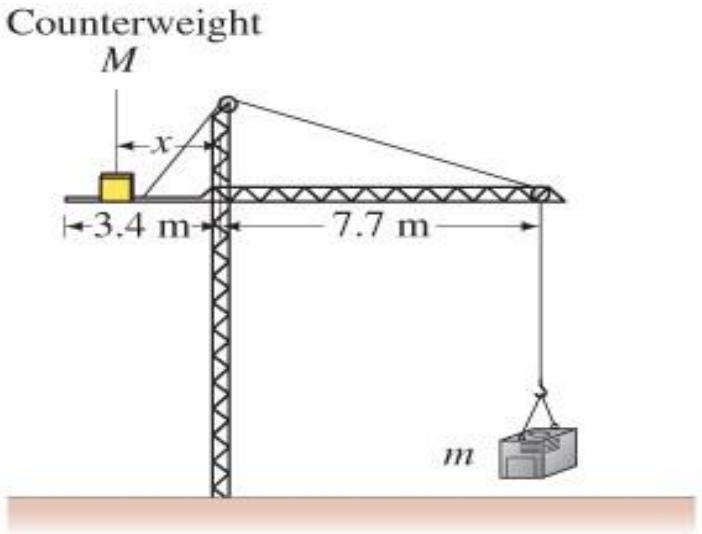
Rotational and Translational Statics

Example 1:

A tower crane must always be carefully balanced so that there is no net torque tending to tip it. A particular crane at a building site is about to lift an air-conditioning unit of mass $m_{ac} = m = 500kg$. The crane's dimensions are shown in the figure below on the right. The crane's counterweight is a movable (which provides stability against tipping) mass $m_{cw} = M = 10{,}000kg$ while the right and left beams have masses $m_{br} = 11{,}000kg$ and $m_{bl} = 10{,}000kg$ respectively. At what distance x from the crane's tower should the counterweight be placed so that the crane is balanced when it just picks up the air conditioner?





Figures are from Knight, Jones and Field, Physics: A Strategic Approach, 4th Edition.

Rotational and Translational Statics

Example 1 Solution:

Taking vertically up as the positive y-direction we have for the forces in the y-direction:

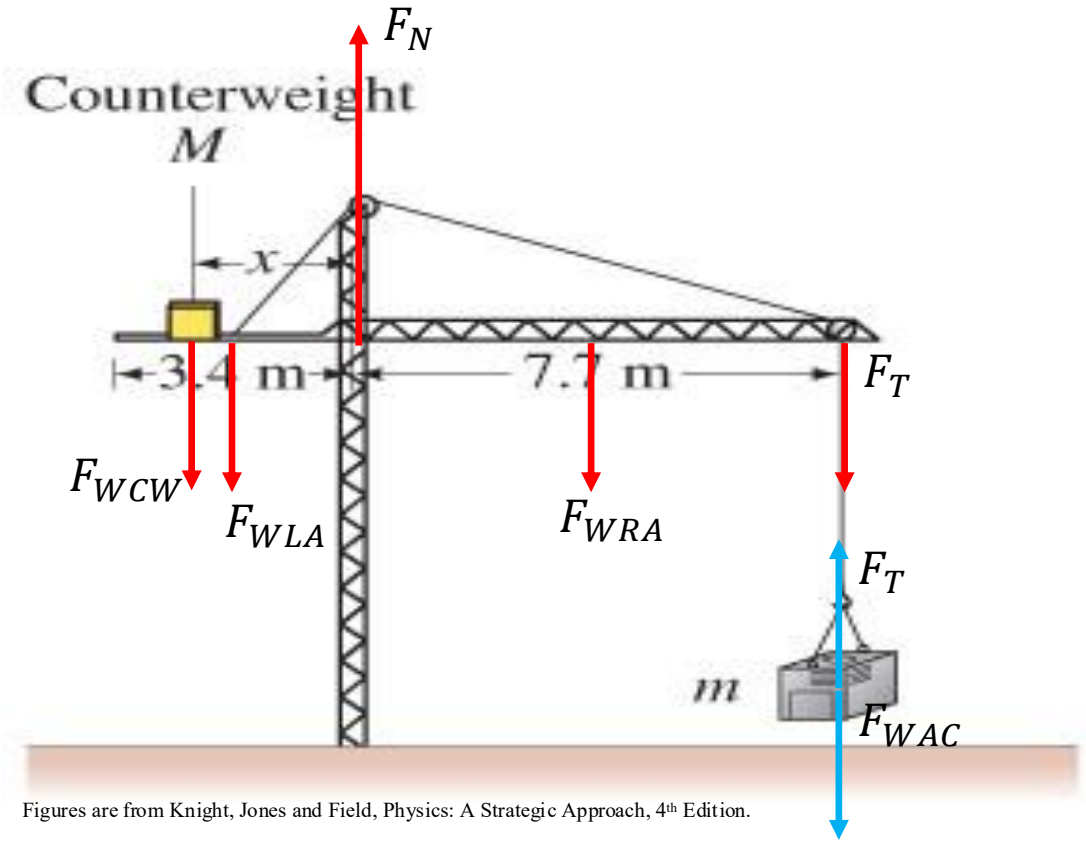
$$F_N - F_{WLA} - F_{WRA} - F_{CW} - F_T = m_{system} a_y^0 = 0$$

$$F_N = F_{WLA} + F_{WRA} + F_{CW} + F_T$$

The tension force in the cable is determined from the forces that act from the AC unit.

$$F_T - F_{WAC} = m_{AC} a_y = 0 \rightarrow F_T = F_{WAC}$$

$$F_N = F_{WLA} + F_{WRA} + F_{CW} + F_{WAC}$$



Figures are from Knight, Jones and Field, Physics: A Strategic Approach, 4th Edition.

Rotational and Translational Statics

Example 1 Solution:

Placing the pivot at F_N we have for the sum of the torques about the pivot:

$$\tau_{net} = I_{system}\alpha = 0 = +\tau_{CW} + \tau_{LA} - \tau_{RA} - \tau_{AC}$$

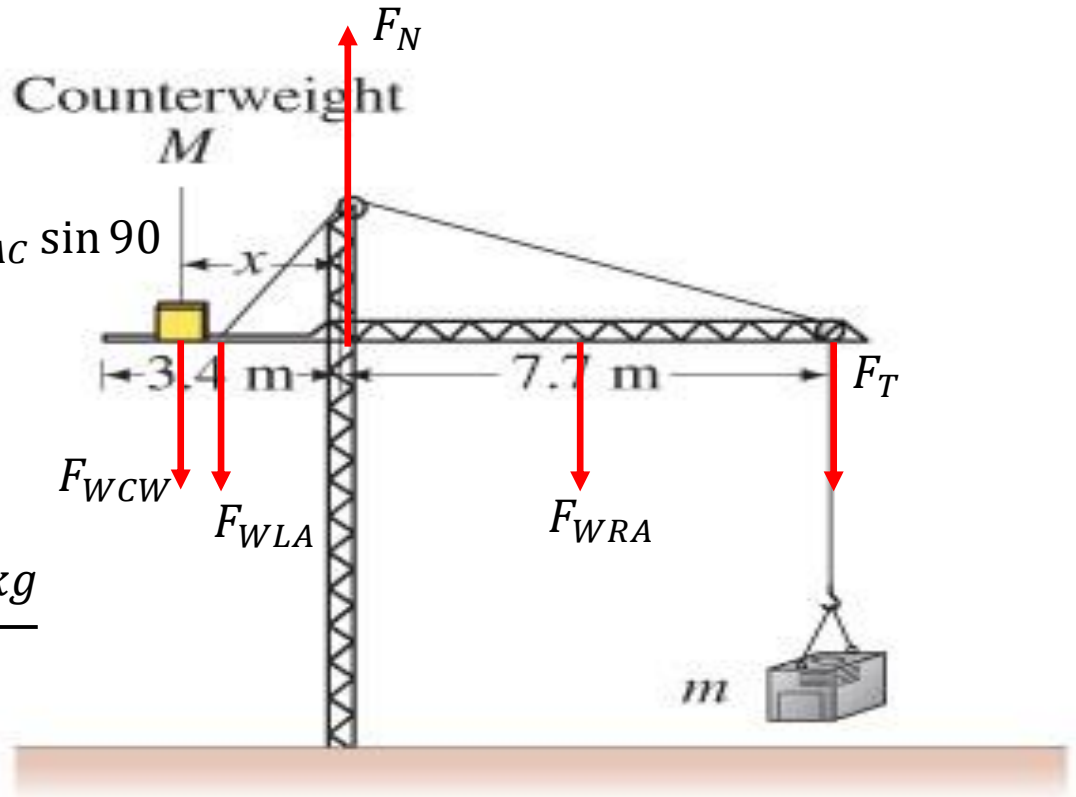
$$0 = +r_{CW}F_{CW}\sin 90 + r_{LA}F_{WLA}\sin 90 - r_{RA}F_{WRA}\sin 90 - r_{AC}F_{WAC}\sin 90$$

$$r_{CW} = \left(\frac{-r_{LA}m_{LA}g + r_{RA}m_{RA}g + r_{AC}m_{AC}g}{m_{CW}g}\right)$$

$$r_{CW} = \frac{-\left(\frac{3.4m}{2}\right)\cdot 10000kg + \left(\frac{7.7m}{2}\right)\cdot 11000kg + (7.7m)\cdot 500kg}{10000kg}$$

$$r_{CW} = x = 2.92m$$

This seems reasonable. The counterweight has to be somewhere between the pivot and the end of the left arm of the crane. This is in between those two values.



Figures are from Knight, Jones and Field, Physics: A Strategic Approach, 4th Edition.

Rotational and Translational Statics

Example 2:

A person of mass $m_p = 70kg$ stands on the edge of a table of mass $m_t = 55kg$. How far from the right table leg can the person stand before the table tips over?

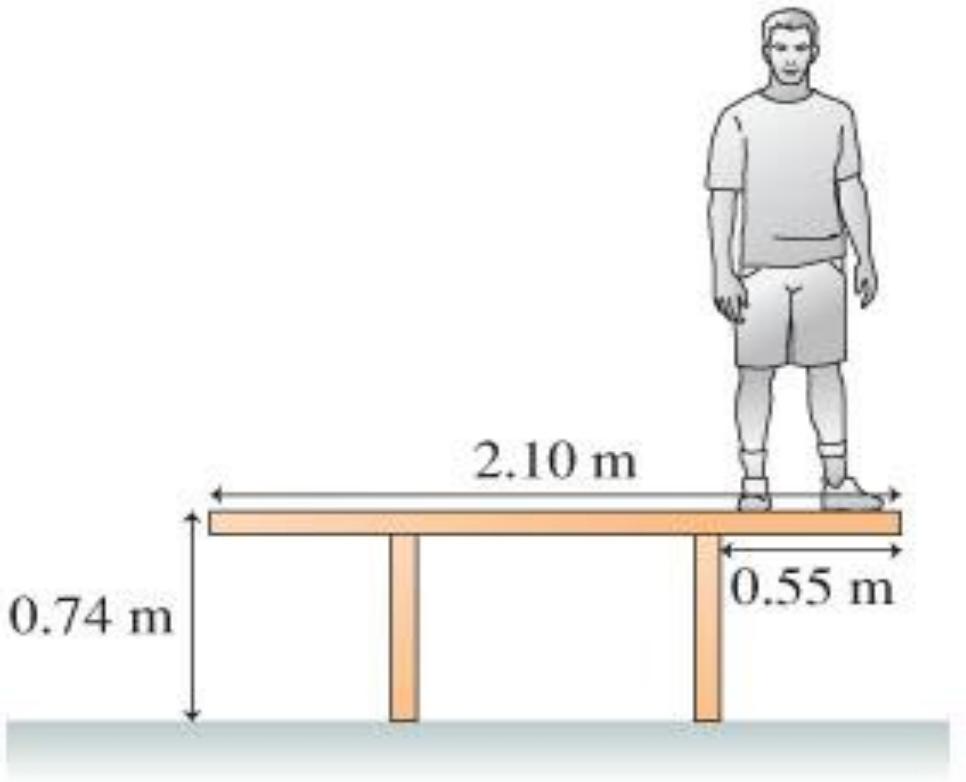


Figure from Knight, Jones and Field, Physics: A Strategic Approach, 4th Edition.

Rotational and Translational Statics

Example 2 Solution:

Here we have the table just about to tip. We'll look at the forces and the torques about the right leg.

Taking vertically up as the positive y-direction, we have:

$$F_N - F_{WT} - F_{Wp} = m_{system}\cancel{a_y}^{0} = 0$$

$$F_N = F_{WT} + F_{Wp}$$

Taking the pivot at the right table leg and assuming counterclockwise torques are positive, we have:

$$+\tau_{F_{WT}} - \tau_{F_{Wp}} = I_{system}\cancel{\alpha}^{0} = 0$$

$$+r_{F_{WT}}F_{WT}\sin\beta - r_{F_{Wp}}F_{Wp}\sin\gamma = 0$$

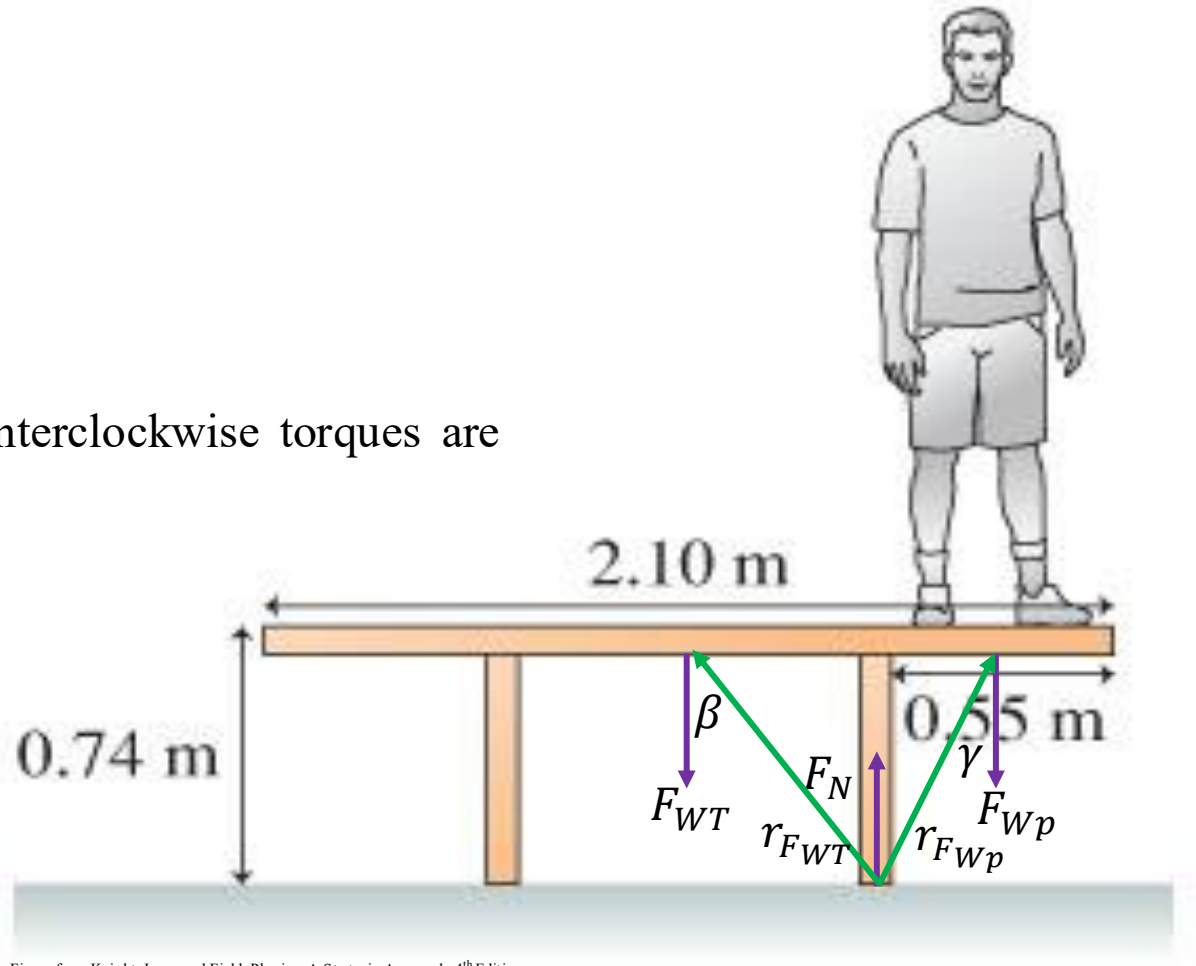


Figure from Knight, Jones and Field, Physics: A Strategic Approach, 4th Edition.

