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Physics 120 Quiz #7, March 9, 2007

1. A *50g* object connected to a spring with a force constant of *35 N/m* oscillates on a horizontal frictionless surface with amplitude *4cm*.

a. What is the total energy of the system?

$$E_T = \tfrac{1}{2}kA^2 = \tfrac{1}{2}\left(35\tfrac{N}{m}\right)(0.04m)^2 = 0.028J$$

b. What is the speed of the object when the position is 1cm?

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \rightarrow v = \sqrt{\frac{kA^2}{m}\left(1-\left(\frac{x}{A}\right)^2\right)}$$

$$v = \sqrt{\frac{35\tfrac{N}{m}\times(0.04)^2}{0.05kg}\left(1-\left(\frac{0.01m}{0.04m}\right)^2\right)} = 1.03\tfrac{m}{s}$$

c. What is the kinetic energy when the position is 3 cm?

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \rightarrow v = \sqrt{\frac{kA^2}{m}\left(1-\left(\frac{x}{A}\right)^2\right)}$$

$$v = \sqrt{\frac{35\tfrac{N}{m}\times(0.04)^2}{0.05kg}\left(1-\left(\frac{0.03m}{0.04m}\right)^2\right)} = 0.7\tfrac{m}{s}$$

$$\therefore KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.05kg)\left(0.7\tfrac{m}{s}\right)^2 = 0.012J$$

d. What is the potential energy when the position is 3cm?

$$U_s = \tfrac{1}{2}kx^2 = \tfrac{1}{2}\left(35\tfrac{N}{m}\right)(0.03m)^2 = 0.016J$$

2. A sinusoidal wave of frequency *f* is traveling along a stretched string. The string is brought to rest and a second traveling wave of frequency $^3/_2 f$ is established on the same string. The wave speed of the second wave is

a. slower than the first wave by a factor of $^2/_3$.
b. faster than the first wave by a factor of $^2/_3$.
c. the same as that of the first wave.
d. unable to be determined.

Motion in the x-direction

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$
$$v_{fx} = v_{ix} + a_x t$$
$$v_{fx}^{\ 2} = v_{ix}^{\ 2} + 2a_x \Delta x$$
$$F_x = ma_x$$

Motion in the y-direction

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2$$
$$v_{fy} = v_{iy} + a_y t$$
$$v_{fy}^{\ 2} = v_{iy}^{\ 2} + 2a_y \Delta y$$
$$F_y = ma_y$$

Vectors

$$\text{magnitude of a vector} = \sqrt{v_x^{\ 2} + v^2_{\ y}}$$
$$\text{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$$g = 9.8\,\tfrac{m}{s^2}$$
$$\text{Quadratic Equation}: ax^2 + bx + c = 0; \quad \text{solutions}: x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Forces

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$
$$\vec{F}_s = -k\Delta\vec{x}$$
$$F_c = m\frac{v^2}{r}$$

Work – Energy-Momentum

$$W = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \Delta KE$$
$$KE = \tfrac{1}{2}mv^2$$
$$PE_g = mgy$$
$$PE_s = \tfrac{1}{2}kx^2$$
$$\vec{p} = m\vec{v}$$
$$\vec{I} = \int \vec{F}dt = \Delta\vec{p}$$

Relativity

$$L = \frac{L_p}{\gamma}$$
$$\Delta t = \gamma \Delta t_p$$
$$p = \gamma mv$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$E_r = mc^2$$
$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$
$$KE = (\gamma - 1)mc^2$$

Rotational Motion

$$s = r\theta; \quad v = r\omega; \quad a_t = r\alpha$$
$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$
$$\omega_f = \omega_i + \alpha t$$
$$\omega_f^2 = \omega_{\mathbf{i}}^2 + 2\alpha\Delta\theta$$
$$\left|\vec{\tau}\right| = I\alpha = rF\sin\theta = \left|\frac{d\vec{L}}{dt}\right|$$
$$\vec{L} = I\vec{\omega}$$
$$KE_{rot} = \tfrac{1}{2}I\omega^2$$

Oscillatory Motion

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 y; \quad \frac{\partial^2 \theta}{\partial t^2} = -\omega^2\theta$$
$$\frac{d^2x}{dt^2} = -\omega^2 x \rightarrow x(t) = A\cos(\omega t) \text{ or } x(t) = A\sin(\omega t)$$
$$v(t) = -A\omega\sin(\omega t) \text{ or } v(t) = A\omega\cos(\omega t); \quad v_{\max} = \omega A$$
$$a(t) = -A\omega^2\cos(\omega t) \text{ or } a(t) = -A\omega^2\sin(\omega t); \quad a_{\max} = \omega^2 A$$
$$\omega = 2\pi f; \quad T = \frac{1}{f}; \quad T_{\text{mass on a spring}} = 2\pi\sqrt{\frac{m}{k}}; \quad T_{\text{simple pendulum}} = 2\pi\sqrt{\frac{l}{g}}$$

Simple Harmonic Motion/Mechanical Waves

$$y = A\sin(kx - \omega t + \phi)$$
$$v = \frac{\omega}{k} = f\lambda = \sqrt{\frac{F_T}{\mu}}; \quad k = \frac{2\pi}{\lambda}$$
$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \tfrac{\phi}{2}\right)$$
$$y = 2A\cos(\omega t)\sin(kx)$$
$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L}v$$