Union College *Winter 2022*

Physics 120
Rotational Dynamics

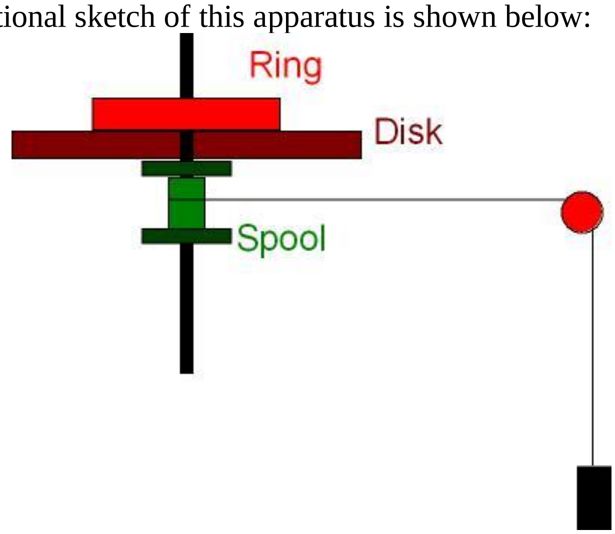
Introduction:

In this laboratory experiment we will investigate several aspects of rotational dynamics by examining torque and rotational energy considerations. In the first part of our experiment we determine the value of the rotational inertia of a solid ring and compare it with the value that we calculate using a theoretically derived expression. In the second part of the experiment we determine the speed of a hanging mass connected to our rotational apparatus by applying conservation of energy relationships for spinning objects.

The primary purpose of this lab is to become more familiar with rotational dynamics and to examine how the rotational and translational variables are related to each other. As a result, we will spend most of our attention on theoretical derivations of the relationships that we will use.

Apparatus:

The experimental apparatus that we use in this experiment is manufactured by Pasco Scientific Company (www.pasco.com). A cross sectional sketch of this apparatus is shown below:



A hanging mass, m_h, is connected to the rotational apparatus by a string that passes over a Smart Pulley. The pulley is connected to our Capstone data collection system and allows us to record the speed, v, of the falling mass m_h (i.e. the tangential speed of the rim of the pulley) as a function of time. The other end of the string is wound around a spool, of radius r_{shaft}, that is coaxial with the disk of the rotational apparatus. So, as the mass m_h falls, it causes the disk to spin. A ring of radius R can be placed on the disk in a groove so that it also is coaxial with the apparatus.

Theory/Pre-Lab Exercises: Derive expressions for the moment of inertia (and uncertainty) in each of the following cases:

1. ***Torque and acceleration analysis*** (consider the mass of the pulley to be negligibly small):

   Analyzing the forces and torques in the system, derive an expression for the moment of inertia I of the system in terms of the hanging mass, the acceleration a of the hanging mass m_h , the acceleration due to gravity g and the radius of the shaft r_{shaft} .

   From the expression you derived for the moment of inertia, derive an expression for the uncertainty in the moment of inertia ΔI.

   The expressions that you determine will be called the moment of inertia (and its uncertainty) by the torque balance method.

2. ***Energy Considerations***:
   Analyzing the energy changes in the system, derive an expression for the moment of inertia I of the system in terms of the hanging mass m_h, the speed v of the hanging mass m_h after it has fallen a distance h, the height h, the acceleration due to gravity g, and the radius of the shaft r_{shaft} .

   From the expression you derived for the moment of inertia, derive an expression for the uncertainty in the moment of inertia ΔI.

   The expressions that you determine will be called the moment of inertia (and its uncertainty) by the energy balance method.

3. ***The Definition of Moment of Inertia***
   From the definition of moment of inertia, derive an expression for the moment of inertia of the system.

   From the expression you derived for the moment of inertia, derive an expression for the uncertainty in the moment of inertia ΔI.

   This expression that you determine will be called the theoretical moment of inertia (and its uncertainty).