Rotational and Translational Statics

Example 2 Solution:

$$+\underbrace{\left(r_{F_{WT}}\sin\beta\right)}_{r_{\perp,F_{WT}}}F_{WT}-\underbrace{\left(r_{F_{Wp}}\sin\gamma\right)}_{r_{\perp,F_{WT}}}F_{Wp}=0$$

In here, I don't know $r_{F_{WT}}$ or β (as well as $r_{F_{Wp}}$ or γ). But $r_{F_{WT}}\sin\beta$ is the perpendicular distance to the force F_{WT} (as $r_{F_{Wp}}\sin\gamma$ is the perpendicular distance to the force F_{Wp}) by using the geometry in the system.

$$r_{\perp,F_{Wp}}=\left(\frac{F_{WT}}{F_{Wp}}\right)r_{\perp,F_{WT}}=\left(\frac{m_T}{m_p}\right)r_{\perp,F_{WT}}$$

$$r_{\perp,F_{Wp}}=\left(\frac{m_T}{m_p}\right)r_{\perp,F_{WT}}=\left(\frac{55kg}{70kg}\right)\cdot\left(\frac{2.10m}{2}-0.55m\right)$$

$$r_{\perp,F_{Wp}}=0.39m$$

With respect to the right edge of the table, the person can walk to a distance

$$x=0.55m-0.39m=0.16m=16cm$$

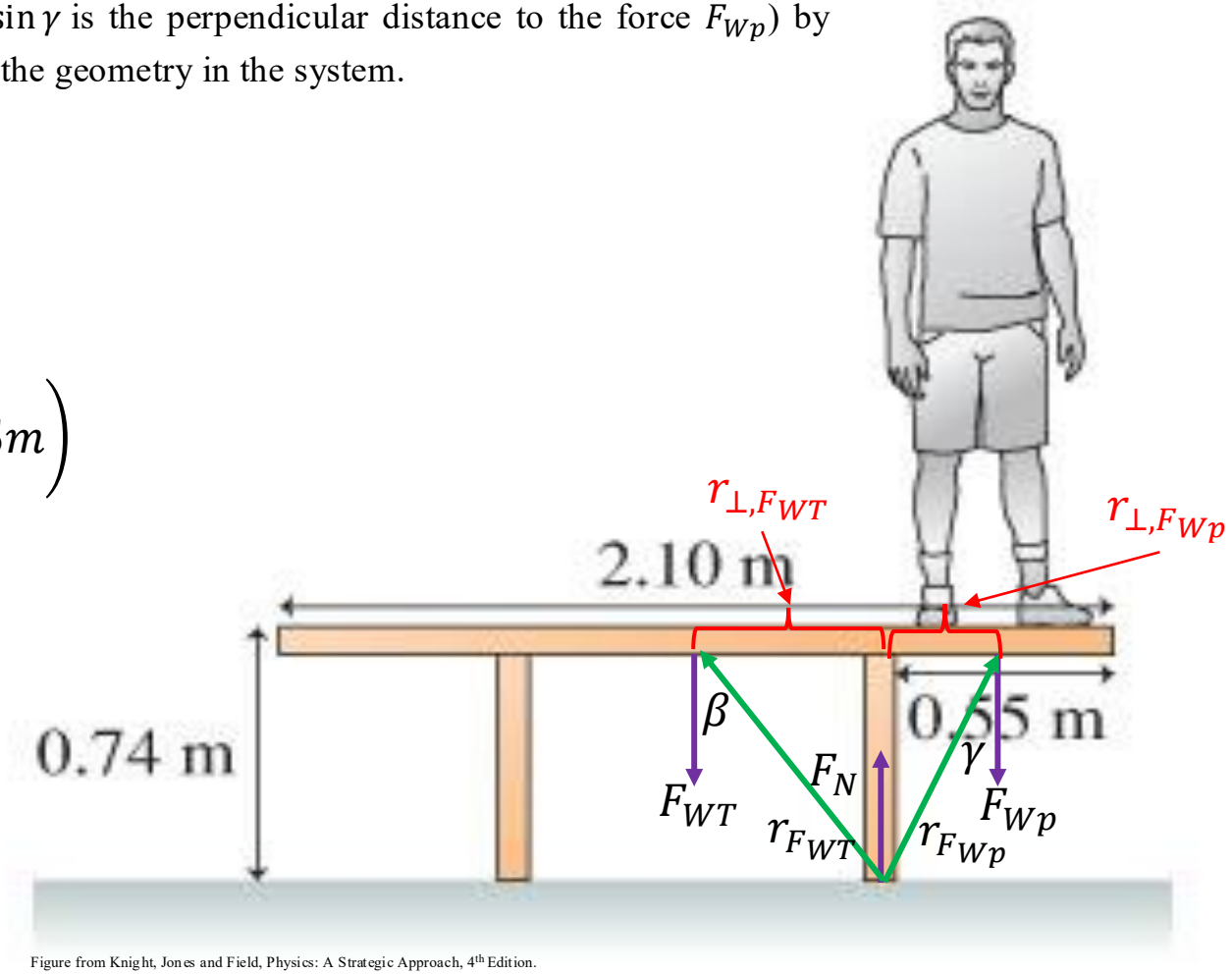


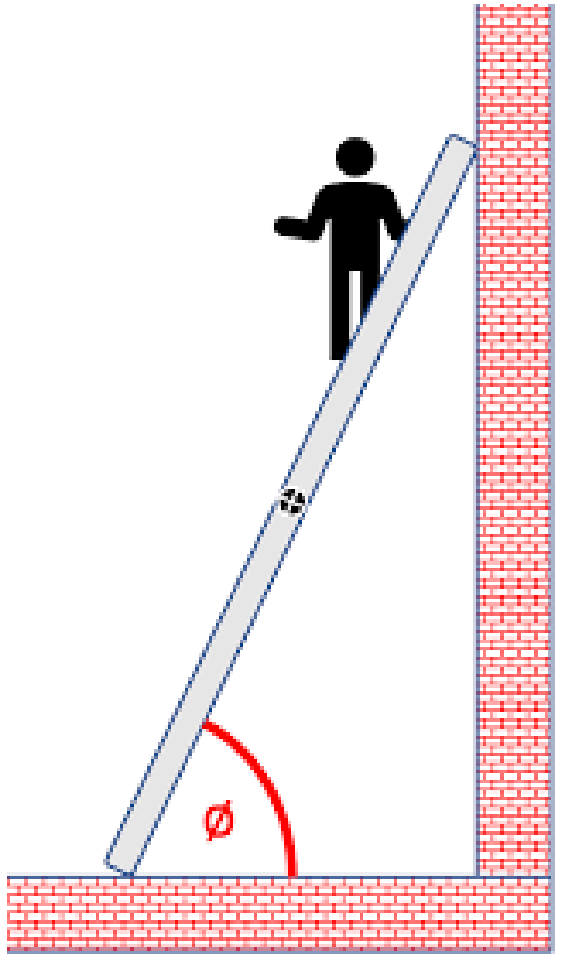
Figure from Knight, Jones and Field, Physics: A Strategic Approach, 4th Edition.

Rotational and Translational Statics

Example 3:

A $L = 5.0m$ long ladder with mass $m_L = 100kg$ is laid against a low friction wall at an angle ϕ measured with respect to the floor as shown on the right.

Suppose that the coefficient of friction between the floor and ladder is $\mu = 0.09$ and that a painter of mass $m_p = 60kg$ has climbed up the ladder and has made it to a point 70% of the length of the ladder when the ladder begins to slip. At what angle ϕ was the ladder placed against the wall?

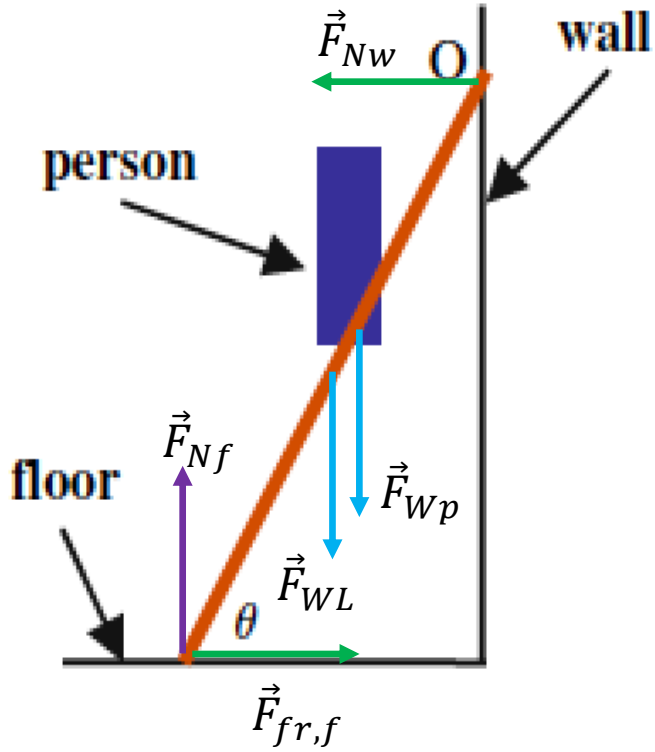


https://www.physicsclassroom.com/calcpad/launch/CPRT14

Rotational and Translational Statics

Example 3 Solution:

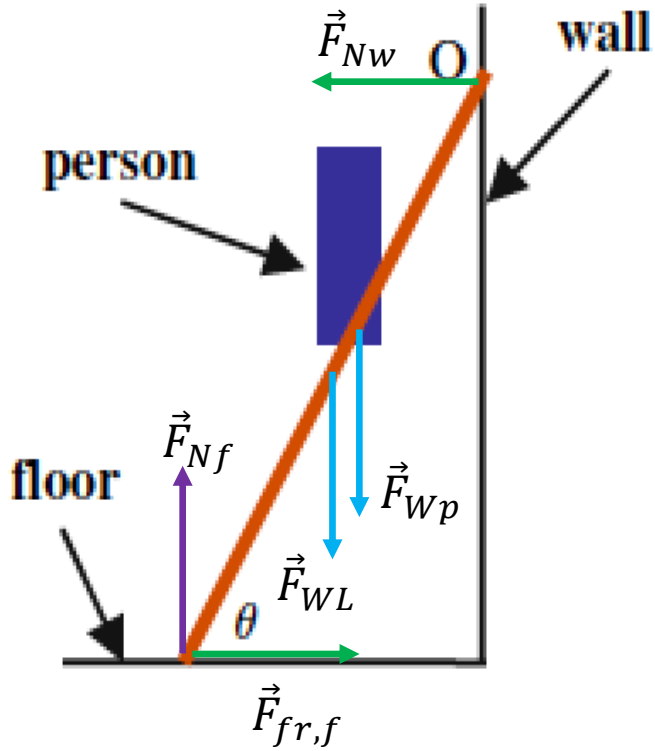
- Let's start by adding the forces that act on the ladder/painter system and take the pivot where the ladder makes contact with the wall.
- The ladder with painter is resting on the floor and against the wall and thus the floor and wall have to react to the ladder and painter. Let's call these forces $\vec{F}_{Nf}$ (the reaction force of the floor) and $\vec{F}_{Nw}$ (the reaction force of the wall).
- If the ladder slips, then the bottom of the ladder will move left and the top of the ladder moves down the wall. Let's assume that there is no friction on between the wall and the ladder but there is between the floor and the ladder $\vec{F}_{fr,f}$.
- The ladder has a weight $\vec{F}_{WL}$ and assuming a uniform distribution of mass acts at the midpoint of the ladder ($\frac{L}{2}$), assuming the ladder has length L.
- The painter has a weight $\vec{F}_{Wp}$ and is located at a distance of 70% of the length of the ladder measured from the floor (at $0.7L$).



Rotational and Translational Statics

Example 3 Solution:

- Assuming a standard cartesian coordinate system break the forces up into their horizontal and vertical components and apply the condition that the horizontal and vertical forces have to vanish.



Horizontal forces

$$F_{fr,f} - F_{Nw} = \mu F_{Nf} - F_{Nw} = m_{system} a_x^{\,0} = 0$$

$$\longrightarrow \quad F_{Nw} = \mu F_{Nf} = \mu(m_L + m_p)g$$

We could evaluate this if we wanted. I'm lazy and I'm not going to but you can if you want.

Vertical forces

$$F_{nf} - F_{WL} - F_{Wp} = F_{nf} - m_L g - m_p g = m_{system} a_y^{\,0} = 0$$

$$\longrightarrow \quad F_{Nf} = m_L g + m_p g$$

We could evaluate this if we wanted. I'm lazy and I'm not going to but you can if you want.

- Unfortunately, the forces in the horizontal and vertical directions don't shed any light on the angle at which the ladder begins to slip. Let's look at the torques in the system about the pivot.

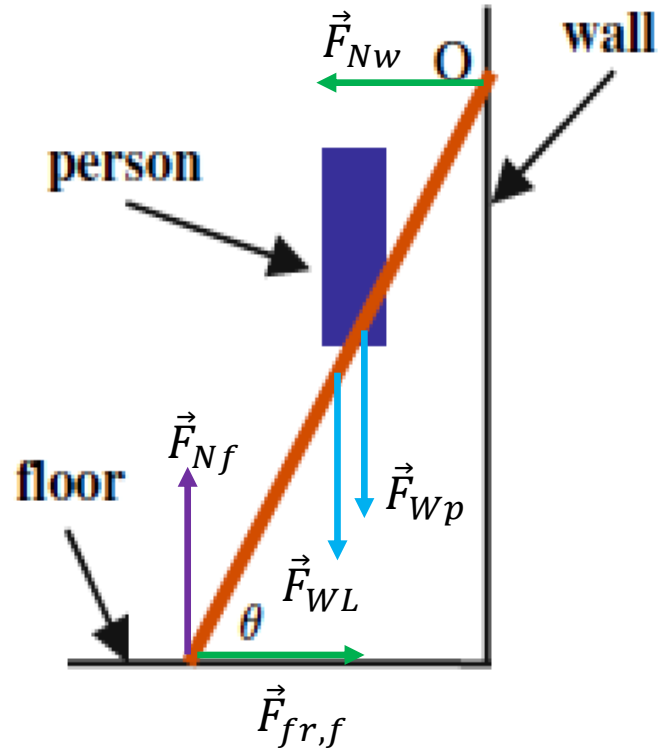
Rotational and Translational Statics

Example 3 Solution continued:

- To calculate angle θ at which the ladder slips when the painter gets to 70% of the way up the ladder, we'll determine the torques in the system.
- You can take the pivot anywhere you'd like, but this problem wants us to take the pivot where the ladder touches the wall.
- Assume clockwise rotations are negative and counterclockwise rotations are positive.

$$\tau_{F_{Nf}} + \overset{0}{\cancel{\tau_{F_{Nw}}}} + \tau_{F_{fr,f}} + \tau_{F_{WL}} + \tau_{F_{Wp}} = I_{system}\overset{0}{\cancel{\alpha}} = 0$$

- All of the torques look like a product of the distance from the pivot to the applied force, the applied force, and the sine of the angle between the moment arm (distance from the pivot to the applied force) and the applied force, ϕ.
- Let's look at the torques created from each of the forces one by one so that we know what's going on.



$$\tau = r_{\perp}F = rF_{\perp} = rF\sin\phi$$

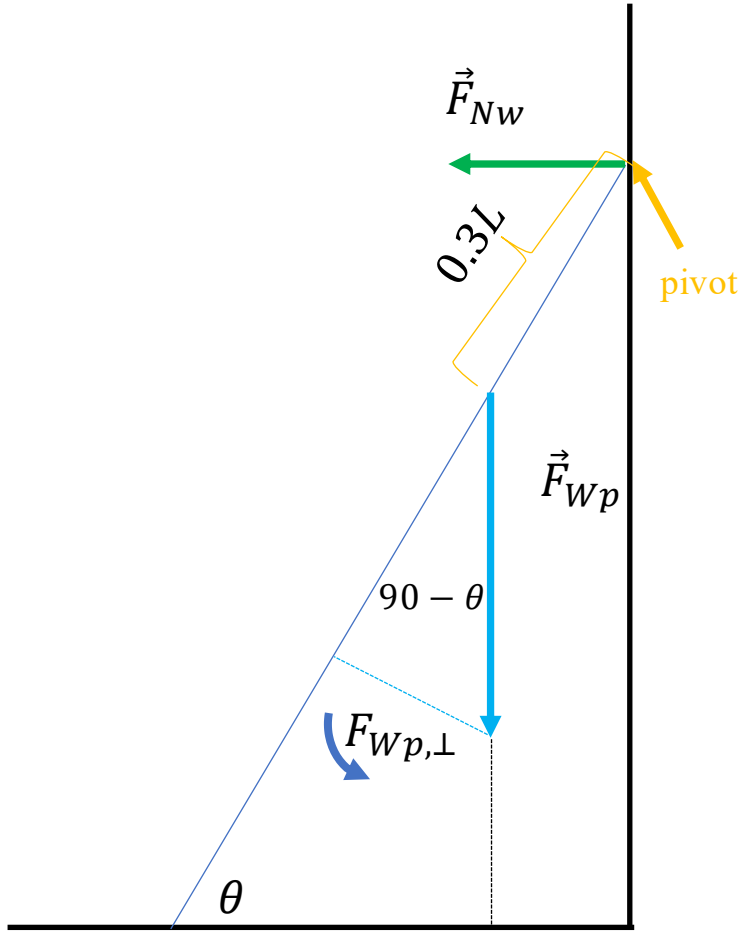
How much of r is perpendicular to F.

How much of F is perpendicular to r.

Rotational and Translational Statics

Example 3 Solution continued:

- Let's start at the pivot and work our way through each force moving towards the bottom of the ladder at the floor.
- $\vec{F}_{Nw}$ produces no torque since it is at the pivot.
- The weight of the painter $\vec{F}_{Wp}$ produces a counterclockwise rotation of the ladder about the pivot. You can see this by "grabbing" the weight vector and pulling it straight down. The bottom of the ladder would get lower (if the floor were not in the way) and rotate counterclockwise about the pivot.
- The moment arm is from the pivot to the location of the painter and points from the pivot down the ladder to the floor and is $L - 0.7L = 0.3L$ of the length of the ladder.
- The amount of the weight of the painter that is perpendicular to this moment arm is $F_{Wp,\perp} = F_{Wp}\sin(90 - \theta)$.
- The torque due to the weight of the painter is:



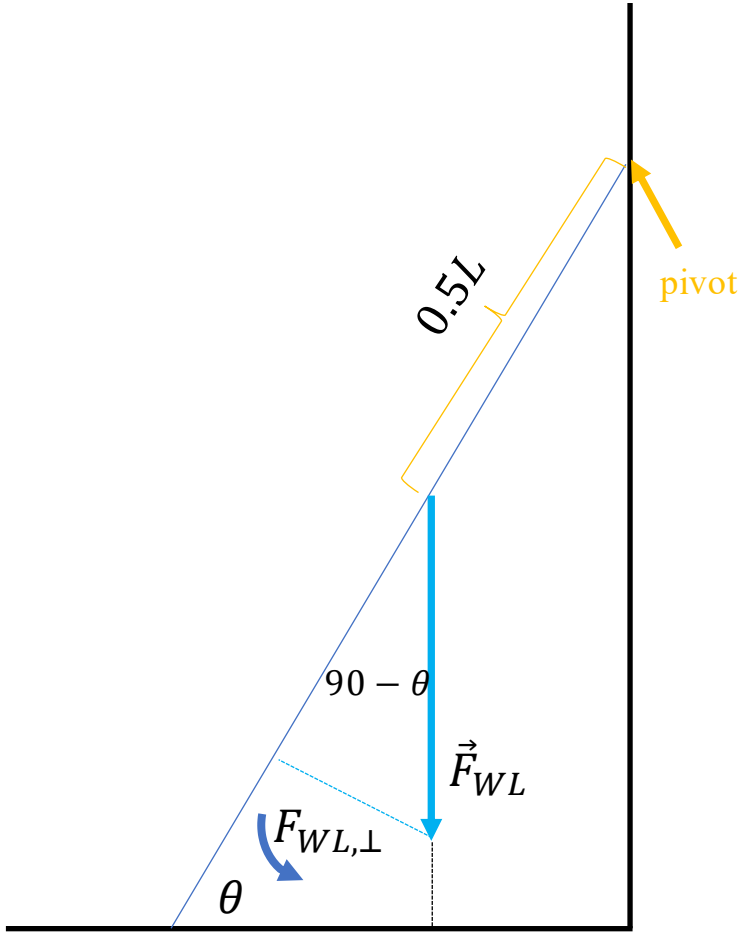
$$\tau = r_\perp F = rF_\perp = rF\sin\phi$$

$$\tau_{F_{Wp}} = r_\perp F = rF_\perp = 0.3LF_{Wp}\sin(90 - \theta)$$

Rotational and Translational Statics

Example 3 Solution continued:

- The weight of the ladder $\vec{F}_{WL}$ produces a counterclockwise rotation of the ladder about the pivot. You can see this by "grabbing" the weight vector and pulling it straight down. The bottom of the ladder would get lower (if the floor were not in the way) and rotate counterclockwise about the pivot.
- The moment arm is from the pivot to the location of the weight of the ladder and points from the pivot down the ladder towards the floor and is at half of the length of the ladder.
- The amount of the weight of the ladder that is perpendicular to this moment arm is $F_{WL,\perp} = F_{WL}\sin(90-\theta)$.
- The torque due to the weight of the ladder is:



$$\tau = r_\perp F = rF_\perp = rF\sin\phi$$

$$\tau_{F_{WL}} = r_\perp F = rF_\perp = 0.5LF_{WL}\sin(90-\theta)$$

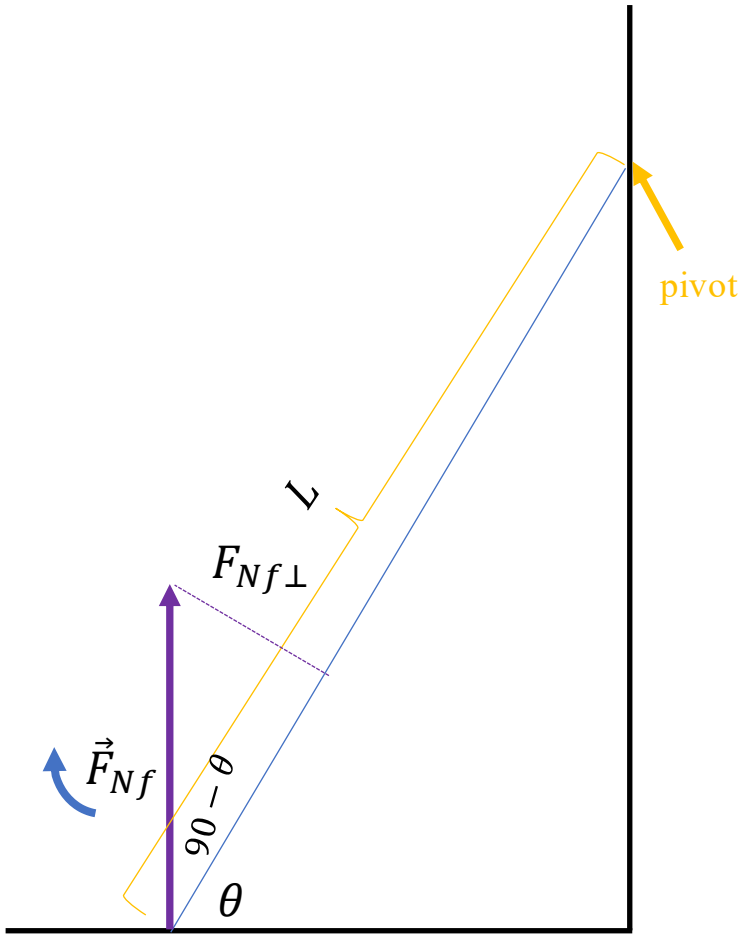
Rotational and Translational Statics

Example 3 Solution continued:

- The reaction force from the floor $\vec{F}_{Nf}$ produces a clockwise rotation of the ladder about the pivot. You can see this by "grabbing" the normal force vector and pulling it straight up. The bottom of the ladder would get higher and move away from the floor.

