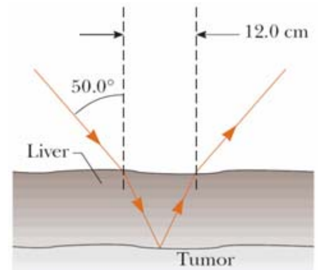
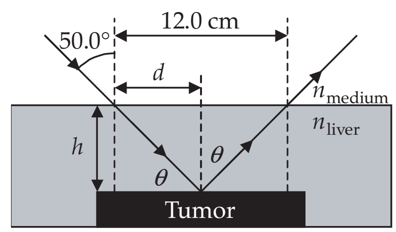
Physics 111
Fall 2007
Reflection, Refraction and Optical Instruments

1. A narrow beam of ultrasonic waves reflects off the liver tumor shown on the right. The speed of the wave is 10.0% less in the liver than in the surrounding medium. Determine the depth of the tumor.



From Snell's law, $\sin\theta = \left(\frac{n_{\text{medium}}}{n_{\text{liver}}}\right)\sin 50.0°$



But,

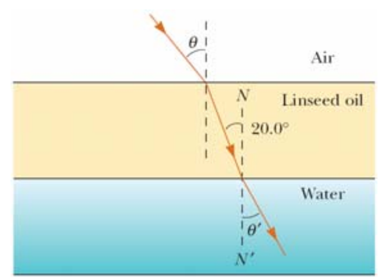
$$\frac{n_{\text{medium}}}{n_{\text{liver}}} = \frac{c/v_{\text{medium}}}{c/v_{\text{liver}}} = \frac{v_{\text{liver}}}{v_{\text{medium}}} = 0.900\ ,$$

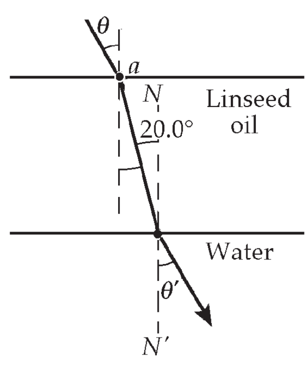
so, $\theta = \sin^{-1}\left[(0.900)\sin 50.0°\right] = 43.6°$.

From the law of reflection,

$$d = \frac{12.0\text{ cm}}{2} = 6.00\text{ cm}\ , \text{ and } h = \frac{d}{\tan\theta} = \frac{6.00\text{ cm}}{\tan(43.6°)} = \boxed{6.30\text{ cm}}$$

2. The light beam shown in the figure on the right makes an angle of 20.0° with the normal line *NN*' in the linseed oil. Determine the angles θ and θ'. (*Note:* The index of refraction of linseed oil is 1.48.)





Applying Snell's law at the air-oil interface,

$n_{\text{air}}\sin\theta = n_{\text{oil}}\sin 20.0°$ yields $\boxed{\theta = 30.4°}$.

Applying Snell's law at the oil-water interface

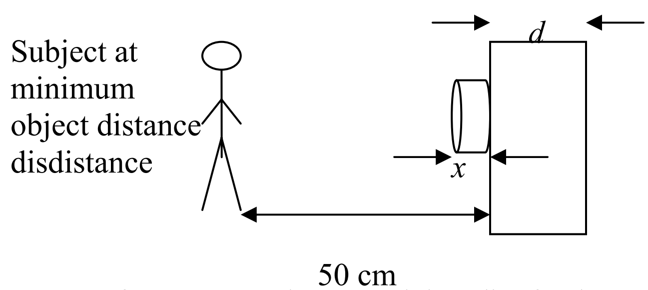
$n_w \sin\theta' = n_{\text{oil}}\sin 20.0°$ yields $\boxed{\theta' = 22.3°}$.

3. Consider a convex lens of focal length 20 cm. Calculate the image distance for each of the following object distances: ∞, 4m, 2 m, 1m, 80 cm, 60 cm, 40 cm, 20 cm.

Here we use the thin lens equation to calculate the image distances

object distance (m)	image distance (m)
infinity	0.20
4	0.21
2	0.22
1	0.25
0.8	0.27
0.6	0.30
0.4	0.40
0.2	infinity

4. A camera has a lens with adjustable position. The camera depth $d = 4$ cm. Determine the focal length of the lens and the necessary allowable extension of the lens, x, in order that the camera be able to take sharp photographs of objects positioned anywhere from 50 cm to infinity, measured from the front surface of the camera body.



