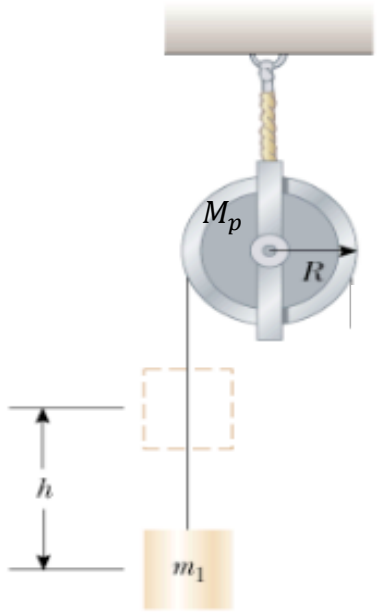
Rotational Dynamics

Example 1:

Consider a pulley of radius $R_P = R$ and mass M_p. A light string is wound around the outer edge of the pulley and a mass m_1 is suspended.

The mass m_1 is released from rest and is allowed to fall through a height of $|\Delta y| = h$.



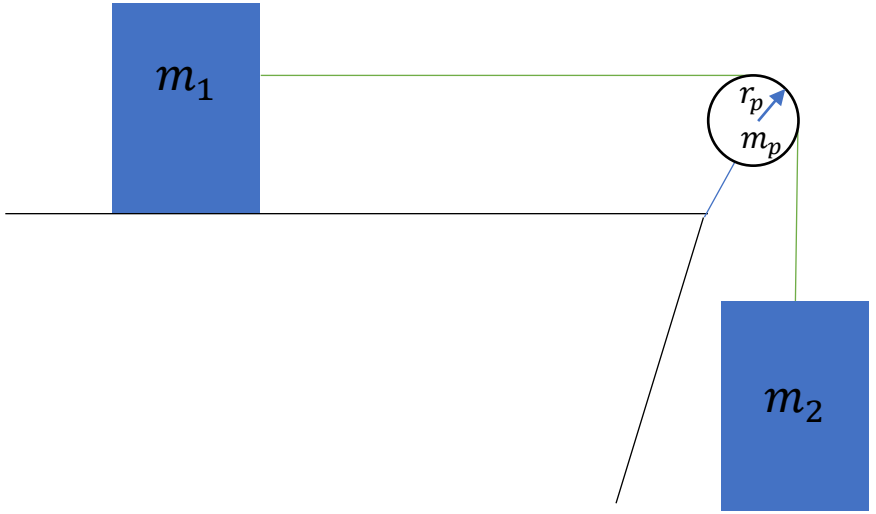
- What is the torque created about the axis of rotation of the pulley due to the falling mass?
- What is the angular acceleration α of the pulley about the axis of rotation due to the falling mass?
- What is the rotational speed ω of the pulley about its center if the mass m_1 has fallen through a height h from rest?
- Through how much angle θ has the pulley spun if m_1 has fallen through a height h from rest?

Rotational Dynamics

Example 2:

Suppose a box of mass $m_1 = 2kg$ is connected to a box of mass $m_2 = 1kg$ by a light rope that is passed over a massive pulley ($m_p = 250g$ and $r_p = 2.5cm$). The system is released from rest and m_2 is allowed to fall through a height of $|\Delta y| = 1m$.

- Using forces/torques what is the translational acceleration of the system. ?
- How long does it take for mass m_1 to move a horizontal distance of $1m$ from rest?
- Using this time, what is the translational speed of mass m_1 and the rotational speed of the pulley?
- Through how many radians did the pulley turn? How many revolutions is this?
- Using the distance moved by mass m_1, what is the is the translational speed of mass m_1 and the rotational speed of the pulley?