- The moment arm is from the pivot to the location of the normal force from the floor and points from the pivot down the ladder towards the floor and is at the full length of the ladder.

- The amount of the normal force that is perpendicular to the moment arm is $F_{Nf\perp} = F_{Nf}\sin(90-\theta)$.



- The torque due to the reaction force of the floor on the ladder is:

$$\tau = r_\perp F = rF_\perp = rF\sin\phi$$

$$\tau_{F_{Nf}} = r_\perp F = rF_\perp = -LF_{Nf}\sin(90-\theta)$$

Rotational and Translational Statics

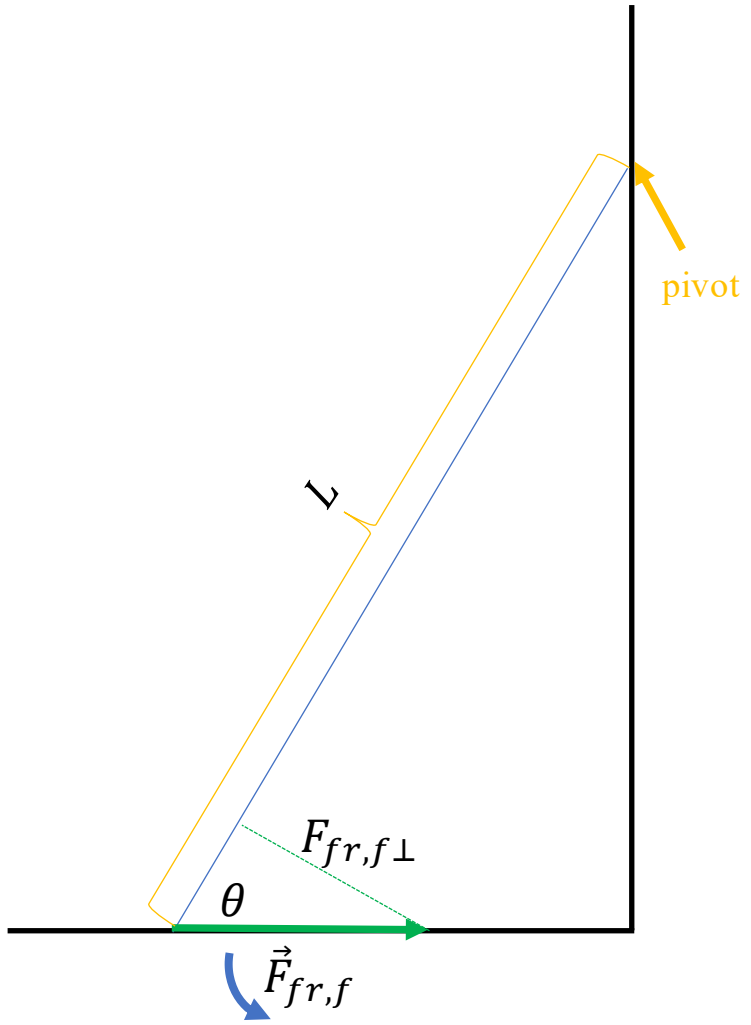
Example 3 Solution continued:

- The frictional force between the ladder and the floor $\vec{F}_{fr,f}$ produces a counterclockwise rotation of the ladder about the pivot. You can see this by "grabbing" the frictional force vector and pulling it to the right. The bottom of the ladder would get lower (if the floor wasn't there).

- The moment arm is from the pivot to the location of the frictional force and points from the pivot down the ladder towards the floor and is at the full length of the ladder.

- The amount of the frictional force that is perpendicular to the moment arm is $F_{fr,f\perp} = F_{fr,f} \sin\theta$.

- The torque due to the frictional force from the floor on the ladder is:



$$\tau = r_\perp F = rF_\perp = rF\sin\phi$$

$$\tau_{F_{fr,f}} = r_\perp F = rF_\perp = LF_{fr,f}\sin\theta$$

Rotational and Translational Statics

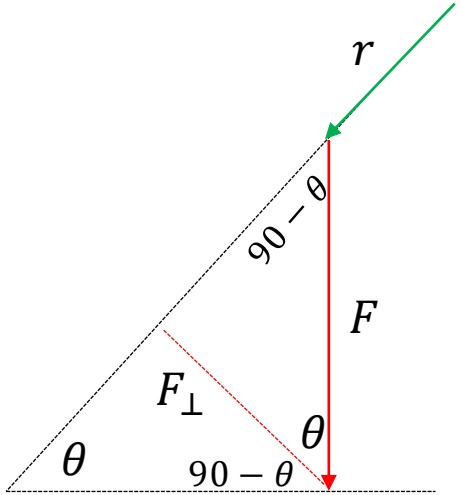
Example 3 Solution continued:

- Adding all of the torques together and setting the sum equal to zero we get:

$$\tau_{F_{Nf}} + \tau_{F_{Nw}}^{0} + \tau_{F_{fr,f}} + \tau_{F_{WL}} + \tau_{F_{Wp}} = I_{system}\alpha^{0} = 0$$

$$-LF_{Nf}\sin(90-\theta) + LF_{fr,f}\sin\theta + 0.5LF_{WL}\sin(90-\theta) + 0.3LF_{Wp}\sin(90-\theta) = 0$$

- We need to get rid of the terms $\sin(90-\theta)$ and re-express them into something more manageable.



- Consider the torque produced by some r and some F. We need the amount of the force F that is perpendicular to the moment arm r. Dropping a perpendicular from the force to the moment arm we get that $F_{\perp} = F\sin(90-\theta)$.

- Using the geometry we see that $F_{\perp} = F\sin(90-\theta) = F\cos\theta$.

- Equivalently, we could have also calculated the amount of the the moment arm r that is perpendicular to the force F, and we would have obtained $r_{\perp} = r\sin(90-\theta) = r\cos\theta$

Rotational and Translational Statics

Example 3 Solution continued:

- Thus we can rewrite our expression for the sum of the torques as:

$$-LF_{Nf}\sin(90-\theta)+LF_{fr,f}\sin\theta+0.5LF_{WL}\sin(90-\theta)+0.3LF_{Wp}\sin(90-\theta)=0$$

$$-LF_{Nf}\cos\theta+LF_{fr,f}\sin\theta+0.5LF_{WL}\cos\theta+0.3LF_{Wp}\cos\theta=0$$

$$\longrightarrow \quad \tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{LF_{Nf}-0.5LF_{WL}-0.3LF_{Wp}}{LF_{fr,f}}=\frac{F_{Nf}-0.5F_{WL}-0.3F_{Wp}}{\mu F_{Nf}}$$

- Substituting the expression for the reaction force from the floor that we found earlier:

$$\tan\theta=\frac{F_{Nf}-0.5F_{WL}-0.3F_{Wp}}{\mu F_{Nf}}=\frac{F_{WL}+F_{Wp}-0.5F_{WL}-0.3F_{Wp}}{\mu F_{Nf}}=\frac{0.5F_{WL}+0.7F_{Wp}}{\mu\left(F_{WL}+F_{Wp}\right)}$$

This result is independent of the length of the ladder and only depends on the weights of the ladder and painter.

$$\tan\theta=\frac{(0.5\times100kg+0.7\times60kg)\times9.8\frac{m}{s^2}}{0.09\times(100kg+60kg)\times9.8\frac{m}{s^2}}=6.389$$

$$\boxed{\theta=81^0}$$

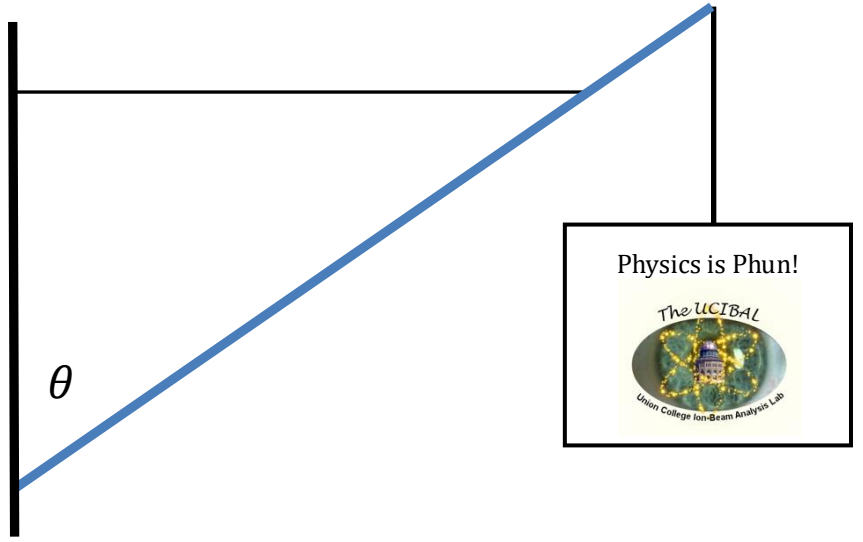
This is a pretty steep angle, but it's greater than zero (the ladder flat on the ground) and less than 90^0 (the ladder completely vertical and unclimbable).

Rotational and Translational Statics

Example 4:

The Union College Physics Department has decided to hang a sign outside of the department office so that all passers by can read how much fun physics can be. Suppose that the sign (with a mass $m_s = 220kg$) is suspended by a massless wire from a uniform boom ($m_b = 110kg$) and that the boom makes an angle of $\theta = 30^0$ measured with respect to the vertical, as shown below. The boom is attached to the wall by a light horizontal support wire located $\frac{2}{3}$ of the way from the wall to the end of the boom.

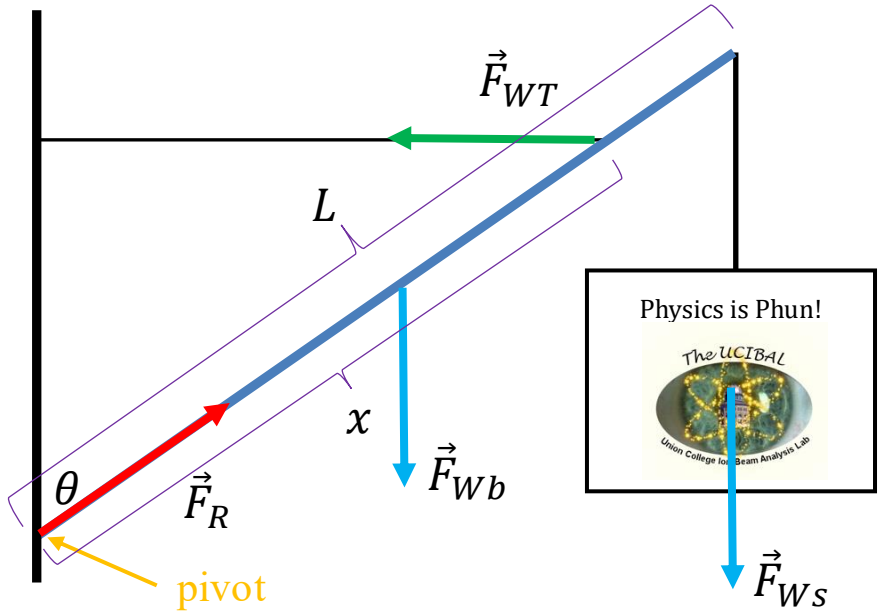
- What is the magnitude and direction of the reaction force of the wall on the boom?
- What is the tension force in the horizontal wire?



Rotational and Translational Statics

Example 4 Solution:

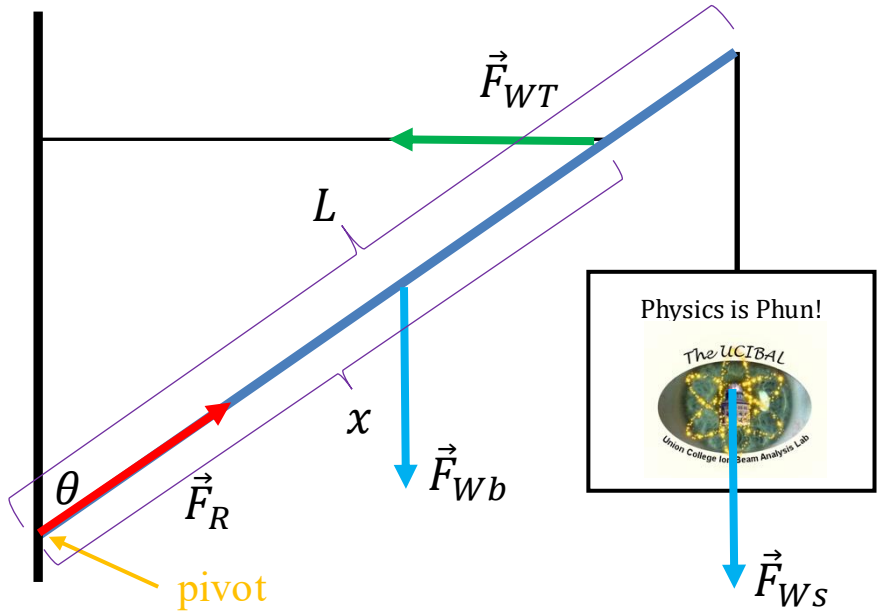
- Let's start by adding the forces that act on the boom/sign system and take the pivot where the boom attaches to the wall.
- The boom is attached to the wall and thus the wall has to react to the boom. Let's call this $\vec{F}_R$.
- The boom has a weight $\vec{F}_{Wb}$ and assuming a uniform distribution of mass acts at the midpoint of the boom ($\frac{L}{2}$), assuming the boom has length L.
- The sign has a weight $\vec{F}_{Ws}$ and acts at the end of the boom (at L) and at some fraction of the length of the boom there is a horizontal support wire of negligible mass. We'll call this length where the support wire attaches to the boom x. Since the boom/sign are attached to the support wire, tension $\vec{F}_{WT}$ is created in the wire.



Rotational and Translational Statics

Example 4 Solution continued:

- Assuming a standard cartesian coordinate system break the forces up into their horizontal and vertical components and apply the condition that the horizontal and vertical forces have to vanish. This will let us solve for the reaction force of the boom from the wall and the tension force in the horizontal wire.



Horizontal forces

$$F_{Rx} - F_T = F_R \sin\theta - F_T = m_{system} \cancel{a_x}^{\,0} = 0$$

$$\rightarrow \boxed{F_{Rx} = F_R \sin\theta = F_T} \rightarrow F_T = F_R \sin\theta = 3734N \sin 30 = 1867N$$

Vertical forces

$$F_{Ry} - F_{Wb} - F_{Ws} = F_R \cos\theta - m_b g - m_s g = m_{system} \cancel{a_y}^{\,0} = 0$$

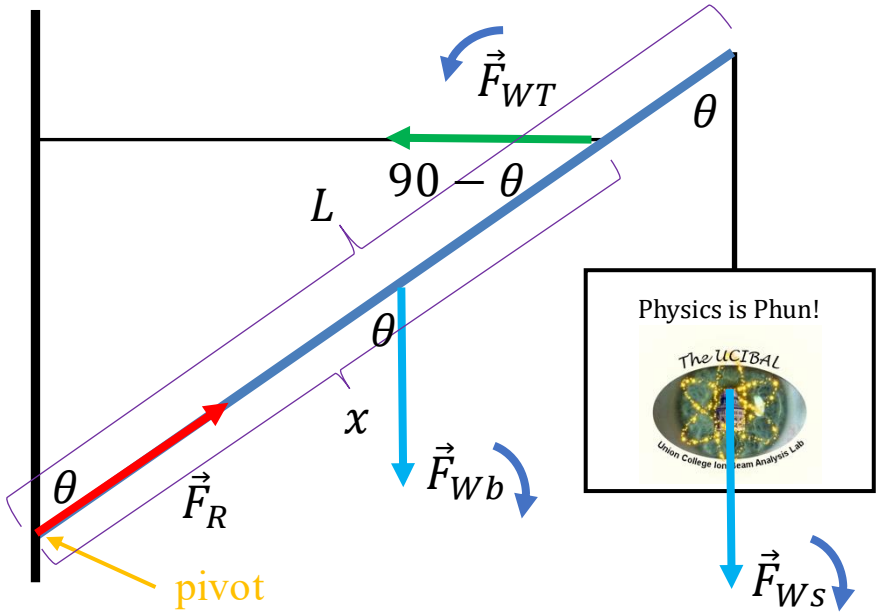
$$\rightarrow \boxed{F_{Ry} = F_R \cos\theta = m_b g + m_s g} \rightarrow F_R = \frac{(m_b + m_s)g}{\cos\theta} = \frac{(110kg + 220kg)9.8\frac{m}{s^2}}{\cos 30} = 3734N$$

Rotational and Translational Statics

Example 4 Solution continued:

- To calculate where the horizontal wire is connected to the boom, we take the torques about the pivot and use the condition that the sum of the torques must vanish.