To focus on objects very far away, use do = ∞ and then di = f = d = 4 cm. So the camera is designed to focus at infinity with no extension of the lens. Then to focus on an object at do = 50 cm, we need $\frac{1}{50} + \frac{1}{4+x} = \frac{1}{4}$, where we have used that f = 4 cm and the image is now located at 4 + x. Solving for x, we find x = 0.35 cm.

a. The focal length is given by the thin lens equation
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.36m} + \frac{1}{4.5m} \rightarrow f = 0.33m\ .$$

b. The velocity of the creature is magnified by the lens. Thus the magnification is $M = \frac{-d_i}{d_0} = \frac{-4.5}{0.36} = -12.5$. Thus the magnitude of the velocity on the screen is magnified by this same factor. In the dish the velocity is 1 cm/s therefore the velocity on the screen is 12.5cm/s.

5. A movie star catches a reporter shooting pictures of her at home. She claims the reporter was trespassing. To prove her point, she gives as evidence the film she seized. Her 1.75-m height is 8.25 mm high on the film, and the focal length of the camera lens was 210 mm. How far away from the subject was the reporter standing?

We will use two equations: $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$ (1)

and $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$. (2)

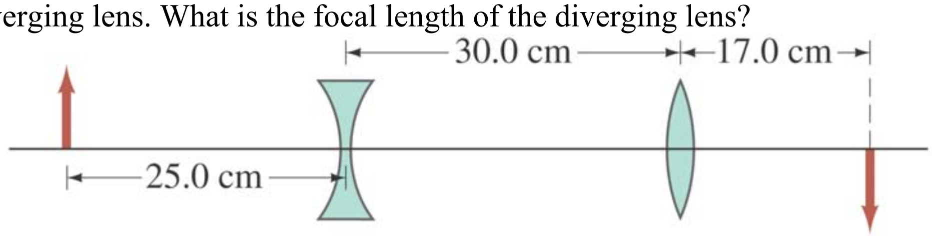
From (1), $-\frac{1}{m} = -\frac{h_o}{h_i} = \frac{d_o}{d_i}$, and from (2), $\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$.

Therefore,

$$
\begin{aligned}
-\frac{h_o}{h_i} &= d_o\left(\frac{1}{f} - \frac{1}{d_o}\right) \\
&= \frac{d_o}{f} - 1; \text{ and so} \\
d_o &= f\left(1 - \frac{h_o}{h_i}\right) \\
&= (210 \text{ mm})\left(1 - \frac{1.75 \text{ m}}{-0.00825 \text{ m}}\right) \\
&= 44{,}755 \text{ mm.}
\end{aligned}
$$

The reporter was standing $\boxed{45 \text{ m}}$ from the subject.

6. A small object is 25.0 cm from a diverging lens as shown in the figure below. A converging lens with a focal length of 12.0 cm is 30.0 cm to the right of the diverging lens. The two-lens system forms a real inverted image 17.0 cm to the right of the converging lens. What is the focal length of the diverging lens?



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Working backwards, we use

$$\frac{1}{d_{o2}} + \frac{1}{d_{i2}} = \frac{1}{f_2}$$

with $d_{i2} = 17.0 \text{ cm}$ and $f_2 = 12.0 \text{ cm}$:

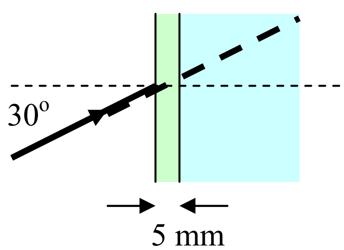
$$\frac{1}{d_{o2}} + \frac{1}{17.0 \text{ cm}} = \frac{1}{12.0 \text{ cm}} \text{ gives } d_{o2} = 40.8 \text{ cm.}$$

This means that $d_{i1} = 30.0 \text{ cm} - 40.8 \text{ cm} = -10.8 \text{ cm}$ (10.8 to the left of the diverging lens). So for the diverging lens,

$$\left[\frac{1}{d_{o1}} + \frac{1}{d_{i1}} = \frac{1}{f_1};\right]$$

$$\frac{1}{25.0 \text{ cm}} + \frac{1}{-10.8 \text{ cm}} = \frac{1}{f_1}, \text{ which gives } f_1 = \boxed{19 \text{cm.}}$$

7. A narrow pencil of light strikes the side of a rectangular fish tank at an angle of 30^o below the horizontal as shown.
   (a) What angle does the light ray make with the horizontal in the glass, assuming a 1.55 index of refraction?
   (b) What angle does it make in the water?
   (c) If the glass wall is 5 mm thick, by what distance is the exit spot inside the glass wall displaced from the location at which the incident beam is aimed?



