Physics 111
Fall 2007
Radioactive Decay Problems

1. The ${}^{3}_{1}\mathrm{H}$ isotope of hydrogen, which is called *tritium* (because it contains three nucleons), has a half-life of 12.33 yr. It can be used to measure the age of objects up to about 100 yr. It is produced in the upper atmosphere by cosmic rays and brought to Earth by rain. As an application, determine approximately the age of a bottle of wine whose ${}^{3}_{1}\mathrm{H}$ radiation is about $\frac{1}{10}$ that present in new wine.

2. Strontium-90 is produced as a nuclear fission product of uranium in both reactors and atomic bombs. Look at its location in the periodic table to see what other elements it might be similar to chemically, and tell why you think it might be dangerous to ingest. It has too many neutrons, and it decays with a half-life of about 29 yr. How long will we have to wait for the amount of ${}^{90}_{38}\mathrm{Sr}$ on the Earth's surface to reach 1% of its current level, assuming no new material is scattered about? Write down the decay reaction, including the daughter nucleus. The daughter is radioactive: write down its decay.

3. An old wooden tool is found to contain only 6.0% of ${}^{14}_{6}\mathrm{C}$ that a sample of fresh wood would. How old is the tool?

4. An amateur archeologist finds a bone that she believes to be from a dinosaur and she sends a chip off to a laboratory for 14C dating. The lab finds that the chip contains 5g of carbon and has an activity of 0.5 Bq. How old is the bone and could it be from a dinosaur?

5. Calculate the decay energy for the β^- decay of ^{24}Na given the following data: m(^{24}Na) = 23.98492 u, m(^{24}Mg) = 23.97845 u, m(^{24}Ne) = 23.98812 u, m(β^-) = 5.49×10^{-4} u. What is the range of the possible energies of the emitted beta particle?

6. Show that in alpha decay from a stationary parent nuclide that the conservation of energy and momentum lead to a relation between the decay energy for the nuclear reaction and the kinetic energy gained by the alpha particle, KE, given by $Q = KE\left(1+\frac{m\left({}^{4}_{2}He\right)}{m(daughter)}\right)$. What is the kinetic energy of the alpha particle emitted in the decay of ^{238}U?

7. Calculate the binding energies of radium-226 (m = 225.97709 u), radium-228 (m = 227.98275 u), and thorium-232 (m = 231.98864 u).

8. What is the activity of 1gram of radium-226?

9. Strontium is chemically similar to calcium and can replace calcium in bones. The radiation from ^{90}Sr can damage bone marrow where blood cells are produced, and lead to serious health problems. How long would it take for all but 0.01% of a sample of ^{90}Sr to decay?

10. After the sudden release of radioactivity from the Chernobyl nuclear reactor accident in 1986, the radioactivity of milk in Poland rose to 2 000 Bq/L due to iodine-131 present in the grass eaten by dairy cattle. Radioactive iodine, with half-life 8.04 days, is particularly hazardous because the thyroid gland concentrates iodine. The Chernobyl accident caused a measurable increase in thyroid cancers among children in Belarus. (a) For comparison, find the activity of milk due to potassium. Assume that one liter of milk contains 2.00 g of potassium, of which 0.011 7% is the isotope ^{40}K with half-life 1.28×10^9 yr. (b) After what time interval would the activity due to iodine fall below that due to potassium?

11. A small building has become accidentally contaminated with radioactivity. The longest-lived material in the building is strontium-90. ($^{90}_{38}\mathrm{Sr}$ has an atomic mass 89.907 7 u, and its half-life is 29.1 yr. It is particularly dangerous because it substitutes for calcium in bones.) Assume that the building initially contained 5.00 kg of this substance uniformly distributed throughout the building and that the safe level is defined as less than 10.0 decays/min (to be small in comparison to background radiation). How long will the building be unsafe?

12. Natural uranium must be processed to produce uranium enriched in ^{235}U for bombs and power plants. The processing yields a large quantity of nearly pure ^{238}U as a by-product, called "depleted uranium." Because of its high mass density, it is used in armor-piercing artillery shells. (a) Find the edge dimension of a 70.0-kg cube of ^{238}U. The density of uranium is 18.7×10^3 kg/m^3. (b) The isotope ^{238}U has a long half-life of 4.47×10^9 yr. As soon as one nucleus decays, it begins a relatively rapid series of 14 steps that together constitute the net reaction

$$^{238}_{92}\mathrm{U} \rightarrow 8(^{4}_{2}\mathrm{He}) + 6(^{0}_{-1}\mathrm{e}) + {}^{206}_{82}\mathrm{Pb} + 6\bar{\nu} + Q_{\text{net}}$$

Find the net decay energy. (Refer to Table A.3.) (c) Argue that a radioactive sample with decay rate R and decay energy Q has power output $\mathcal{P} = QR$. (d) Consider an artillery shell with a jacket of 70.0 kg of ^{238}U. Find its power output due to the radioactivity of the uranium and its daughters. Assume that the shell is old enough that the daughters have reached steady-state amounts. Express the power in joules per year. (e) A 17-year-old soldier of mass 70.0 kg works in an arsenal where many such artillery shells are stored. Assume that his radiation exposure is limited to absorbing 45.5 mJ per year per kilogram of body mass. Find the net rate at which he can absorb energy of radiation, in joules per year.

13. A sealed capsule containing the radiopharmaceutical phosphorus-32 ($^{32}_{15}\mathrm{P}$), an e^- emitter, is implanted into a patient's tumor. The average kinetic energy of the beta particles is 700 keV. The initial activity is 5.22 MBq. Determine the energy absorbed during a 10.0-day period. Assume that the beta particles are completely absorbed within the tumor.

14. To destroy a cancerous tumor, a dose of gamma radiation totaling an energy of 2.12 J is to be delivered in 30.0 days from implanted sealed capsules containing palladium-103. Assume that this isotope has half-life 17.0 d and emits gamma rays of energy 21.0 keV, which are entirely absorbed within the tumor. (a) Find the initial activity of the set of capsules. (b) Find the total mass of radioactive palladium that these "seeds" should contain.

15. A living specimen in equilibrium with the atmosphere contains one atom of ^{14}C (half-life = 5 730 yr) for every 7.7×10^{11} stable carbon atoms. An archeological sample of wood (cellulose, $C_{12}H_{22}O_{11}$) contains 21.0 mg of carbon. When the sample is placed inside a shielded beta counter with 88.0% counting efficiency, 837 counts are accumulated in one week. Assuming that the cosmic-ray flux and the Earth's atmosphere have not changed appreciably since the sample was formed, find the age of the sample.

16. In an experiment on the transport of nutrients in the root structure of a plant, two radioactive nuclides X and Y are used. Initially 2.50 times more nuclei of type X are present than of type Y. Just three days later there are 4.20 times more nuclei of type X than of type Y. Isotope Y has a half-life of 1.60 d. What is the half-life of isotope X?