Procedure and analysis:

1. First make sure that the Smart Pulley is at the right height so that the string winding around the spool is perfectly horizontal (why?).
2. Keep a tension in the string and rotate the disk manually to wind the string around the spool. Again, take care for the string to remain horizontal throughout the winding process.
3. Open and configure Capstone according to: Smart Pulley and pick Graph (velocity versus time)
4. Hang your mass m_h and start taking data using the Capstone as the hanging mass m_h is to just hit the floor.
5. Record the height h through which the mass m_h fell.
6. Fit a line to the velocity versus time data in Capstone and record the acceleration in Table 1for m_h.
7. Determine the maximum speed v of the hanging mass and record this in Table 2.
8. Repeat steps 4-7 for all the other hanging masses.
9. Now, place the ring on the disk and, for the same set of hanging masses m_h, repeat steps 4 - 7 above to determine the new acceleration value and speed of m_h. Record your data in Tables 1 & 2.
10. Use a pair of Vernier calipers to measure the diameter, 2r, of the spool. Calculate r and record it in the space provided.
11. Using the two equations you derived from considering torques and energy balances, determine the rotational inertia of the system, once with and once without the ring, for each hanging mass m_h. Subtract these values and calculate the rotational inertia of the ring for each trial of the hanging mass m_h.
12. Measure the mass, M, of the ring, its inner and outer radii, R_{inner} and R_{outer}, and calculate its rotational inertia using the expression you derived for the theoretical moment of inertia. Record this in Table 3.
13. Repeat steps 4 – 7 for the piece of art located in the room. Do this one time for a hanging mass of 0.250kg.

Table 1

m(kg)	**a ( ) disk only**	**I ( ) disk only**	**a ( ) disk and ring**	**I ( ) disk and ring**	**I ( ) ring**
0.050					
0.100					
0.200					
0.300					
0.500					

You should have five results for the moment of inertia of the ring listed in Table 1. Calculate the average value of the moment of inertia of the ring. $I_{ring,avg}$ = ___________________________

Determine the uncertainty in the moment of inertia of the ring. $\Delta I_{ring,avg}$ = ___________________________

Table 2

m(kg)	h (m)	v ( ) disk only	I ( ) disk only	v ( ) disk and ring	I ( ) disk and ring	I ( ) ring
0.050						
0.100						
0.200						
0.300						
0.500						

You should have five results for the moment of inertia of the ring listed in Table 2. Calculate the average value of the moment of inertia of the ring. I_{ring} = ___________________________

Determine the uncertainty in the moment of inertia of the ring. ΔI_{ring} = ___________________________

Radius of Spool: r = _________________

Mass of ring: M = ________________

Inner radius of ring: R_{inner}=_______________ Outer radius of ring: R_{outer}=_______________

From table 1, enter your value of $I_{ring} = I_{ring,avg} \pm \Delta I_{ring,avg}$ in the torque balance method in table 3.

From table 2, enter your value of $I_{ring} = I_{ring,avg} \pm \Delta I_{ring,avg}$ in the energy balance method in table 3.

From the definition of moment of inertia, derive an expression for the moment of inertia of a ring and then evaluate your expression (with uncertainty). Enter this value into the theory section in table 3.

Determine the moment of inertia (with uncertainty) for the piece of art and enter it in table 3, under Art.

Table 3

Method	I_{ring} ( )
Torque balance	
Theory	
Energy balance	
Art	

Due on Wednesday 3/9/2022: This is an individual assignment and each person should hand in their own copy. Write the derivations and explanations on separate sheets of paper, answer in complete English sentences and answers should be your own work.

Derivations using torques, energy balance and the definition of moment of inertia, the moments of inertia of the system.

Derivations of the uncertainties associated with the moment of inertia from the torques, energy balance, and theoretical derivations.

Data and Tables, 1, 2, & 3.

Calculations using your derived expressions for the moments of inertia for the ring for each method showing the values that you input into the equation and then the numerical answer.

A written paragraph discussion of the results and answers to the following questions. What are you values for the moment of inertia of the ring along with their uncertainties. Which method gives a better way to determine the moment of inertia of an object and why? Of the method that you do not believe gives a better method of determining the moment of inertia, what explicitly is the reason or variable or measurement that makes it fail. What change would make it a better method and one that is comparable to the one you felt was the best method to determine I. How confident are you in your value for the moment of inertial for the piece of art? Why are you or why are you not confident in your measurement? In the problems we've done so far, we've assumed masses add. Thus, for example $m_{total} = m_1 + m_2$. What can you say about moments of inertia? Are they too additive? Justify your answer.