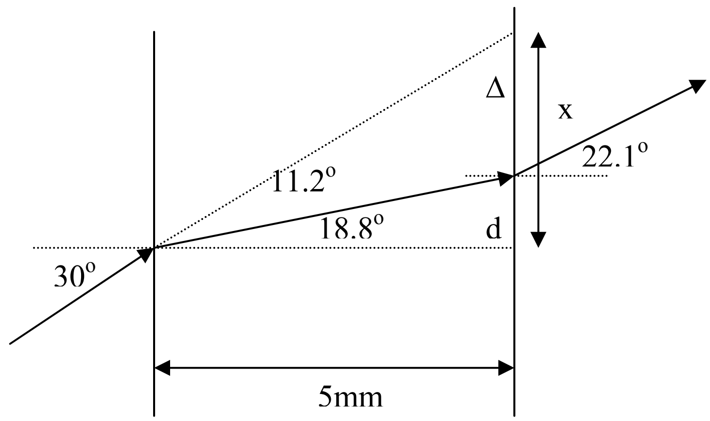
a. Assuming that the index of refraction is 1.55, the angle of refraction is given for the light ray going from air into glass as:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\sin(30) = 1.55 \sin\theta_2$$
$$\theta_2 = 18.8^o$$

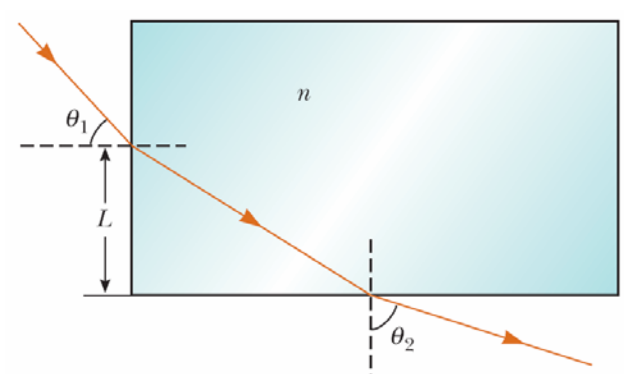
b. As the ray passes through the glass it will eventually strike the interface between the glass and the water at 18.8^o. For water the index of refraction is 1.33 and the angle of refraction in the water is given as:

$$n_2 \sin\theta_2 = n_3 \sin\theta_3$$
$$1.55 \sin(18.8) = 1.33 \sin\theta_3$$
$$\theta_3 = 22.1^o$$

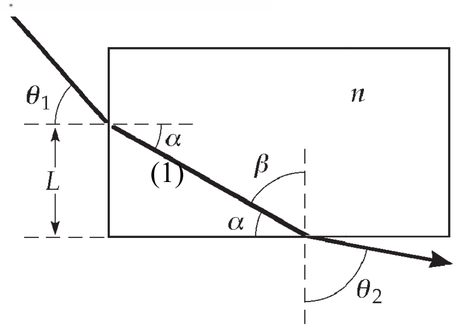
c. From the drawing we can see that x = 5mm tan(30) = 2.88mm and d = 5mm tan (18.8) = 1.70mm so that the difference between where the beam strikes and where it is aimed Δ = 1.19mm



8. A light ray enters a rectangular block of plastic at an angle $\theta_1 = 45.0°$ and emerges at an angle $\theta_2 = 76.0°$ as shown in the figure below.
   (a) Determine the index of refraction of the plastic.
   (b) If the light ray enters the plastic at a point $L = 50.0$ cm from the bottom edge, how long does it take the light ray to travel through the plastic?



(a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$.



Snell's law at the first surface gives

$n \sin \alpha = 1.00 \sin 45.0°$.

Observe that the angle of incidence at the second surface is $\beta = 90.0° - \alpha$.

Thus, Snell's law at the second surface yields

$n \sin \beta = n \sin (90.0° - \alpha) = 1.00 \sin 76.0°$

or $n\cos\alpha = \sin 76.0°$. Dividing Equation (1) by Equation (2),

$$\tan\alpha = \frac{\sin 45.0°}{\sin 76.0°} = 0.729 \text{ or } \qquad \alpha = 36.1°.$$

Then, from Equation (1), $n = \frac{\sin 45.0°}{\sin\alpha} = \frac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$.