- Assume clockwise rotations are negative and counterclockwise rotations are positive.



$$\tau_{Wb} + \tau_{Ws} + \tau_{F_T} + \tau_{F_N}^{0} = I_{system}\alpha^{0} = 0$$

$$-\frac{L}{2}F_{Wb}\sin\theta - LF_{Ws}\sin\theta + xF_T\sin(90-\theta) = 0$$

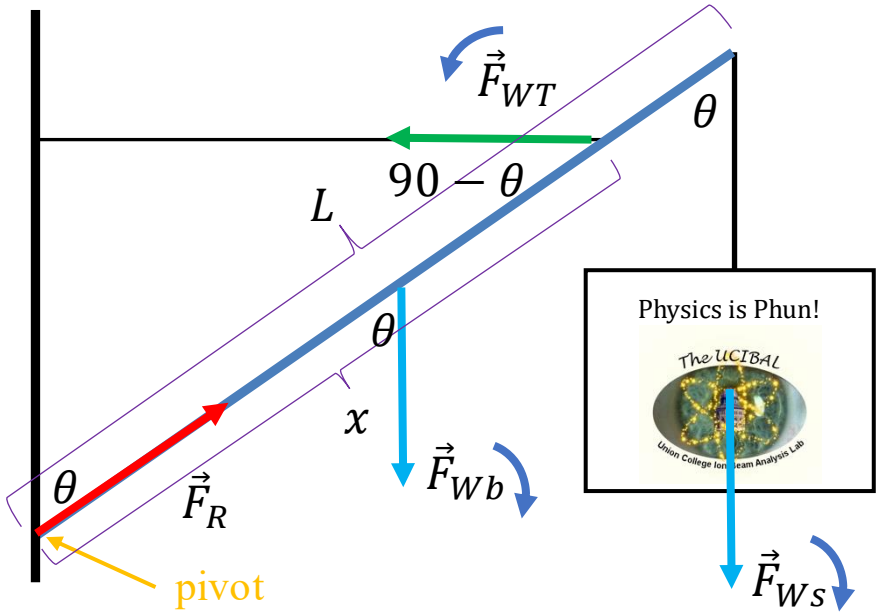
$$x = \frac{\frac{L}{2}F_{Wb}\sin\theta + LF_{Ws}\sin\theta}{F_T\sin(90-\theta)} = \left(\frac{\frac{F_{Wb}}{2}\sin\theta + F_{Ws}\sin\theta}{F_T\cos\theta}\right)L = \left(\frac{\frac{110kg\times 9.8\frac{m}{s^2}}{2}\sin 30 + 220kg\times 9.8\frac{m}{s^2}\sin 30}{1867N\cos 30}\right)L$$

$$\rightarrow \boxed{x = 0.83L}$$

Independent of the length L, the horizontal wire is attached at 83% of the way from the pivot to the end of the boom.

Rotational and Translational Statics

Example 4 Solution continued:



- If the wire breaks the boom/sign will rotate about the pivot. The angular acceleration changes with angle, but we can calculate the initial angular acceleration.

- To calculate the initial angular acceleration of the system we need to know the total moment of inertia of the system.

- Assume that the sign acts as a point mass ($I = MR^2$) at the end of the boom and that the boom is a uniform rod spun about one end ($I = \frac{1}{3}MR^2$).

$$I_{system} = I_b + I_s = \frac{1}{3}m_b L^2 + m_s L^2 = \left(\frac{110kg}{3} + 220kg\right)L^2 = (256.7L^2)kg$$

- When the cable breaks the system rotates clockwise. The net torque is negative, which means the initial angular acceleration is negative as I would expect.

$$\tau_{net} = -\frac{L}{2}F_{Wb}\sin\theta - LF_{Ws}\sin\theta = I_{system}\alpha \rightarrow \alpha = -\frac{\left(\frac{F_{Wb}}{2} + F_{Ws}\right)L\sin\theta}{I_{system}} = -\frac{\left(\frac{F_{Wb}}{2} + F_{Ws}\right)L\sin\theta}{\frac{1}{3}m_b L^2 + m_s L^2}$$

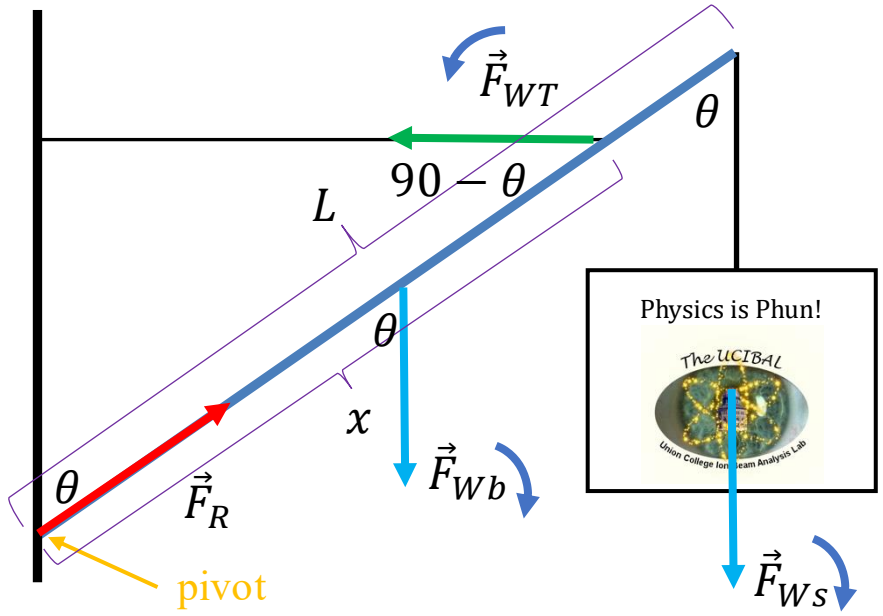
Rotational and Translational Statics

Example 4 Solution continued:

- To calculate a value for the initial angular acceleration we need a length for the boom. Let $L = 1m$.

$$\alpha = -\frac{\left(\frac{F_{Wb}}{2} + F_{Ws}\right) L \sin\theta}{\frac{1}{3} m_b L^2 + m_s L^2}$$

$$\alpha = -\frac{\left(\frac{110kg}{2} + 220kg\right) \times 9.8\frac{m}{s^2} \times 1m \sin 30}{\left(\frac{1}{3} \times 110kg + 220kg\right)(1m)^2} = 5.24\frac{rad}{s^2}$$

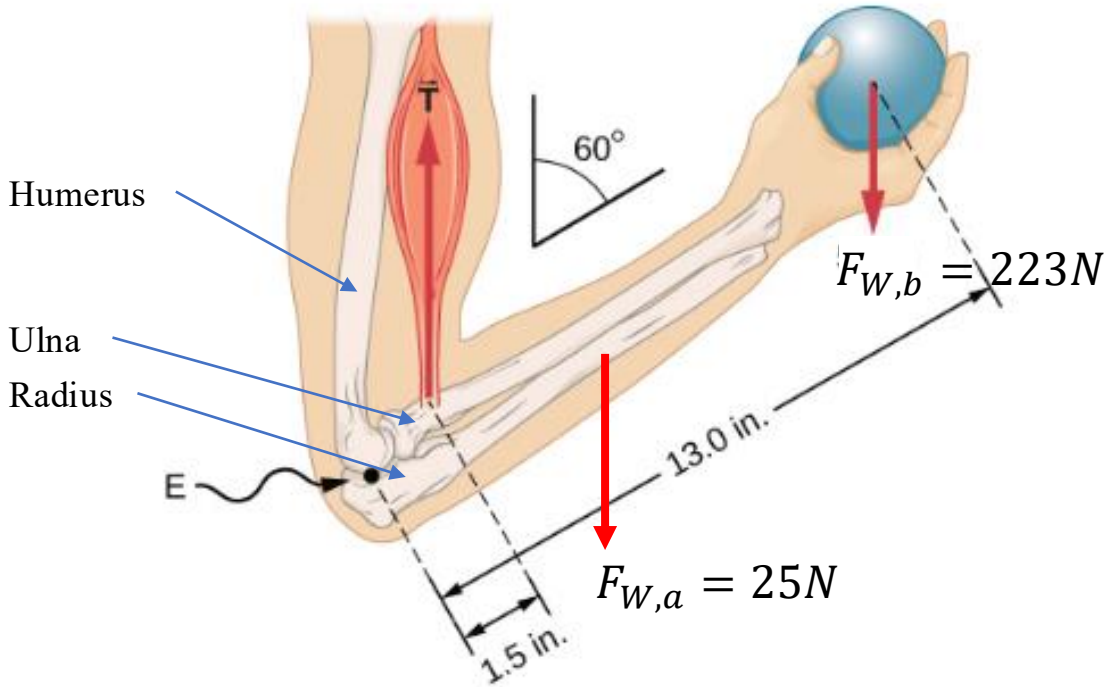


- This is the angular acceleration of the system which is the same for all points on the boom since the system must remain whole.

Rotational and Translational Statics

Example 5:

A person holds a $50lb$ weight in their hand by exerting an upward force through the bicep's tendon/muscle. What is the magnitude of the force exerted by the biceps muscle? What is the magnitude and direction of the force at the pivot, E?



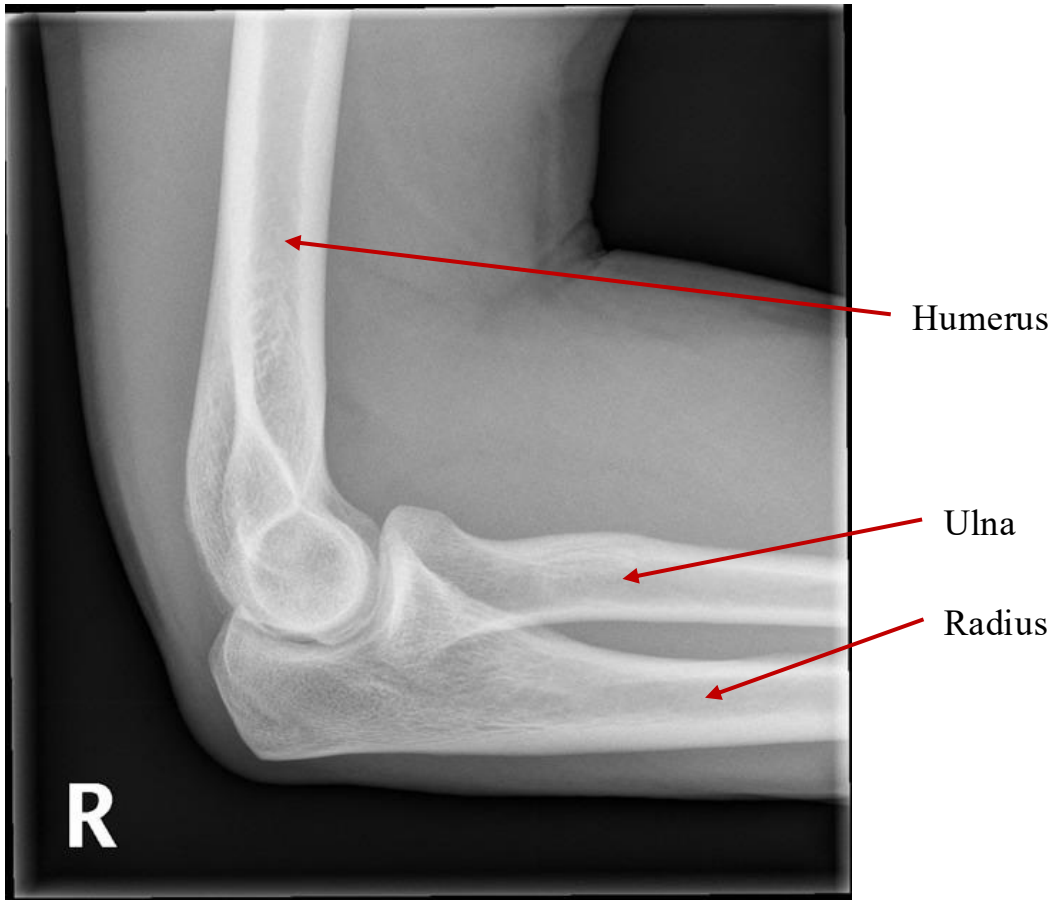


Figure are from OpenStax, University Physics Volume 1, https://openstax.org/books/university-physics-volume-1/pages/preface

https://radiopaedia.org/cases/normal-elbow-radiographs

Rotational and Translational Statics

Example 5 Solution:

From the forces in the vertical direction, assuming that the reaction force of the elbow points vertically up (if it does not, the math will work itself out to tell me) we have:

$$F_E + F_T - F_{W,a} - F_{W,b} = m_{system} \cancel{a_y}^{0} = 0$$

$$F_E = -F_T + F_{W,a} + F_{W,b}$$

To determine the force on the elbow joint, we need to know the force that the tension force in the biceps muscle.

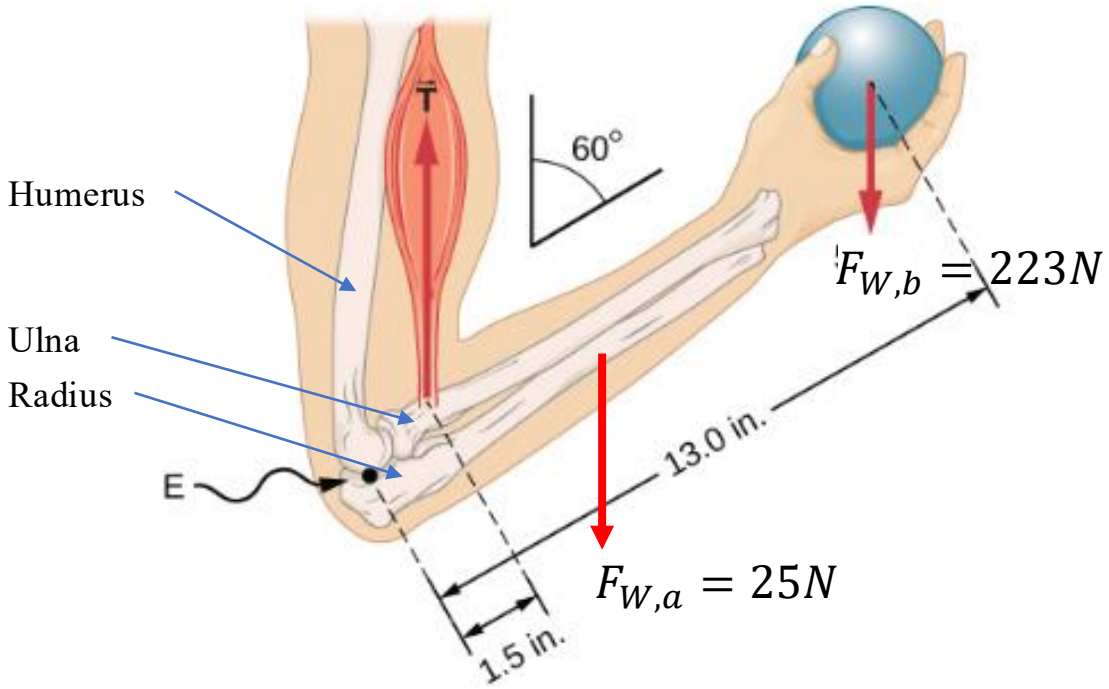


Figure are from OpenStax, University Physics Volume 1, https://openstax.org/books/university-physics-volume-1/pages/preface

Rotational and Translational Statics

Example 5 Solution:

Taking the pivot to be at the elbow joint and looking at the sum of the torques we have for the tension force in the biceps muscle:

$$+\tau_{F_T} - \tau_{F_{W,a}} - \tau_{F_{W,b}} = I_{system}\cancel{\alpha}^{0}$$

$$r_{F_T}F_T \sin 60 - r_{F_{W,a}}F_{W,a} \sin 60 - r_{F_{W,b}}F_{W,b} \sin 60 = 0$$

$$F_T = \frac{r_{F_{W,a}}F_{W,a} + r_{F_{W,b}}F_{W,b}}{r_{F_T}} = \frac{6.5” \cdot 25N + 13” \cdot 223N}{1.5”}$$

$$F_T = 2041N$$

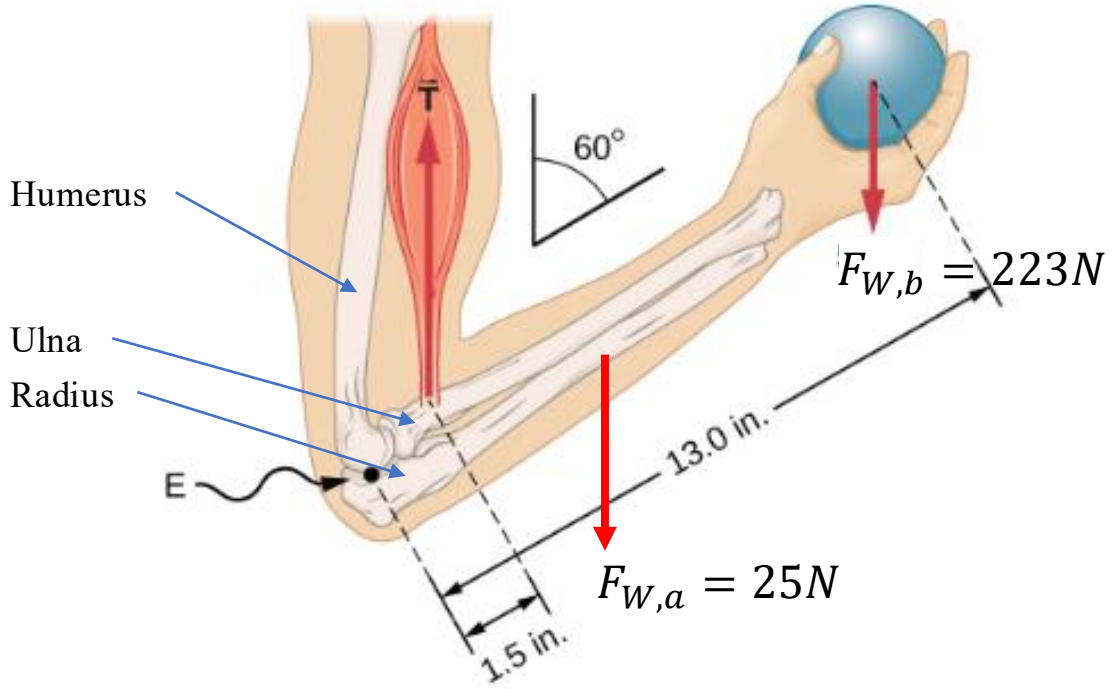


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Rotational and Translational Statics

Example 5 Solution:

The reaction force of the elbow joint:

$$F_E = -F_T + F_{W,a} + F_{W,b}$$

$$F_E = -2041N + 25N + 223N = -1793N$$

The reaction force of the elbow joint points in the negative y-direction. The upwards tension force is huge and pulls the arm up into the the bottom of the humerus. The humerus must react to that and exerts a downward force (by Newton's 3rd law) onto the elbow.