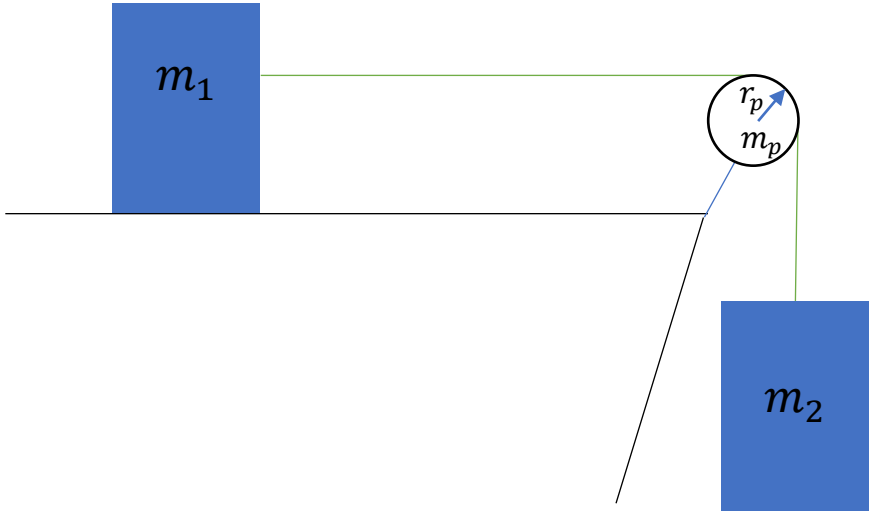


Rotational Dynamics

Example 3:

Suppose a box of mass $m_1 = 2kg$ is connected to a box of mass $m_2 = 1kg$ by a light rope that is passed over a massive pulley ($m_p = 250g$ and $r_p = 2.5cm$). The system is released from rest and m_2 is allowed to fall through a height of $|\Delta y| = 1m$.

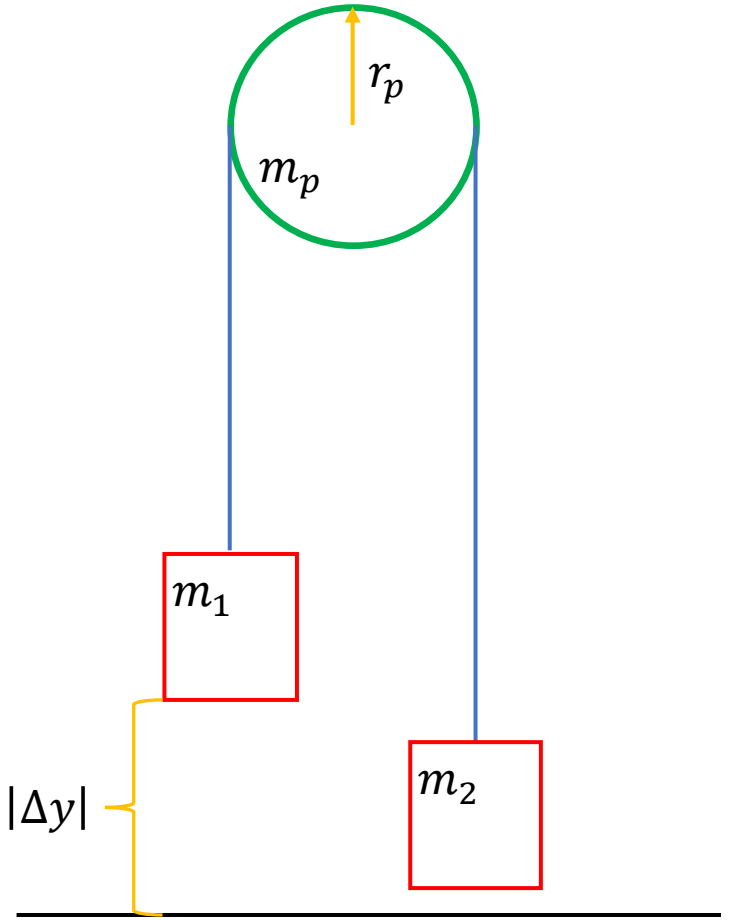
- How much work was done on mass m_1?
- How much work was done on mass m_2?
- How much work did it take to spin the pulley?
- Using the work-kinetic energy theorem, what is the translational speed of mass m_1?
- Using energy ideas what is the translational speed of mass m_1?



Rotational Dynamics

Example 4:

Suppose a box of mass $m_1 = 2kg$ is released from rest and allowed to fall through a height of $|\Delta y|$. Assume m_1 is connected to a mass $m_2 = 1kg$ by a massless rope that passes over a massive pulley with mass $m_p = 0.25kg$ and radius $r_p = 0.1m$. This arrangement of masses is called an Atwood machine.