(b) From the sketch, observe that the distance the light travels in the plastic is $d = \frac{L}{\sin\alpha}$. Also, the speed of light in the plastic is $v = \frac{c}{n}$, so the time required to travel through the plastic is

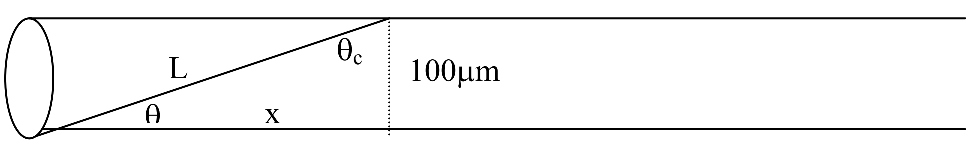
$$\Delta t = \frac{d}{v} = \frac{nL}{c\sin\alpha} = \frac{1.20(0.500 \text{ m})}{(3.00\times10^{8} \text{ m/s})\sin 36.1°} = 3.40\times10^{-9} \text{ s} = \boxed{3.40 \text{ ns}}.$$

9. A major problem with larger diameter fibers is the difference in travel times of rays along a fiber. In traveling a distance *d*, the shortest time is that of the axial beam $t_1 = d/\text{v}$, while the longest time t_2 is that of a ray bouncing back and forth along the fiber just at the critical angle. Compute the time difference between these two rays for a 1.5 index fiber that is 10 km long, surrounded by 1.49 index cladding. This effectively limits the frequency of a signal that can be transmitted without significant degradation in larger diameter fibers. Small diameter (~10 μm diameter) single-mode fibers, in which the light travels as a wave and not as a geometrical ray, overcome this problem.

The time for the ray to travel straight through along the axis of the waveguide is:

$$t_1 = \frac{d}{v} = \frac{10{,}000m}{3\times10^{8}\, {}^{m}\!/\!_{s} / 1.5} = 50\mu s.$$

The time for the ray to bounce off of the upper and lower surfaces as it travels down the waveguide is found by observing the following.



The critical angle is given as: $\theta_c = \sin^{-1}(\frac{1.49}{1.50}) = 83.4^o$, and the angle with respect to the axis of the guide is θ = 90 – 83.4 = 6.6°. Thus the distance $x = \frac{100\times10^{-6}m}{\tan 6.6} = 8.64\times10^{-4}m = 864\mu m$ and the distance $L = \frac{100\times10^{-6}m}{\sin 6.6} = 8.70\times10^{-4}m = 870\mu m$. The ray bounces off of a surface in a distance of 864μm and in 10,000m there are $11.57\text{x}10^6$ bounces. Thus the ray travels a total zig-zag distance along L of 11.57x10^6 bounces times 870μm per bounce = 10,069m. The total time needed to travel this distance is $t_2 = \frac{d'}{v} = \left(\frac{10{,}069m}{3\times10^8\ {}^m\!/\!_s / 1.5}\right) = 50.3\mu s.$

Therefore the difference between the two times is 0.3μs.

10. (*a*) Show that the lens equation can be written in the *Newtonian form*

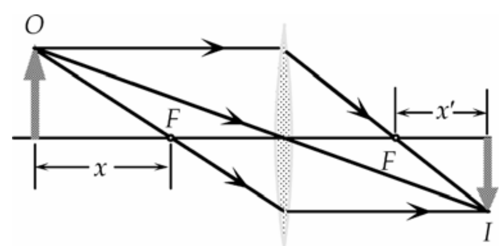
$$xx' = f^2,$$

where *x* is the distance of the object from the focal point on the front side of the lens, and x' is the distance of the image to the focal point on the other side of the lens. Calculate the location of an image if the object is placed 45.0 cm in front of a convex lens with a focal length *f* of 32.0 cm using (*b*) the standard form of the thin lens equation, and (*c*) the Newtonian form, stated above.

(*a*) For the thin lens we have

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left[\frac{1}{(f+x)}\right]+\left[\frac{1}{(f+x')}\right]=\frac{1}{f},$$



which can be written as

$$2f+x+x'=\frac{(f+x)(f+x')}{f}$$

$$=f+(x+x')+\left(\frac{xx'}{f}\right),\ \text{or}\ xx'=f^2.$$

(*b*) For the standard form we have

$$\left(\frac{1}{d_o}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{f};$$

$$\left(\frac{1}{45.0\,\text{cm}}\right)+\left(\frac{1}{d_i}\right)=\frac{1}{32.0\text{ cm}},\ \text{which gives}\ d_i=\boxed{+110.8\text{ cm.}}$$

(*c*) For the Newtonian form we have

$$xx'=f^2;$$

$$(45.0\text{ cm}-32.0\text{ cm})x'=(32.0\text{ cm})^2,\ \text{which gives}\ x'=78.7\,\text{cm}.$$

Thus the distance from the lens is

$$d_i=x'+f=78.7\text{ cm}+32.0\text{ cm}=\boxed{110.8\text{ cm.}}$$

11. You are designing an endoscope for use inside an air-filled body cavity. A lens at the end of the endoscope will form an image covering the end of a bundle of optical fibers. This image will then be carried by the optical fibers to an eyepiece lens at the outside end of the fiberscope. The radius of the bundle is 1.00 mm. The scene within the body that is to appear within the image fills a circle of radius 6.00 cm. The lens will be located 5.00 cm from the tissues you wish to observe.
   (a) How far should the lens be located from the end of an optical fiber bundle?
   (b) What is the focal length of the lens required?