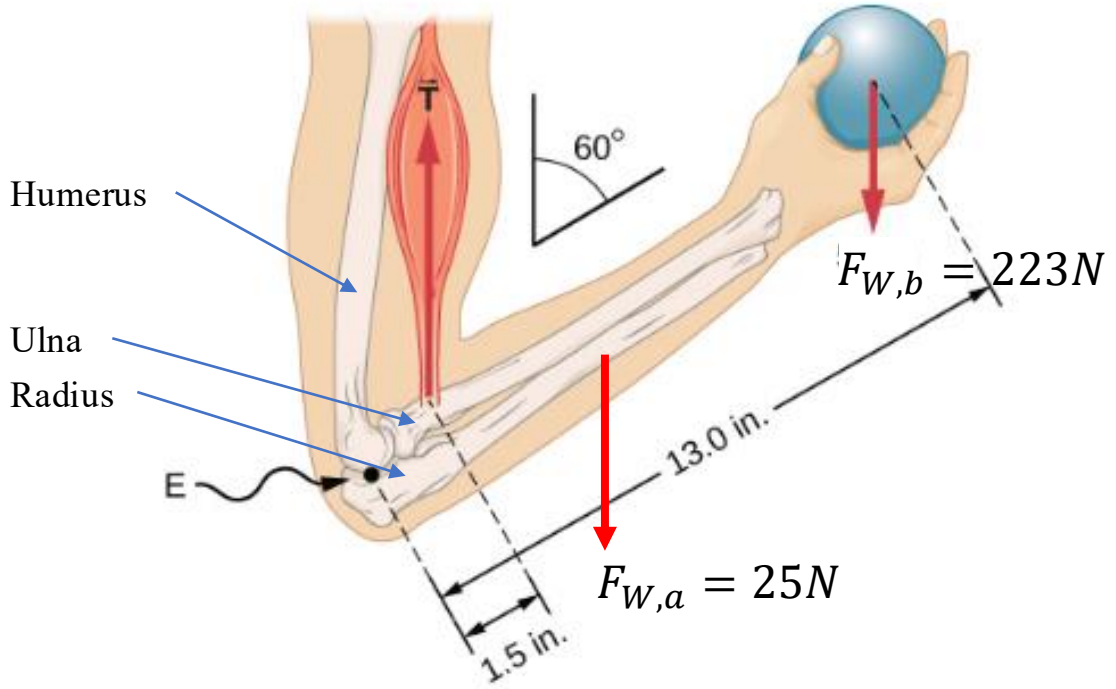
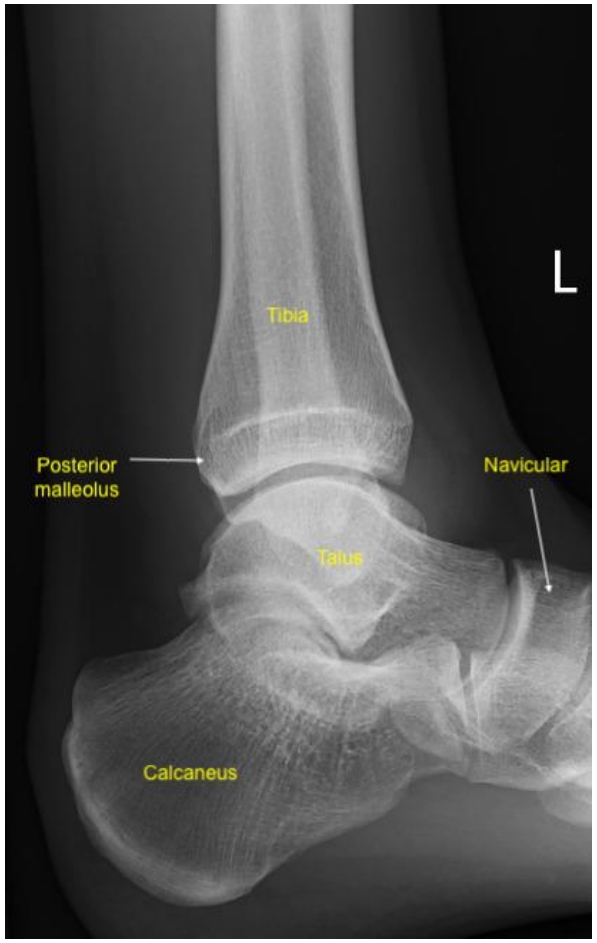


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Rotational and Translational Statics

Example 6:

An $80kg$ person stands on their toes by exerting an upward force through their Achilles tendon. What is the magnitude of the force through the Achilles tendon? What is the magnitude of the force at the pivot, F_P?



https://www.ncbi.nlm.nih.gov/books/NBK542324/figure/article-17597.image.f3/

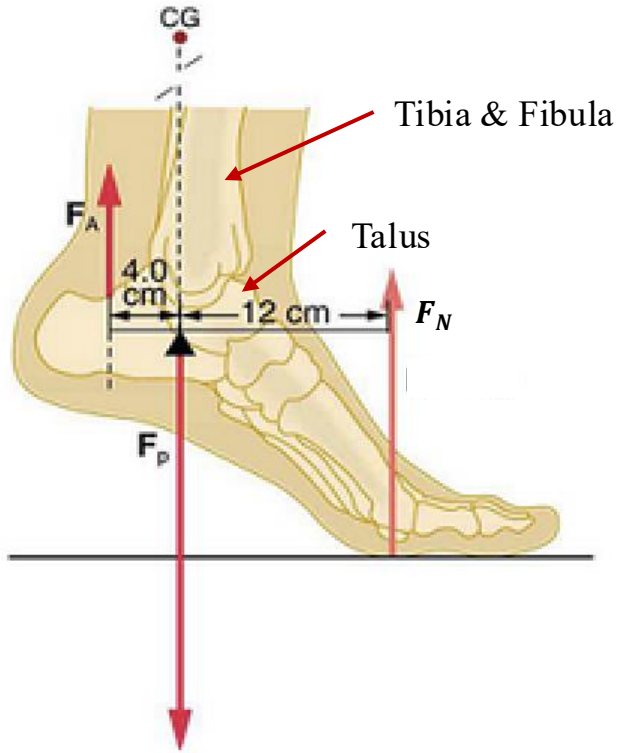


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Rotational and Translational Statics

Example 6 Solution:

From the forces in the vertical direction, assuming that the reaction force of the talus points vertically down (if it does not, the math will work itself out to tell me) we have:

$$-F_T + F_N + F_A = m_{system} a_y^0 = 0$$

$$F_T = F_N + F_A$$

To determine the force exerted on the talus, we need to know the force that the Achillies tendon exerts.

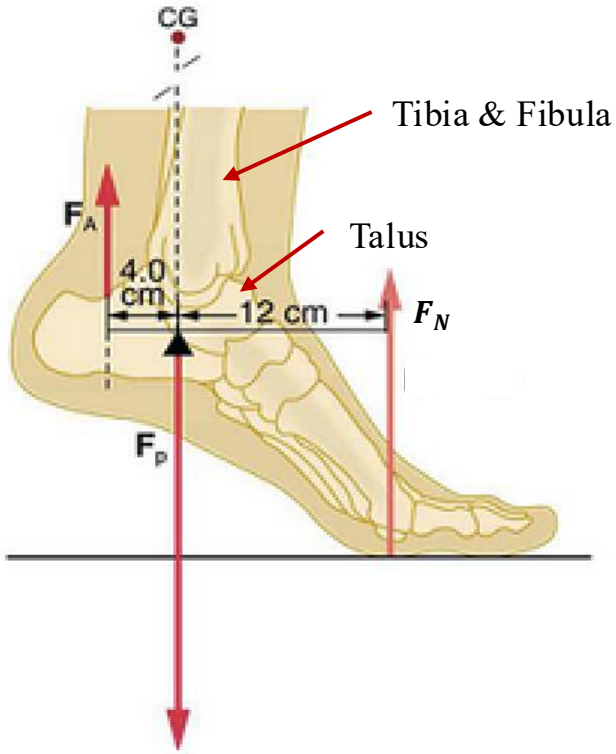


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Rotational and Translational Statics

Example 6 Solution:

Taking the pivot to be at the talus and looking at the sum of the torques we have for the tension force in the Achilles tendon:

$$-\tau_{F_A} + \tau_{F_N} = I_{system}\cancel{\alpha}^{0}$$

$$-r_{F_A}F_A \sin\beta + r_{F_N}F_N \sin\gamma = -\underbrace{\left(r_{F_A}\sin\beta\right)}_{r_{\perp,F_A}}F_A + \underbrace{\left(r_{F_N}\sin\gamma\right)}_{r_{\perp,F_N}}F_N = 0$$

$$F_A = \left(\frac{r_{\perp,F_N}}{r_{\perp,F_A}}\right)F_N = \left(\frac{r_{\perp,F_N}}{r_{\perp,F_A}}\right)\frac{F_W}{2} = \left(\frac{12cm}{4cm}\right)\cdot\frac{80kg\cdot 9.8\frac{m}{s^2}}{2}$$

$$F_A = 1176N$$

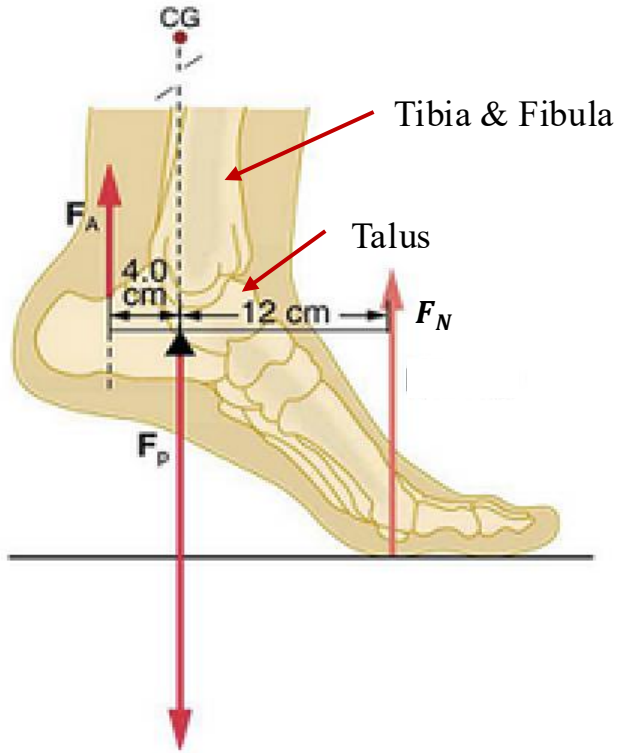


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Rotational and Translational Statics

Example 6 Solution:

To force exerted on the talus:

$$F_T = F_N + F_A = \frac{F_W}{2} + F_A$$

$$F_T = \frac{80kg \cdot 9.8\frac{m}{s^2}}{2} + 1176N = 1568N$$

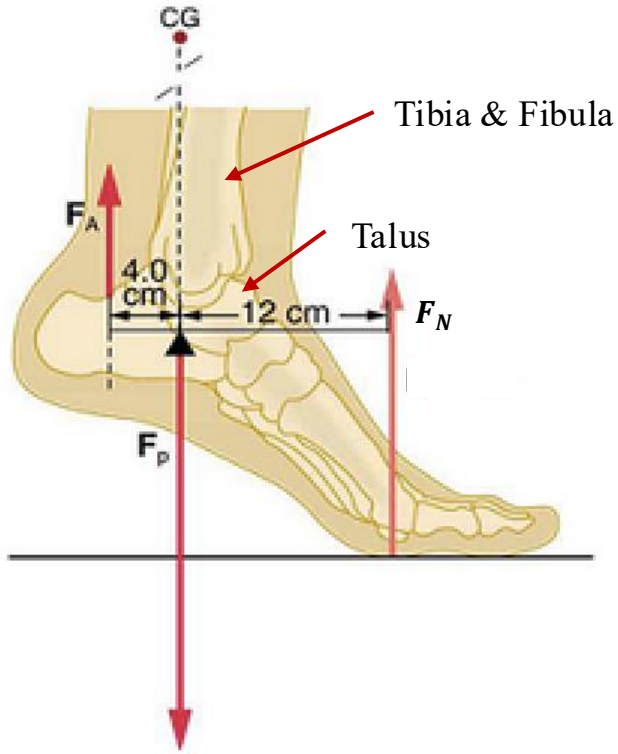


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Rotational and Translational Statics

Example 7:

As high as eighty percent of the population at one time or another will suffer from some form of lower back pain, especially during bending and lifting activities.

The stresses, which the mussels apply to the disks located between each of the vertebrae, can be very large and these stresses can, along with degeneration of the structure of the disks produce pain, muscle spasm, and immobilization of the lower back. We'll specifically look at the lumbo-sacral intersection of the spine as the point at which we bend, to say pick something up when you keep your legs straight.

We'll look at a specific case of just bending over with your arms hanging vertically to determine how large the reaction force on the lumbo-sacral disk can be in this case in terms of your weight.

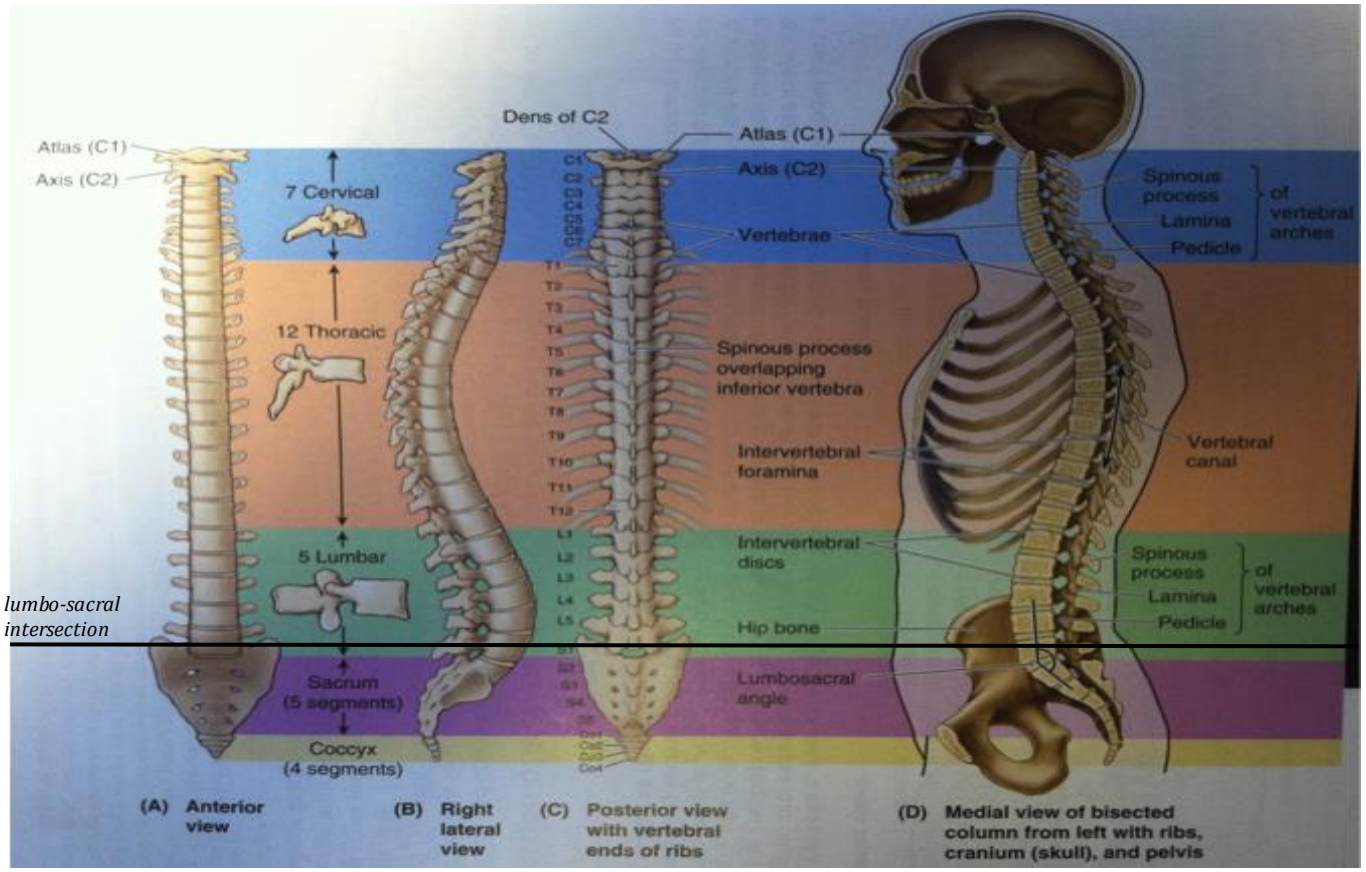


Figure 1: Views of the human spine. Figure from Clinically Oriented Anatomy, by Moore & Dalley.

Rotational and Translational Statics

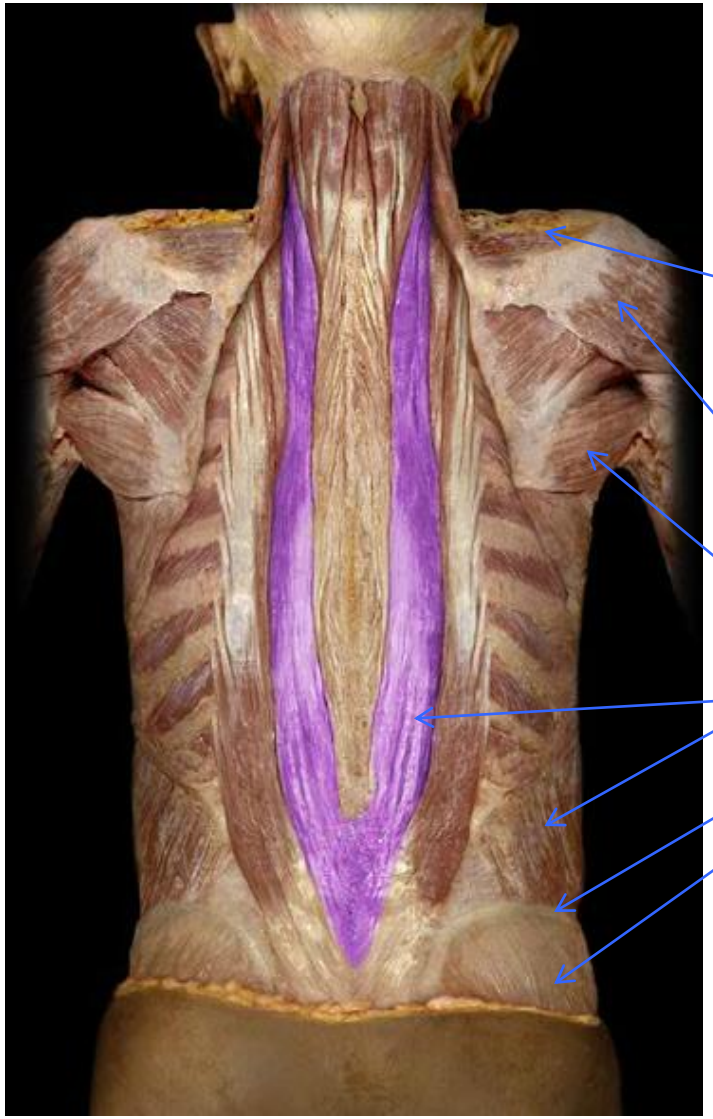
- *The Major Muscle Groups and Vertebral Column of the Human Back*

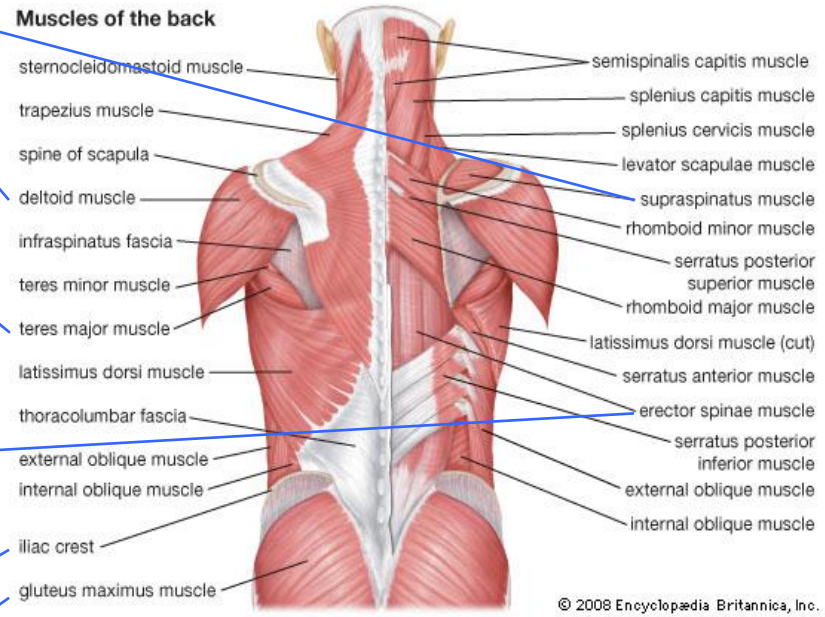
Example 7:

Cadaver dissection showing the location of the *erector-spinae* muscles (stained purple for visualization).