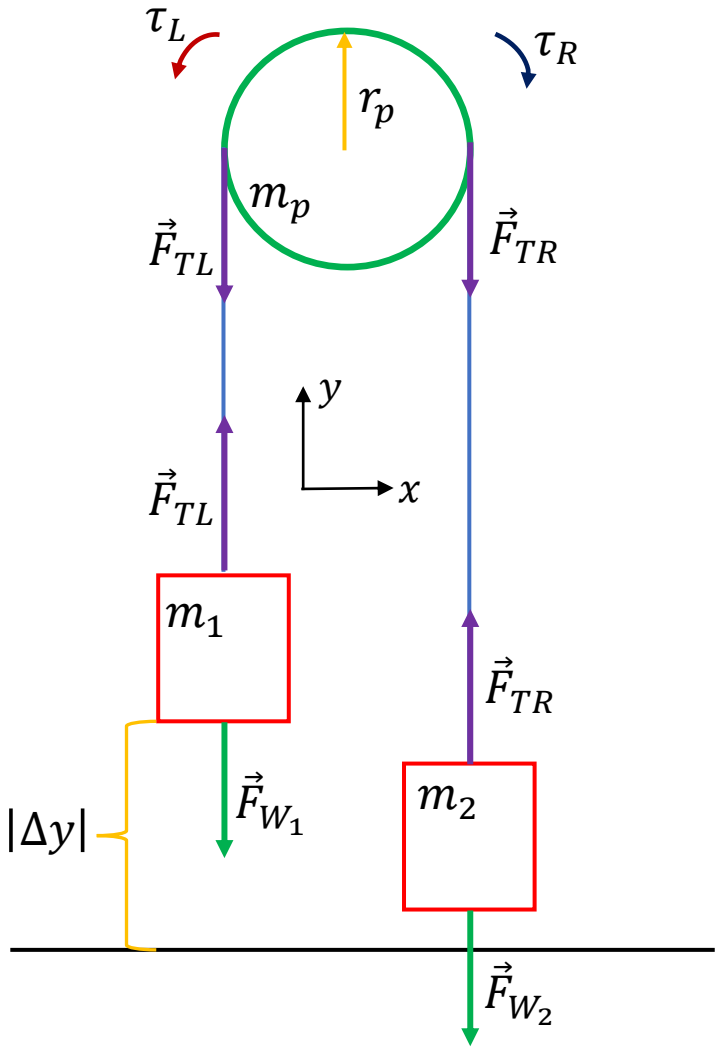
- Using forces/torques, what is the magnitude of the translational acceleration of the system of masses?
- How does this compare this to the case if the pulley was massless. What does this imply about the tension forces on the right and left side of the pulley?
- Using energy ideas, what is the translational speed of the masses m_1 and m_2 after m_1 has fallen by an amount $|\Delta y| = y = 1.0m$, from rest?
- What is the rotational speed ω of the pulley after after m_1 has fallen by an amount $|\Delta y| = y = 1.0m$, from rest?
- Through how much angle $\Delta\theta$ (in radians) has the pulley rotated after m_1 has fallen by an amount $|\Delta y| = y = 1.0m$, from rest?

Rotational Dynamics

Example 5:

Suppose a box of mass $m_1 = 2kg$ is released from rest and allowed to fall through a height of $|\Delta y| = 1.0m$. Assume m_1 is connected to a mass $m_2 = 1kg$ by a massless rope that passes over a massive pulley with mass $m_p = 0.25kg$ and radius $r_p = 0.1m$.

