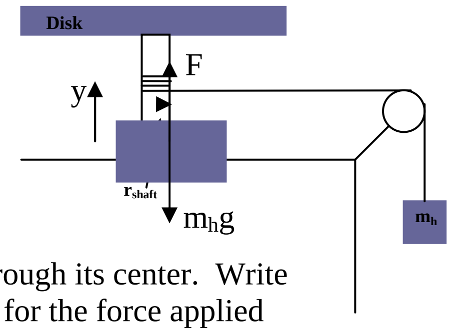
Physics 120 Quiz #6, March 2, 2007

Below is a moment of inertia device which consists of a shaft of radius $r_{shaft} = 0.5cm$ around which a massless string is wound and a disk of moment of inertial I is placed. This string is passed over a massless pulley where a mass m_h is hung and allowed to fall from rest through a height $h = 1m$ acquiring a speed of $v = 1.84m/s$.



a. Draw a carefully labeled free body diagram for the hanging mass and solve for the tension force in the string.

$$F_T - m_h g = -m_h a \rightarrow F_T = m_h(g-a)$$

b. The hanging mass causes the disk to rotate about an axis through its center. Write an equation for the torque produced and solve this equation for the force applied to the disk and shaft.

$$\tau = I\alpha = r_{shaft}F_T \rightarrow F_T = \frac{I\alpha}{r_{shaft}}$$

c. Using the fact that the acceleration of the hanging mass is proportional to the angular acceleration of the shaft, what is the ***expression*** for the moment of inertia of the disk using parts b and c above?

$$F_T = m_h(g-a) = \frac{I\alpha}{r_{shaft}} = \frac{Ia}{r_{shaft}^2}, \text{ Since } a = r_{shaft}\alpha \rightarrow \alpha = \frac{a}{r_{shaft}}.$$

Therefore, $I = m_h r_{shaft}^2\left(\frac{g}{a} - 1\right)$

d. If a mass of $200g$ was suspended and allowed to fall what is the value for the moment of inertia of the disk, I?

The mass falls from rest through a height h. Its acceleration is given by $v_f^2 = 2ah \rightarrow a = \frac{v_f^2}{2h} = \frac{(1.84\frac{m}{s})^2}{2\times 1m} = 1.69\frac{m}{s^2}$. Therefore its moment of inertia is $I = m_h r_{shaft}^2\left(\frac{g}{a} - 1\right) = 0.20kg\times(0.005m)^2\left(\frac{9.8\frac{m}{s^2}}{1.69\frac{m}{s^2}} - 1\right) = 2.3\times 10^{-5}kgm^2$.

2. A grandfather clock depends on the period of a pendulum to keep correct time. Suppose that a grandfather clock is calibrated correctly and then a mischievous child slides the bob of the pendulum downward on the oscillating rod. This causes the grandfather clock to run

a. slower. b. faster. c. correctly. d. in a manner that cannot be determined.

Motion in the x-direction

$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$

$v_{fx} = v_{ix} + a_x t$

$v_{fx}^{\ 2} = v_{ix}^{\ 2} + 2a_x \Delta x$

$F_x = ma_x$

Motion in the y-direction

$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$

$v_{fy} = v_{iy} + a_y t$

$v_{fy}^{\ 2} = v_{iy}^{\ 2} + 2a_y \Delta y$

$F_y = ma_y$

Vectors

$\text{magnitude of a vector} = \sqrt{v_x^{\ 2} + v^2{}_y}$

$\text{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8 \frac{m}{s^2}$

$\text{Quadratic Equation}: ax^2 + bx + c = 0; \quad \text{solutions}: x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Forces

$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

$\vec{F}_s = -k\Delta\vec{x}$

$F_c = m\frac{v^2}{r}$

Work – Energy-Momentum

$W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \Delta KE$

$KE = \frac{1}{2}mv^2$

$PE_g = mgy$

$PE_s = \frac{1}{2}kx^2$

$\vec{p} = m\vec{v}$

$\vec{I} = \int \vec{F}dt = \Delta\vec{p}$

Relativity

$L = \frac{L_p}{\gamma}$

$\Delta t = \gamma \Delta t_p$

$p = \gamma mv$

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$E_r = mc^2$

$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$

$KE = (\gamma - 1)mc^2$

Rotational Motion

$s = r\theta; \quad v = r\omega; \quad a_t = r\alpha$

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega_f^2 = \omega_{\mathbf{i}}^2 + 2\alpha\Delta\theta$

$\left|\vec{\tau}\right| = I\alpha = rF\sin\theta = \left|\frac{d\vec{L}}{dt}\right|$

$\vec{L} = I\vec{\omega}$

$KE_{rot} = \frac{1}{2}I\omega^2$

Oscillatory Motion

$\frac{d^2x}{dt^2} = -\omega^2 x \rightarrow x(t) = A\cos(\omega t) \quad \text{or} \quad x(t) = A\sin(\omega t)$

$v(t) = -A\omega\sin(\omega t) \quad \text{or} \quad v(t) = A\omega\cos(\omega t); \quad v_{\max} = \omega A$

$a(t) = -A\omega^2\cos(\omega t) \quad \text{or} \quad a(t) = -A\omega^2\sin(\omega t); \quad a_{\max} = \omega^2 A$

$\omega = 2\pi f; \quad T = \frac{1}{f}; \quad T_{\text{mass on a spring}} = 2\pi\sqrt{\frac{m}{k}}; \quad T_{\text{simple pendulum}} = 2\pi\sqrt{\frac{l}{g}}$