The major muscle groups and muscles of the back are also shown.

The *erector-spinae* muscles insert at the base of the spine and run the length of your spinal column. These muscles are, in part, responsible for picking you back upright (making your erect) from a prone (or bent over) position.





https://www.studyblue.com/notes/note/n/huaty-1223-study-guide-2011-12-rooney/deck/9730325

Rotational and Translational Statics

- *The Major Muscle Groups and Vertebral Column of the Human Back*

Example 7:

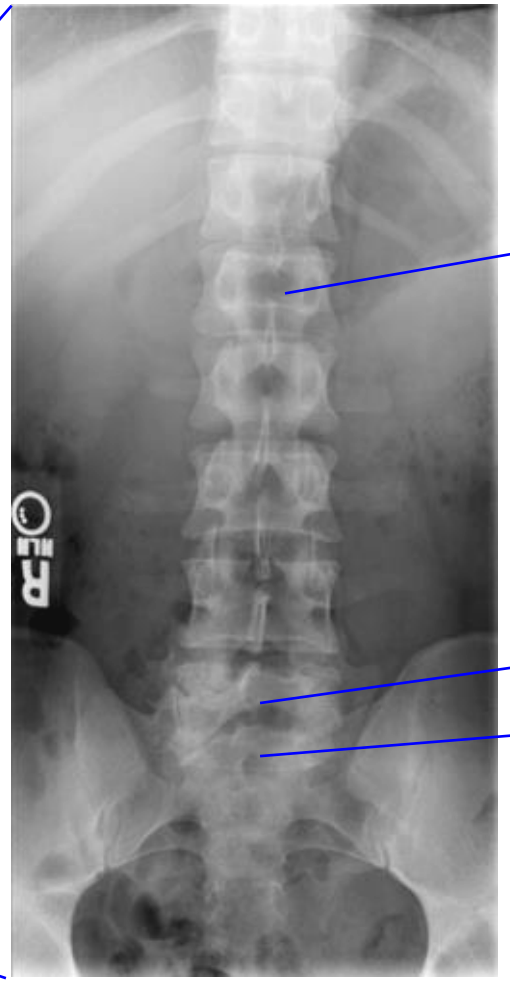
Cadaver dissection showing the location of the *erector spinae* muscles groups on either side of the spine and the vertebra of the spine.

Here it's been superimposed an x-ray image of the spine onto the back showing the erectors lie just to the side of the spinal column.

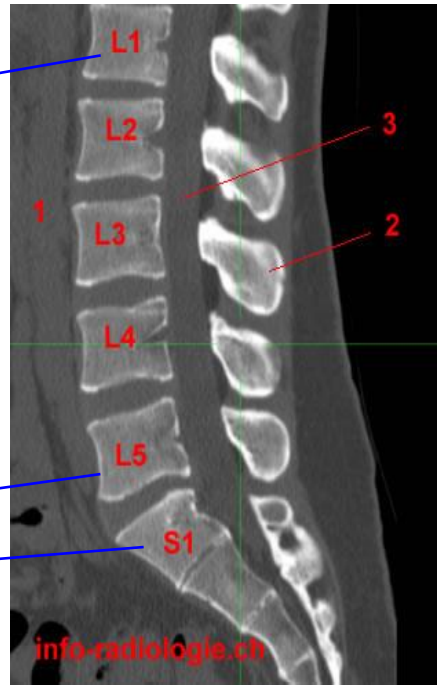
The magnetic resonance (MRI) image of the spinal column show the location of interest at the L5/S1 intersection. This is the disk (between the vertebra) that is often ruptured by lifting heavy loads incorrectly.



https://www.studyblue.com/notes/note/n/huaty-1223-study-guide-2011-12-rooney/deck/9730325



http://medicine.missouri.edu/radiology/iCases/MSK/case1/case01.htm



http://w-radiology.com/ct-lumbar-spine.php

Rotational and Translational Statics

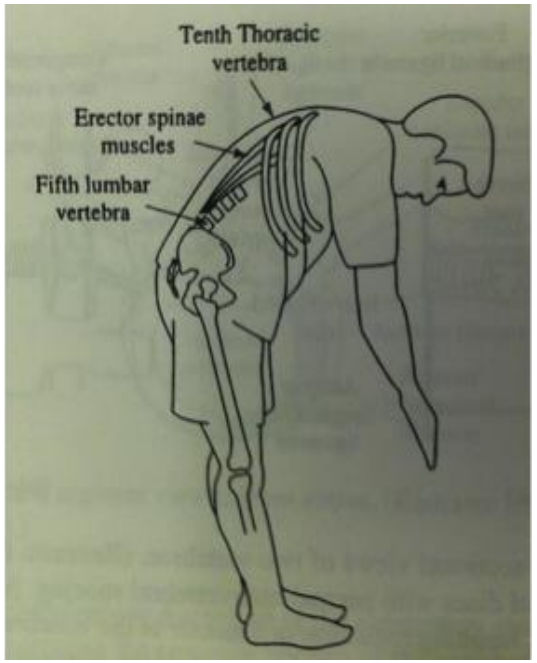
Example 7:

- What are the expressions for the sum of the forces in the vertical and horizontal directions?

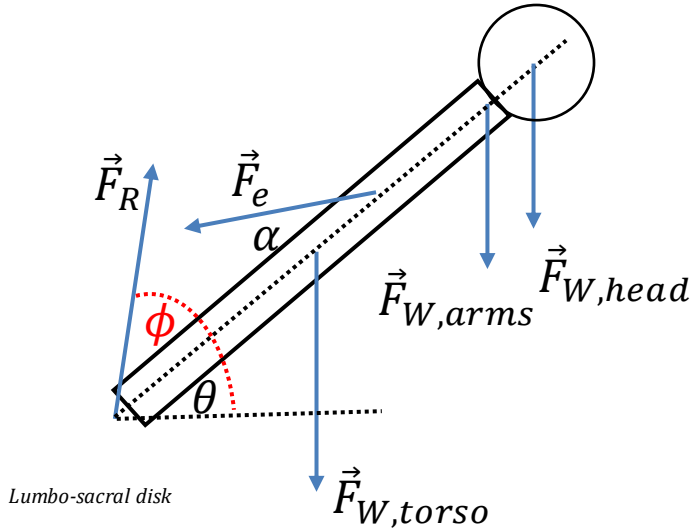
- What is the expression for the sum of the torques about the lumbo-sacral disk? We'll assume that you have bent over so that your spinal column makes an angle θ with respect to the horizontal and are not moving.

- What is the magnitude of the force F_e exerted by the *erector spinae* muscles at a point two-thirds the length (L) of your spine? Assume that $\theta = 30^0$, $\alpha = 12^0$, $F_{W,torso} = 0.5F_W$ located at $0.5L$, $F_{W,arms} = 0.1F_W$ located at $0.75L$, and $F_{W,head} = 0.07F_W$ located at L.

- What is the magnitude and direction of the reaction force F_R at the base of the spine, or at the lumbo-sacral joint, in terms of your weight F_W?



Cartoon illustration of the problem.
Drawing from *Physics with Illustrative Examples from Medicine and Biology*, by Benedek & Villars.

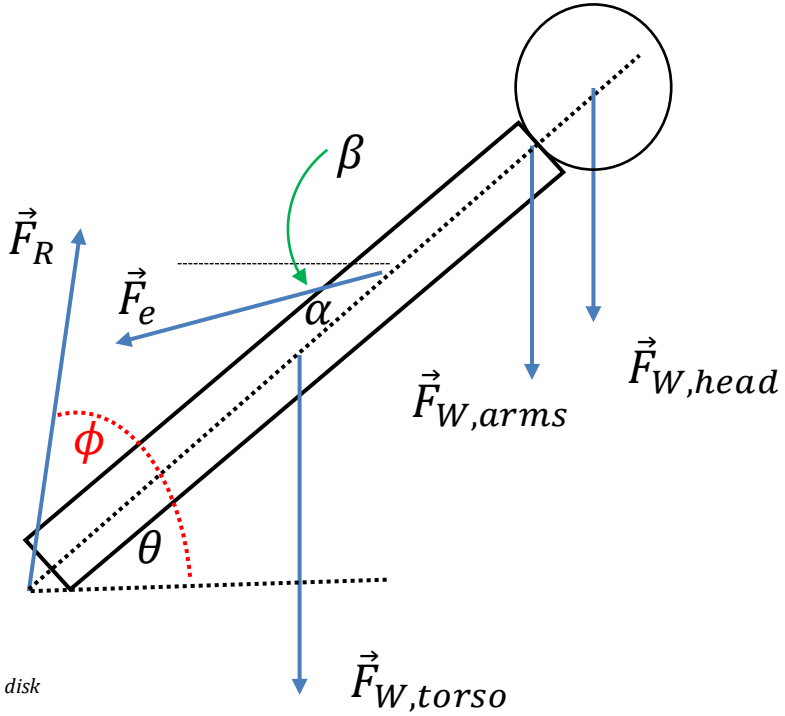


Stick figure drawing of your spinal column showing the various forces that act. The diagram is not to scale

Rotational and Translational Statics

Example 7 Solution:

- Assuming a standard cartesian coordinate system break the forces up into their horizontal and vertical components and apply the condition that the horizontal and vertical forces have to vanish.



Stick figure drawing of your spinal column showing the various forces that act. The diagram is not to scale

Horizontal forces

$$F_{Rx} - F_{ex} = F_R \cos\phi - F_e \cos\beta = m_{system} \overset{0}{a_x} = 0$$

$$\rightarrow \boxed{F_{Rx} = F_R \cos\phi = F_e \cos\beta}$$

This is the horizontal component of the reaction force of the spine. I need to determine F_e and from the geometry, $\theta = \alpha + \beta \rightarrow \beta = \theta - \alpha$.

Vertical forces

$$F_{Ry} - F_{ey} - F_{Wtorso} - F_{Warms} - F_{Whead} = F_R \sin\phi - F_e \sin\beta - F_{Wtorso} - F_{Warms} - F_{Whead} = m_{system} \overset{0}{a_y} = 0$$

$$\rightarrow \boxed{F_{Ry} = F_R \sin\phi = F_e \sin\beta + F_{Wtorso} + F_{Warms} + F_{Whead}}$$

This is the vertical component of the reaction force of the spine. I need to determine F_e and from the geometry, $\theta = \alpha + \beta \rightarrow \beta = \theta - \alpha$.

Rotational and Translational Statics

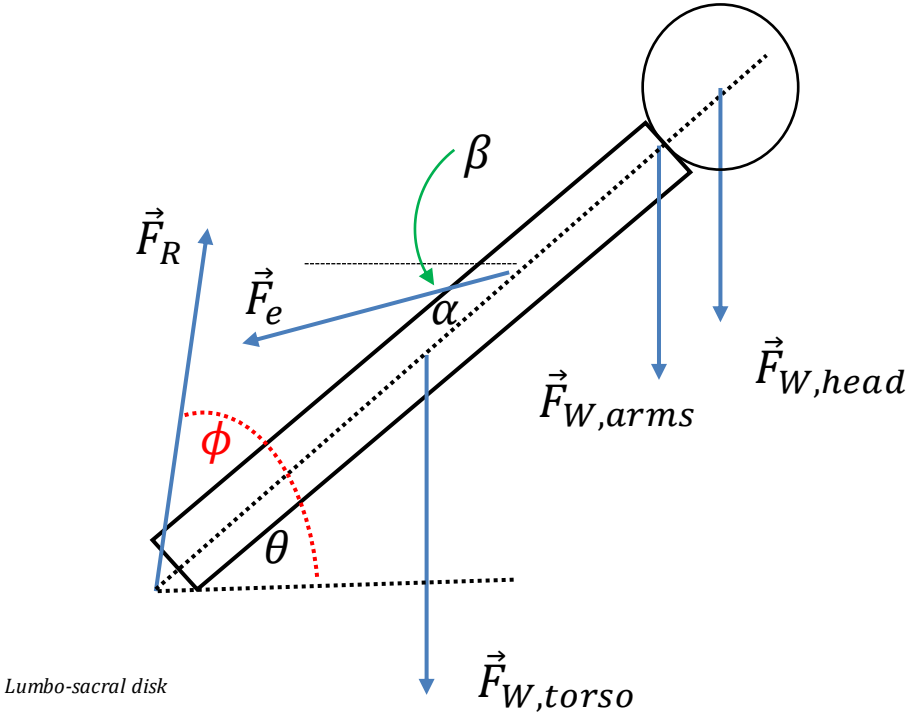
Example 7 Solution continued:

- The last two expressions will allow us to determine the reaction force of the spine, F_R, at the lumbo-sacral disk if we can determine the force the erector spinae muscles pull with. Unfortunately, they do not tell us the magnitude of F_e.

- To determine F_e we'll determine the torques in the system subject to the condition that the net torque has to vanish if you are not moving (rotating).

- You can take the pivot anywhere you'd like, but this problem wants us to take the pivot to be at the base of the spine. This way any equations I write will involve the force due to the erector-spinae muscles.

- Assume clockwise rotations are negative and counterclockwise rotations are positive.



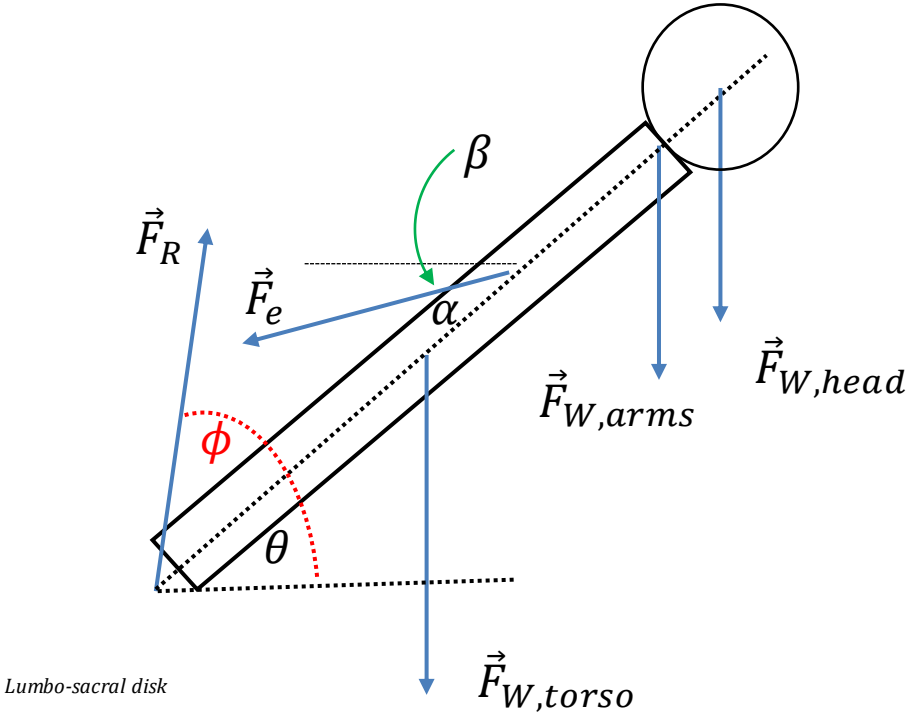
Stick figure drawing of your spinal column showing the various forces that act. The diagram is not to scale

$$\tau_{F_R}^{0} + \tau_{F_e} + \tau_{F_{W torso}} + \tau_{F_{W arms}} + \tau_{F_{W head}} = I_{system}\alpha^{0} = 0$$

Rotational and Translational Statics

Example 7 Solution continued:

- Starting at the base of the spine we'll calculate the torque due to each of the forces.
- The reaction force of the spine acts at the pivot, so $\tau_{F_R} = 0$.
- The weight of the torso acts at one-half of the length $(0.5L)$ of the spine and tends to rotate the system clockwise. The torque due to the weight of the torso is $\tau_{F_{Wtorso}} = -0.5LF_{Wtorso}\sin(90-\theta)$.
- The weight of the arms acts at say 90% of the length of the spine $(0.9L)$ and tends to rotate the system clockwise. The torque due to the weight of the arms is $\tau_{F_{Warms}} = -0.9LF_{Warms}\sin(90-\theta)$.
- The head acts the upper end of your spine (L) and the head tends to rotate the system clockwise. The torque due to your head is $\tau_{F_{Whead}} = -LF_{Whead}\sin(90-\theta)$.



Stick figure drawing of your spinal column showing the various forces that act. The diagram is not to scale

- The *erector-spinae* muscles act at two-thirds of the length of your spine $(0.67L)$ and the erector-spinae muscles tends to rotate the system counterclockwise to stand you back upright and make you erect. The torque due to the erector-spinae muscles is $\tau_{F_e} = 0.67LF_e\sin\alpha$.