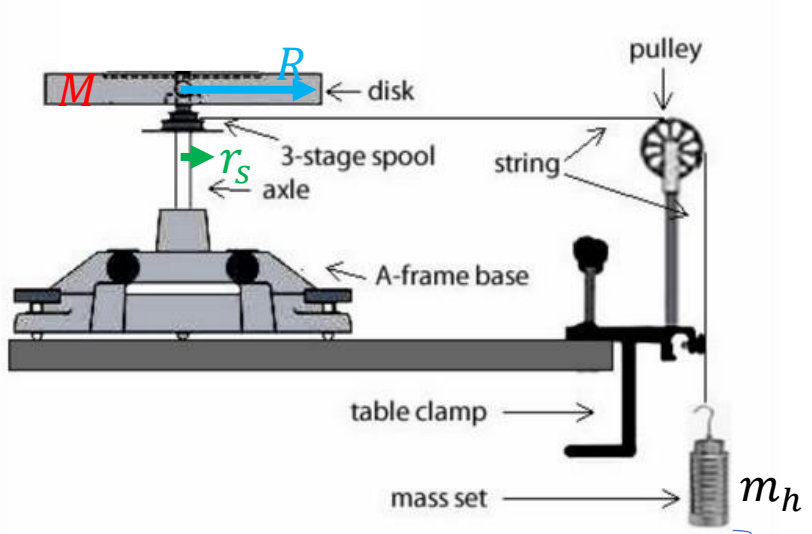
- Using the acceleration calculated in the last problem, how long does it take for mass m_1 to fall through $|\Delta y| = 1.0m$ starting from rest?
- What are the translational speed of the system of masses m_1 and m_2 and the rotational speed of the pulley after this time? Do this two ways.
- How many revolutions has the pulley spun through when mass m_1 has fallen by $|\Delta y| = 1.0m$? Use the time calculated above and do it two ways.



Rotational Dynamics

Example 6:

The expressions that we've been using for the moment of inertia of various objects in solving problems we've taken from Table 7.1 on page 226 of KJF College Physics: A Strategic Approach, Volume 1. These moments of inertia are experimentally determined. To experimentally determine the moment of inertia of an object, we use this apparatus on the right. Here, we'll determine an expression for the moment of inertia of a disk. Consider a disk of radius R and mass M mounted on a frictionless axle. A light string is wound around the three-stage spool and let the spool around which the string is wound has a radius r_s. A set of masses are used to create different torques about the spool. Each separate mass (m_h) is attached to the free end of the string and is released from rest. Each hanging mass on the end of the sting falls through a height y. The pulley in the system is considered massless (or at least so much less in mass than any other mass in the system).



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- Using the forces and torques that act in the system, what is the expression for the moment of inertia of the disk?

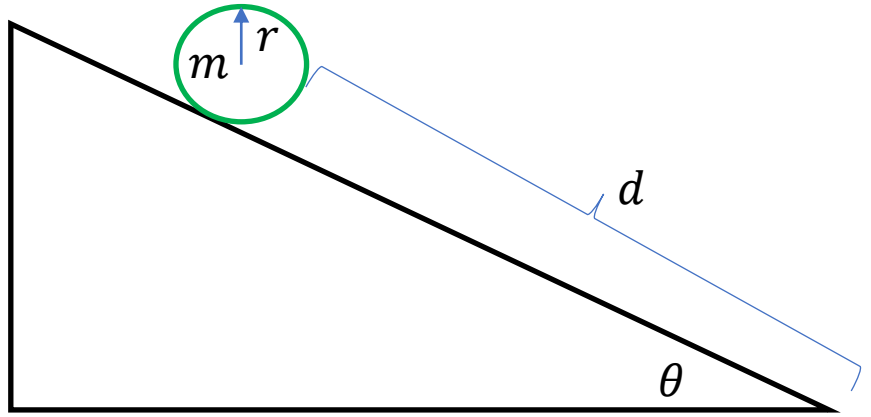
- Using energy ideas, determine an expression for the moment of inertia of the disk.

Rotational Dynamics

Example 7:

We're in a position now to talk about why objects roll. Consider several objects (a hoop, a solid cylinder, and a solid ball) each of mass m and equal radii $r_{hoop} = r_{cylinder} = r_{ball} = r$. All the objects are placed on an incline (oriented at an angle of inclination θ measured with respect to the horizontal) and released from rest both translationally and rotationally. Static friction between the object and the ramp is responsible for the rolling motion of the object, due to a torque created about the center-of-mass of the object. If it were not for friction, there would be no motion except for sliding down the incline by the object. We need static friction (which acts at a point) to cause the object to turn on its axis.

- What is the expression for the translational acceleration of each of the objects in terms of m, r, I, θ and any constants?
- What is the speed of each of the objects at the bottom of the incline if each object starts from rest and covers a distance d along the incline. Do this using forces/torques and then using energy.
- What is the order of arrival of the objects at the bottom of the incline? Which gets there first, second and third? Or do they all get there at the same time?