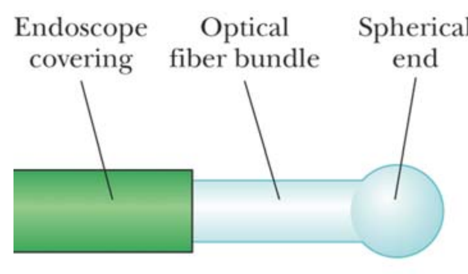
The image will be inverted. With $h = 6\text{ cm}$, we require $h' = -1\text{ mm}$.

(a) $$M = \frac{h'}{h} = -\frac{q}{p}$$

$$q = -p\frac{h'}{h} = -50\text{ mm}\frac{(-1\text{ mm})}{60\text{ mm}} = \boxed{0.833\text{ mm}}$$

(b) $$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{50\text{ mm}} + \frac{1}{0.833\text{ mm}} \qquad f = \boxed{0.820\text{ mm}}$$

12. Consider the endoscope probe used for treating hydrocephalus and shown in the figure on the right. The spherical end, with refractive index 1.50, is attached to an optical fiber bundle of radius 1.00 mm, which is smaller than the radius of the sphere. The center of the spherical end is on the central axis of the bundle. Consider laser light that travels precisely parallel to the central axis of the bundle and then refracts out from the surface of the sphere into air.

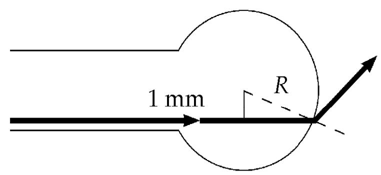


   (a) In the figure, does light that refracts out of the sphere and travels toward the upper right come from the top half of the sphere or from the bottom half of the sphere?
   (b) If laser light that travels along the edge of the optical fiber bundle refracts out of the sphere tangent to the surface of the sphere, what is the radius of the sphere?
   (c) Find the angle of deviation of the ray considered in part (b), that is, the angle by which its direction changes as it leaves the sphere.
   (d) Show that the ray considered in part (b) has a greater angle of deviation than any other ray. Show that the light from all parts of the optical fiber bundle does not refract out of the sphere with spherical symmetry, but rather fills a cone around the forward direction. Find the angular diameter of the cone.

(e) In reality, however, laser light can diverge from the sphere with approximate spherical symmetry. What considerations that we have not addressed will lead to this approximate spherical symmetry in practice?

(a) Light leaving the sphere refracts away from the normal, so light that travels toward the upper right comes from the $\boxed{\text{bottom half}}$ of the sphere.

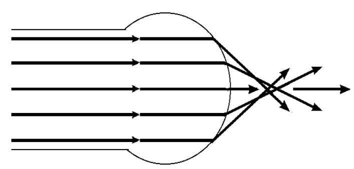
(b) $\sin\theta_1 = \dfrac{1\text{ mm}}{R}$ $\qquad$ $n_1 \sin\theta_1 = n_2 \sin\theta_2$



$$1.50\left(\frac{1\text{ mm}}{R}\right) = 1\sin 90° \qquad R = \boxed{1.50\text{ mm}}$$

(c) $\delta = |\theta_2 - \theta_1| = \left|90° - \sin^{-1}\left(\dfrac{1}{1.5}\right)\right| = \boxed{48.2°}$

(d) No ray can have an angle of refraction larger than 90°, so the ray considered in parts (b) and (c) has the largest possible angle of refraction and then the largest possible deviation.



All other rays, at distances from the axis of less than 1 mm, will leave the sphere at smaller angles with the axis than 48.2°. The angular diameter of the cone of diverging light is $2 \times 48.2° = \boxed{96.4°}$.

(e) Light can be absorbed by a coating on the sphere and reradiated in any direction. The sphere deviates from a perfectly spherical shape, probably macroscopically and surely at the scale of the wavelength of light. Light rays enter the sphere along directions not parallel to the axis of the fiber. Inhomogeneities within the sphere scatter light. Light reflects from the interior surface of the sphere.