Rotational and Translational Statics

Example 7 Solution continued:

- Now, substitute the expressions we determined for the torques into the net toque equation we get:

$$\cancel{\tau_{F_R}}^{0} + \tau_{F_e} + \tau_{F_{Wtorso}} + \tau_{F_{Warms}} + \tau_{F_{Whead}} = I_{system}\cancel{\alpha}^{0} = 0$$

$$0.67LF_e \sin\alpha - 0.5LF_{Wtorso} \sin(90-\theta) - 0.9LF_{Warms} \sin(90-\theta) - LF_{Whead} \sin(90-\theta) = 0$$

- Using the fact that $\sin(90-\theta) = \cos\theta$ and dividing out the common length of the spine (L) term we get:

$$0.67LF_e \sin\alpha - 0.5LF_{Wtorso} \cos\theta - 0.9LF_{Warms} \cos\theta - LF_{Whead} \cos\theta = 0$$

- We'll solve this expression for the magnitude of the force that the erector-spinae muscles exert to stand you back upright.

$$F_e = \frac{0.5F_{Wtorso} \cos\theta + 0.9F_{Warms} \cos\theta + F_{Whead} \cos\theta}{0.67\sin\alpha} = \left\{\frac{[(0.5\times0.5) + (0.9\times0.1) + (0.07)]\cos 30}{0.67 \sin 12}\right\} F_W$$

$$\rightarrow \boxed{F_e = 2.55F_W}$$

The *erector spinae* muscles exert a force F_e that is over two and one-half times your weight to pull you back upright. This is huge and its why you can strain a muscle in your back easily with these large forces.

Rotational and Translational Statics

Example 7 Solution continued:

- Knowing the magnitude of the force the erector-spinae muscles exert we can determine the magnitude and direction of the reaction force at the base of the spine at the lumbo-sacral joint.

The horizontal component of F_R.

$$F_{Rx} = F_R \cos\phi = F_e \cos\beta = 2.55F_W \cos(30 - 12)$$

$$\longrightarrow F_{Rx} = 2.43F_W$$

The vertical component of F_R.

$$F_{Ry} = F_R \sin\phi = F_e \sin\beta + F_{Wtorso} + F_{Warms} + F_{Whead} = 2.55F_W \sin(30 - 12) + 0.5F_W + 0.1F_W + 0.07F_W$$

$$\longrightarrow F_{Ry} = 1.46F_W$$

The magnitude of F_R.

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(2.43F_W)^2 + (1.46F_W)^2} = 2.83F_W$$

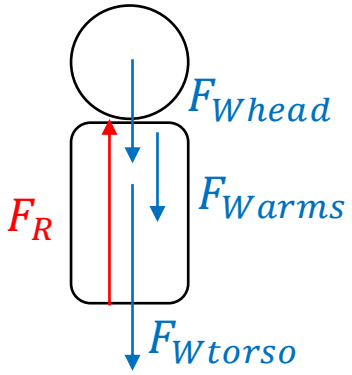
The direction of F_R.

$$\phi = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{1.46F_W}{2.43F_W}\right) = 30^0$$

Rotational and Translational Statics

Example 7 Solution continued:

- From the calculations, the reaction force at the base of your spine (F_R) is almost three times your body weight ($2.8F_W$).
- This result is for everyone since you can plug in your own body weight. This is an incredibly huge number and explains why you should never bend over to pick something up.
- We haven't even included a load yet. Picking up an object would only add to this value and the risk for popping the disk at the lumbo-sacral (L5/S1 joint) intersection in your lower back.
- The direction of the reaction force is in line with your spine which also explains why you should bend with your knees to pick something up. If you bent at your knees with your spine in a vertical line, the reaction force from your spine would be much less.



$$F_R - F_{Wtorso} - F_{Warms} - F_{Whead} = m_{body}a_x = 0$$

$$F_R = F_{Wtorso} + F_{Warms} + F_{Whead} = (0.5 + 0.1 + 0.07)F_W$$

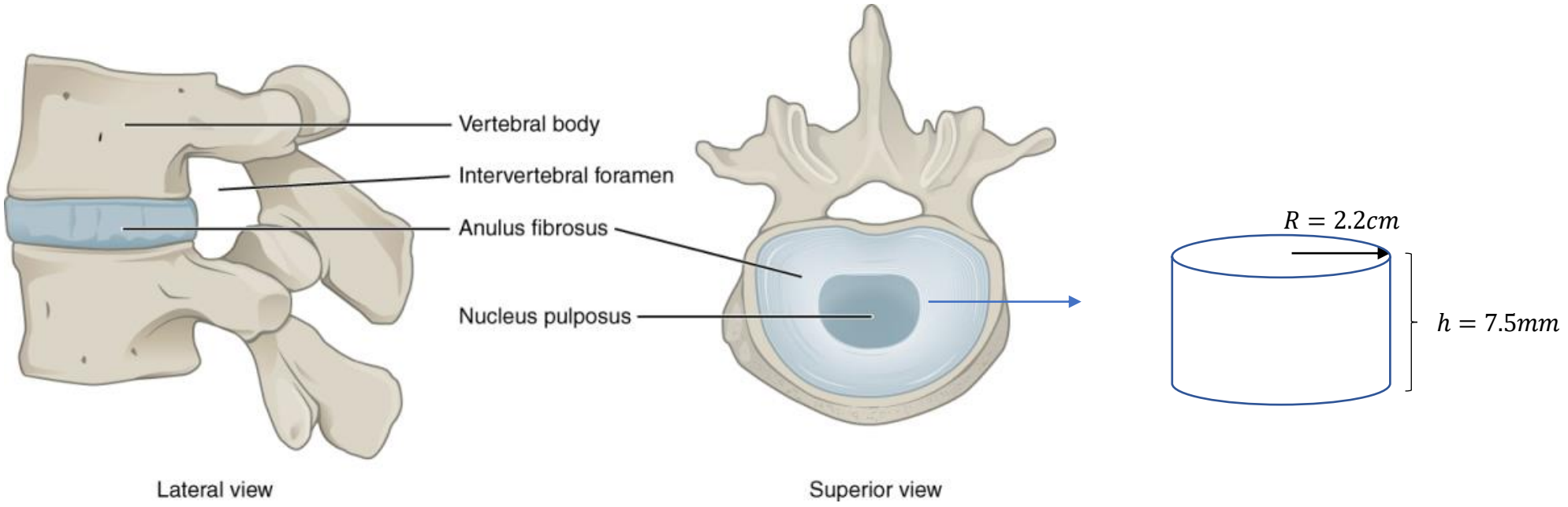
$$\rightarrow \boxed{F_R = 0.67F_W}$$

Bending and lifting with your leg the reaction force of your spine is a fraction of your body weight. The erector-spinae muscles don't have to work to stand you back upright (you're upright already). Picking up a load here puts a much smaller weight on the base of your spine reducing possible injury.

Rotational and Translational Statics

Example 8:

- The intervertebral disk approximates a cylinder with the dimensions shown below. If the elastic modulus of the disk is $E = 25MPa = 25\times10^6\frac{N}{m^2}$, by how much does the disk compress under the weight of your torso/arms/head while standing erect? Under the weight of your torso/arms/head when you're bent over? Assume you have a mass $70kg$ ~$155\ lbs$.



https://www.ncbi.nlm.nih.gov/books/NBK470583/

Rotational and Translational Statics

Example 8 Solution:

Using Hooke's law in its general form we have for the change in length:

$$stress = E \cdot strain \rightarrow \frac{F}{A} = E \cdot \frac{\Delta l}{l} \rightarrow \Delta l = \frac{Fl}{EA}$$

Standing upright:

$$\Delta l = \frac{Fl}{EA} = \frac{F_{R,spine}\, l}{EA} = \frac{0.67 F_W l}{EA} = \frac{0.67 \times 70kg \times 9.8\frac{m}{s^2} \cdot 7.5\times10^{-3}m}{25\times10^6\frac{N}{m^2} \cdot \pi \cdot (0.022m)^2} = 9.1\times10^{-5}m = 91\mu m$$

Bent over:

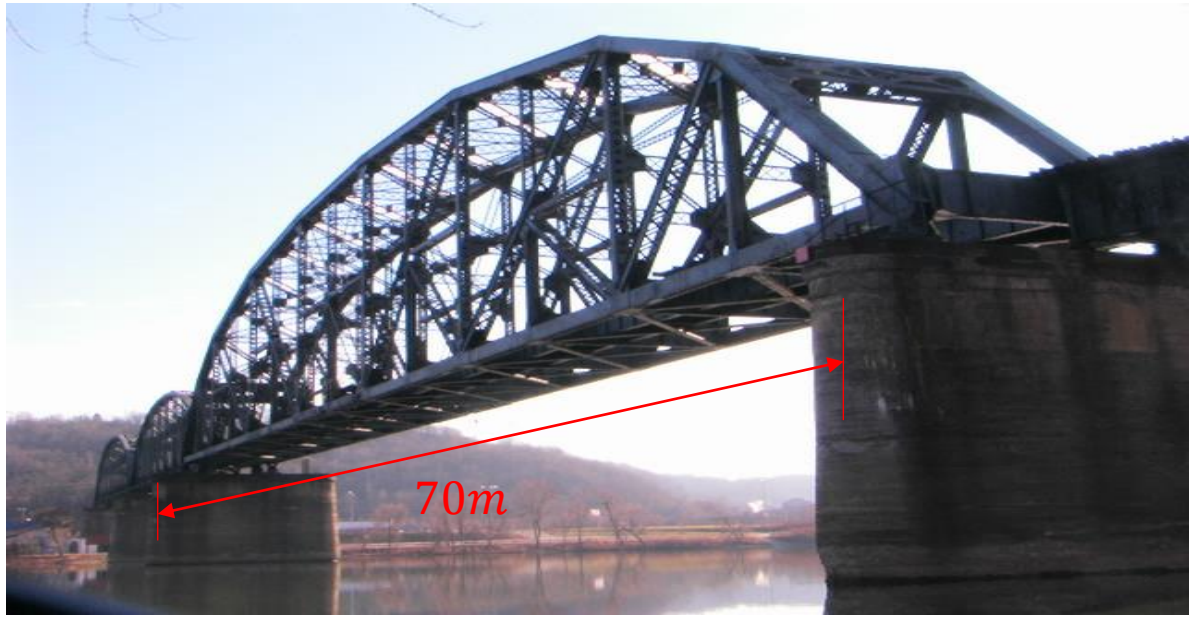
$$\Delta l = \frac{Fl}{EA} = \frac{F_{R,spine}\, l}{EA} = \frac{2.83 F_W l}{EA} = \frac{2.83 \times 70kg \times 9.8\frac{m}{s^2} \cdot 7.5\times10^{-3}m}{25\times10^6\frac{N}{m^2} \cdot \pi \cdot (0.022m)^2} = 3.8\times10^{-4}m = 383\mu m$$

Rotational and Translational Statics

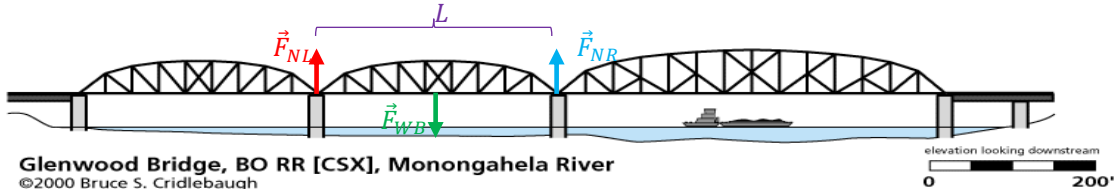
Example 9:

The Glenwood B&O Railroad Bridge in Pittsburgh Pa, that is an example of truss bridge which carries Allegheny Valley Railroad's W&P Subdivision over the Monongahela River.

The span was constructed in 1884 and upgraded in 1915. Suppose that a train engine ($F_{WE} = 220{,}000N$) travels across a section of this truss bridge that is $70m$ long with a weight of $F_{WB} = 230{,}000N$ that is supported by pylons at each end along each section of the span.



https://en.wikipedia.org/wiki/Glenwood_B%26O_Railroad_Bridge



http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm

- What are the maximum and minimum forces exerted on each pylon due to the crossing when there is no train engine on the bridge?

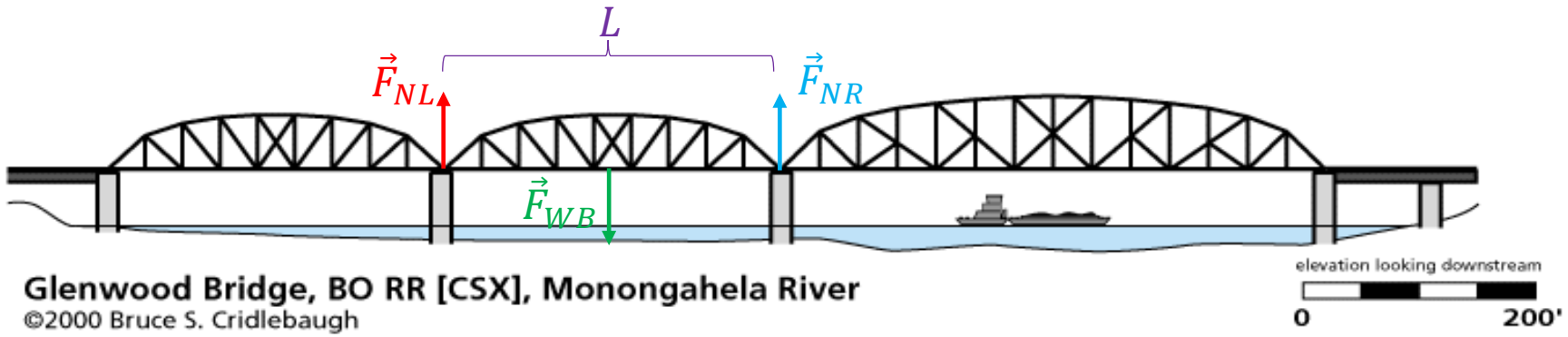
- What are the maximum and minimum forces exerted on each pylon due to the crossing when train engine on both the left and right sides of the bridge?

- Does the location of the pivot affect these forces exerted on the pylons?

Rotational and Translational Statics

Example 9 Solution:

- Consider the center section of the bridge in the elevation drawing below. The span has a length of $L = 70m$.
- Let's first calculate the weight that each pylon has to support due to the weight of the bridge.
- The forces that the pylon have to exert are the reaction forces to the weight of the bridge and so what we're going to calculate are the reaction (or the normal) forces for the pylons. Let's call $\vec{F}_{NL}$ the normal force of the left pylon and $\vec{F}_{NR}$ for the normal force due to the right pylon.
- Assuming a uniform distribution of mass, we'll put the weight of the bridge $\vec{F}_{WB}$ at its center-of-mass.
- Labeling the forces on the diagram below we have:



http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm

Rotational and Translational Statics

Example 9 Solution continued:

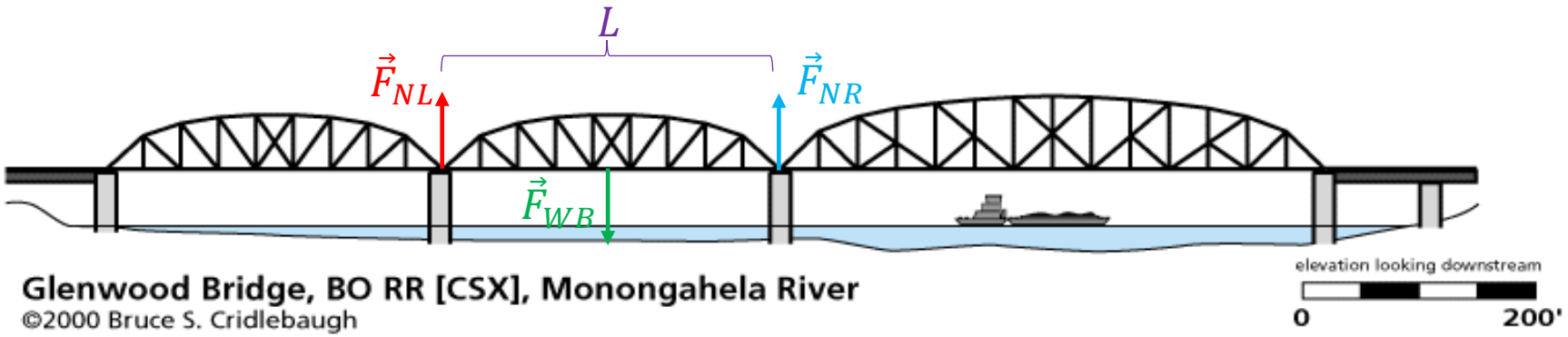
- To calculate the fraction of the weight supported by each pylon we assume a standard cartesian coordinate system and require that the horizontal and vertical forces along with the net torque sum to zero.

$$F_{NL} + F_{NR} - F_{WB} = m_B \cancel{a_y}^{0} = 0 \rightarrow F_{NL} + F_{NR} = F_{WB}$$

This says that the sum of both reactions forces of the pylons support the weight of the bridge. We just don't know how much each supports.

- We need to take the sum of the torques about some convenient point. There are many that you could choose from. Let's start with taking the sum of the torques about the left end of the bridge with clockwise torques being negative and counterclockwise torques being positive.

- Then we'll move the pivot point and see if we get the same result.



http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm

Rotational and Translational Statics

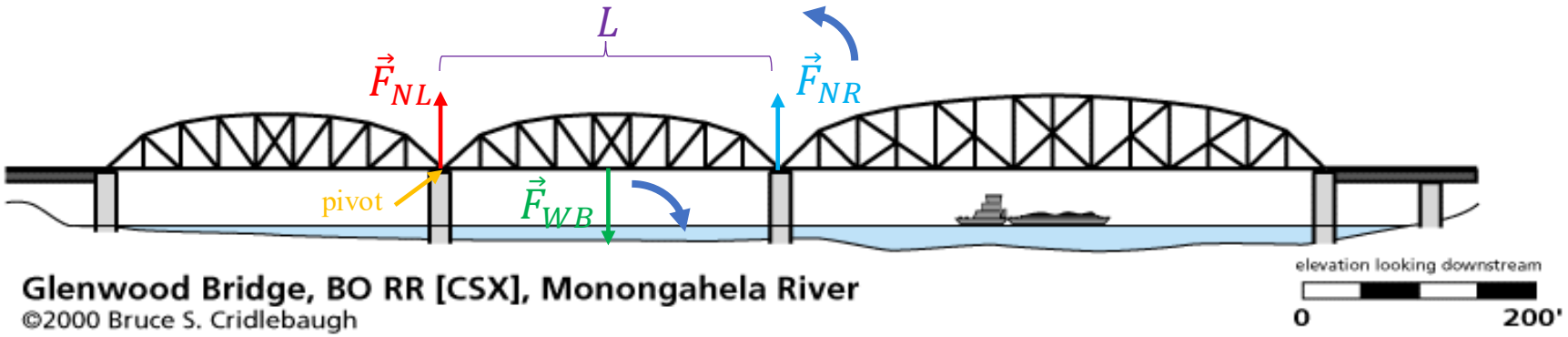
Example 9 Solution continued:

- Taking the pivot about the left end of the span and applying the condition that the net torque is zero we get:

$$\tau_{F_{NL}} + \tau_{F_{NR}} + \tau_{F_{WB}} = I\cancel{\alpha}^{0} = 0$$

- All torques look like a product of a moment arm (distance from the pivot to the applied force), the applied force, and the sine of the angle between the moment arm and the applied force, $\tau = rF\sin\theta$.