Rotational Dynamics

Example 8:

The Nott Memorial is a 16-sided Victorian building in the center of campus and is topped with an approximately hemispherical dome of radius R on top of a base that is a height H above the ground.

Suppose that on a windy night someone has precariously placed a pumpkin (of mass m_p and radius r_p, with $r_p \ll R$) at rest atop the Nott. A sudden gust of wind suddenly gets the pumpkin rolling down the dome of the Nott and at some angle θ_R measured with respect to the vertical the pumpkin will lose contact with the dome and become a projectile in flight.

- At what angle θ_R does the pumpkin leave the dome?
- How does this angle compare to the case where the pumpkin just slides down the dome.



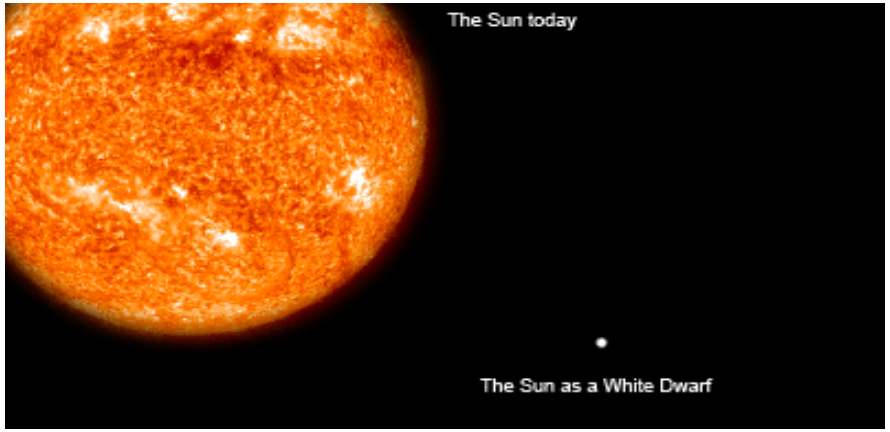
https://commodorebuilders.com/2014/03/24/union-college-nott-memorial/

Rotational Dynamics

Example 9

Stars create energy and pressure (called radiation pressure) inside of their cores by nuclear fusion. The radiation pressure is used to create a force from the core of the star towards its outside to stop the inward squeeze of the force of gravity. The balance of the radiation pressure and gravity is called hydrostatic equilibrium. When the fuel inside of the core is used up, the star dies. The fate of stars, such as our sun, are determined by their mass. Small mass stars, like the sun, will eventually collapse into what is called a white dwarf (the remnant of the stellar core) and lose their outer atmosphere as a nebula. When the sun dies (runs out of fuel in its core) the core collapses and a white dwarf will be formed with the radius of the white dwarf about 1% of the current value of the solar radius (R_s). At the end of its life, the sun will expel its outer atmosphere and collapse into something called a white dwarf. As the sun collapses into a white dwarf, work is done on the collapsing star by the force of gravity. We'll model the sun and the white dwarf as being spheres (balls) spinning about any axis through their centers so that their moments of inertia can be given by $I = \frac{2}{5}mr^2$. Consider the mass of the sun to be $m_s = 2\times10^{30}kg$ and its radius $r_s = 6.96\times10^{8}m$. It takes the sun about 25 days to make one complete revolution on its axis.

- What is the initial rotational speed of the sun?
- What is the final rotational speed of the white dwarf after its collapse? Assume the sun ejects one half of its initial mass, and the rest stays in the white dwarf and that the white dwarf has a radius $r_{wd} = 0.1r_s$.
- What is the change in rotational kinetic energy of the star?
- How much work was done on the star by gravity?



http://migall.fastmail.fm.user.fm/astronomy/stars_and_nebulae/Star_Life_Cycles/page06_wd.htm

Rotational Dynamics

Example 10:

A uniform rod of mass M and length L is attached to a pivot on one end and is held horizontally at the free end. The rod is released from rest and allowed to fall.

- What is the initial angular acceleration of the rod?
- What is the translational acceleration of the pivot, the center-of-mass of the rod, and the tip of the rod farthest from the pivot.
- At what distance from the pivot is the translational acceleration equal to g?