- Since the pivot is at the left pylon, the moment arm to this force is zero and exerts no torque on the system.



http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm

Rotational and Translational Statics

Example 9 Solution continued:

- The weight of the bridge F_{WB} acts at the midpoint $(\frac{L}{2})$ and is perpendicular to the moment arm, so $\theta_{WB} = 90$ and the weight would tend to rotate the system clockwise. Thus the torque due to the weight of the bridge is negative.

- The normal force due to the right pylon F_{NR} acts at the full length of the bridge (L) and is perpendicular to the moment arm, so $\theta_{NR} = 90$ and the normal force would tend to rotate the system counterclockwise. Thus the torque due to the right pylon is positive.

$$\tau_{F_{NL}} + \tau_{F_{NR}} + \tau_{F_{WB}} = -\frac{L}{2}F_{WB}\sin\theta_{WB} + LF_{NR}\sin\theta_{NR} = 0 \rightarrow \boxed{F_{NR} = \frac{F_{WB}}{2}}$$

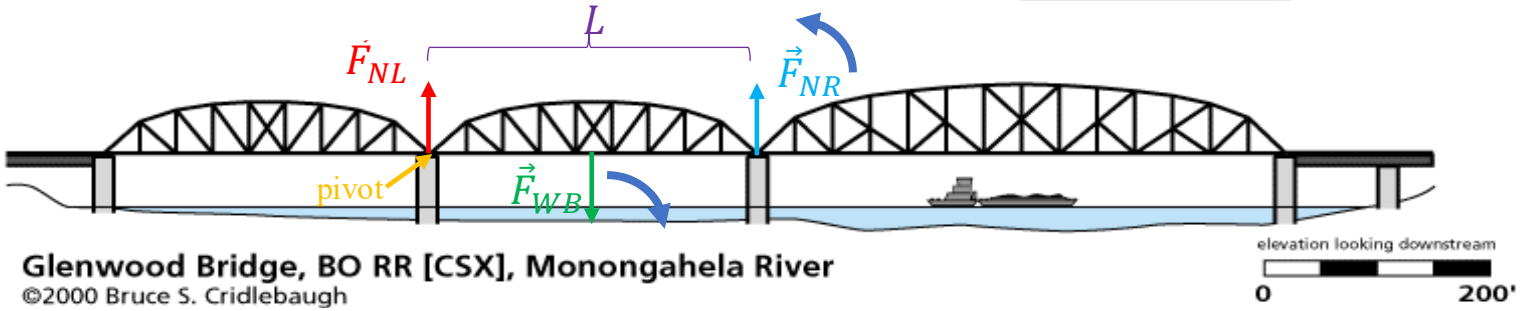
This says that the right pylon has to support one-half of the weight of the bridge.

That seems reasonable. And the left pylon must support the other one-half of the weight of the bridge.

- From the forces we get F_{NL}.

$$F_{NL} + F_{NR} = F_{NL} + \frac{F_{WB}}{2} = F_{WB} \rightarrow \boxed{F_{NL} = \frac{F_{WB}}{2}}$$

The left pylon support the other one-half of the weight of the bridge.



http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm

Rotational and Translational Statics

Example 9 Solution continued:

- What if we redo the problem by taking the pivot about the right end of the span and applying the condition that the net torque is zero, we get:

- The weight of the bridge F_{WB} acts at the midpoint $(\frac{L}{2})$ and is perpendicular to the moment arm, so $\theta_{WB} = 90$ and the weight would tend to rotate the system counterclockwise. Thus, the torque due to the weight of the bridge is positive.

- The normal force due to the right pylon F_{NL} acts at the full length of the bridge (L) and is perpendicular to the moment arm, so $\theta_{NL} = 90$ and the normal force would tend to rotate the system clockwise. Thus, the torque due to the right pylon is negative.

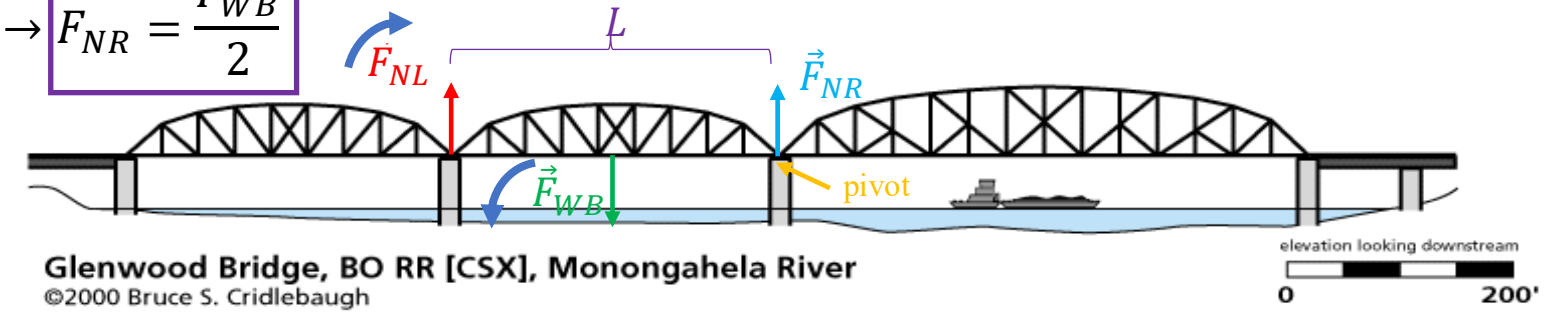
$$\tau_{F_{NL}} + \tau_{F_{NR}} + \tau_{F_{WB}} = -LF_{NL}\sin\theta_{NL} + \frac{L}{2}F_{WB}\sin\theta_{WB} = 0 \rightarrow \boxed{F_{NL} = \frac{F_{WB}}{2}}$$

The left and right pylons each support one-half of the weight of the bridge.

This is the same result taking the pivot on the left end.

- From the forces we get F_{NR}.

$$F_{NL} + F_{NR} = \frac{F_{WB}}{2} + F_{NR} = F_{WB} \rightarrow \boxed{F_{NR} = \frac{F_{WB}}{2}}$$



http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm

Rotational and Translational Statics

Example 9 Solution continued:

- One last time just to make sure the location of the pivot does not matter. Let's take the pivot at the weight of the bridge and apply the condition that the net torque is zero we get:

- The reaction force of the left pylon F_{NL} acts at a point half the length of the bridge away $(\frac{L}{2})$ and is perpendicular to the moment arm, so $\theta_{NL} = 90$ and the weight would tend to rotate the system clockwise. Thus the torque due to the left normal force is negative.

- The reaction force of the right pylon F_{NR} acts at a point half the length of the bridge away $(\frac{L}{2})$ and is perpendicular to the moment arm, so $\theta_{NR} = 90$ and this normal force would tend to rotate the system counterclockwise. Thus the torque due to the right normal force is positive.

$$\tau_{F_{NL}} + \tau_{F_{NR}} + \tau_{F_{WB}} = -\frac{L}{2}F_{NL}\sin\theta_{NL} + \frac{L}{2}F_{NR}\sin\theta_{NR} = 0 \rightarrow F_{NL} = F_{NR}$$

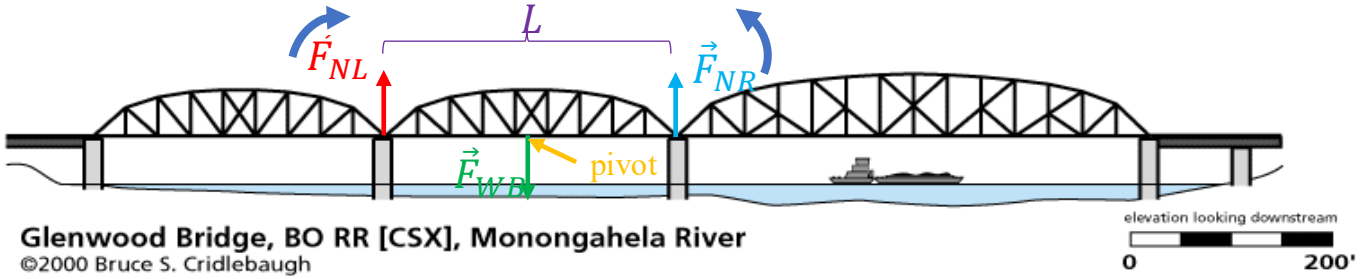
The weight acts at the pivot and exerts no torque.

The left and right reaction forces are equal but we don't know to what. To find out what, we use forces.

- From the forces we get F_{NL} or F_{NR}.

$$F_{NL} + F_{NR} = 2F_{NL} = F_{WB} \rightarrow F_{NL} = \frac{F_{WB}}{2}$$

$$F_{NL} + F_{NR} = 2F_{NR} = F_{WB} \rightarrow F_{NR} = \frac{F_{WB}}{2}$$



http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm

Rotational and Translational Statics

Example 9 Solution continued:

- In the last three parts we put the pivot in three separate locations, and we are able to determine the reaction forces exerted by each pylon in terms of the weight of the bridge.

- From here of course we could calculate the numbers that correspond to the normal forces, but I'm going to not calculate any. I'm going to express all of my answers in terms of the weight of the bridge to see if my answers are reasonable.

- For just the bridge, the answers that the left and right pylons support one-half of the weight of the bridge seems reasonable.

- What if we put the train engine on the left end (and then the right end) of the bridge? What do we expect for the normal forces due to the left and right pylons?

- As a guess, when the train engine is on the left end, the left pylon should support the weight of the engine and one-half of the weight of the bridge. The right pylon should support the other one-half of the weight of the pylon.

- When the train engine is on the right end, the left pylon should support one-half of the weight of the bridge. The right pylon should support the other one-half of the weight of the pylon and the weight of the engine.

- Now, let's apply the net force and the net torque are zero and see if we were right in our guesses.

Rotational and Translational Statics

Example 9 Solution continued:

- To calculate the fraction of the weight supported by each pylon we assume a standard cartesian coordinate system and require that the horizontal and vertical forces along with the net torque sum to zero.

$$F_{NL} + F_{NR} - F_{WB} - F_{WE} = m_{B+T}\cancelto{0}{a_y} = 0 \rightarrow F_{NL} + F_{NR} = F_{WB} + F_{WE}$$

This says that the sum of both reactions forces of the pylons support the weight of the bridge plus the weight of the train engine. We just don't know how much each supports.

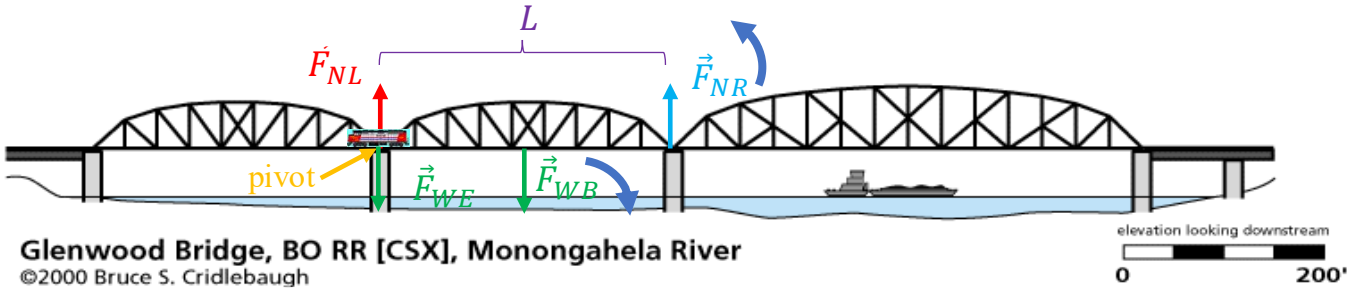
- Let's take the pivot to be at the left end of the bridge since it does not matter where we put it. Thus, we'll take the sum of the torques about the left end of the bridge with clockwise torques being negative and counterclockwise torques being positive.

$$\tau_{F_{NL}} + \tau_{F_{NR}} + \tau_{F_{WB}} + \tau_{F_{WE}} = \cancelto{0}{I\alpha} = 0$$

$$\tau_{F_{NL}} + \tau_{F_{NR}} + \tau_{F_{WB}} + \tau_{F_{WE}} = +LF_{NR}\sin\theta_{NR} - \frac{L}{2}F_{WB}\sin\theta_{WB} = 0 \rightarrow F_{NR} = \frac{F_{WB}}{2}$$

The right normal force acts at the far right end of the bridge while the weight acts at one-half the length of the bridge.

This is as we guessed. The right pylon supports one-half of the weight of the bridge.



Bridge: http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm; train: http://railroadfan.com/gallery/displayimage.php?pos=-68491

Rotational and Translational Statics

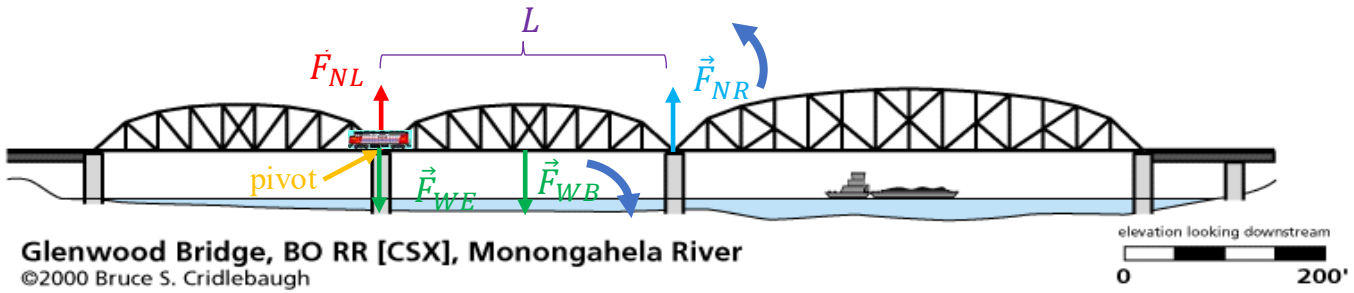
Example 9 Solution continued:

- From the forces we can determine what the left pylon supports now that we know what the right pylon supports.

$$F_{NL} + F_{NR} = F_{WB} + F_{WE}$$

$$F_{NL} + \frac{F_{WB}}{2} = F_{WB} + F_{WE} \rightarrow F_{NL} = \frac{F_{WB}}{2} + F_{WE}$$

The left pylon, as we guessed, supports one-half of the weight of the bridge and the weight of the train engine.



Bridge: http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm; train: http://railroadfan.com/gallery/displayimage.php?pos=-68491

Rotational and Translational Statics

Example 9 Solution continued:

- To find the reaction forces of the pylons if the train engine is on the right end of the bridge take the pivot to be at the left end of the bridge since it does not matter where we put it. Thus, we'll take the sum of the torques about the left end of the bridge again with clockwise torques being negative and counterclockwise torques being positive.

$$\tau_{F_{NL}} + \tau_{F_{NR}} + \tau_{F_{WB}} + \tau_{F_{WE}} = I\alpha^{0} = 0$$

$$\tau_{F_{NL}} + \tau_{F_{NR}} + \tau_{F_{WB}} + \tau_{F_{WE}} = +LF_{NR}\sin\theta_{NR} - \frac{L}{2}F_{WB}\sin\theta_{WB} - LF_{WE}\sin\theta_{WE} = 0 \rightarrow F_{NR} = \frac{F_{WB}}{2} + F_{WE}$$

- From the forces, we can calculate the left reaction force:

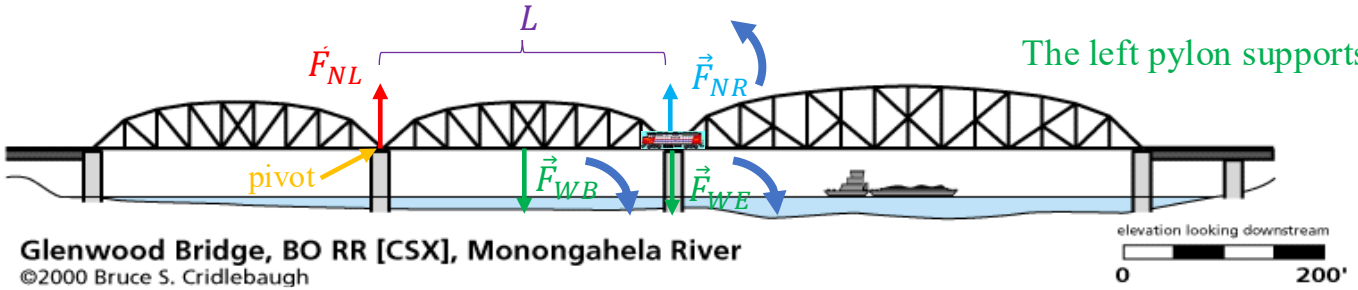
$$F_{NL} + F_{NR} = F_{WB} + F_{WE}$$

$$F_{NL} + \frac{F_{WB}}{2} + F_{WE} = F_{WB} + F_{WE} \rightarrow F_{NL} = \frac{F_{WB}}{2}$$

The right normal force and the weight of the engine act at the far right end of the bridge while the weight acts at one-half the length of the bridge.

The right pylon supports one-half of the weight of the bridge and the full weight of the train engine.

The left pylon supports the other one-half of the weight of the bridge.



Bridge: http://pghbridges.com/pittsburghE/0590-4472/glenwoodBO.htm; train: http://railroadfan.com/gallery/displayimage.php?pos=-68491

Rotational and Translational Statics

Example 9 Solution continued:

- For the train engine on the right side of the bridge, the left pylon must support one-half the weight of the bridge and none of the weight of the engine (when it is on the right side), and the right pylon supports one-half of the weight of the bridge and the full weight of the engine.

- As the train engine moves from the right side of the bridge to the left side of the bridge, the left pylon has to support one-half of the weight of the bridge and an ever-increasing fraction of the weigh of the train engine. When the train engine gets to the left pylon, the left pylon must support the full weight of the train engine and one-half of the weight of the bridge.

- Conversely, as the train engine moves from the right side of the bridge to the left side of the bridge, the right pylon has to support one-half of the weight of the bridge and an ever-decreasing fraction of the weigh of the train engine. When the train engine gets to the left pylon, the right pylon has to support none of the weight of the train engine, only one-half of the weight of the bridge.

- In any of these torque problems it does not matter where we take the pivot, we just need to be consistent with our directions for the torques produced.

Rotational and Translational Statics

Example 9 Solution continued:

- As a last problem, redo the problem putting the railroad crew, F_{WRC}, at a point $\frac{L}{4}$ from the left end. The solutions are:

- For the train engine on the left end of the bridge:

$$F_{NL} = \frac{F_{WB}}{2} + F_{WE} + \frac{3F_{WRC}}{4}$$

$$F_{NR} = \frac{F_{WB}}{2} + \frac{F_{WRC}}{4}$$

- For the train engine on the right end of the bridge:

$$F_{NL} = \frac{F_{WB}}{2} + \frac{3F_{WRC}}{4}$$

$$F_{NR} = \frac{F_{WB}}{2} + F_{WE} + \frac{F_{WRC}}{4}$$