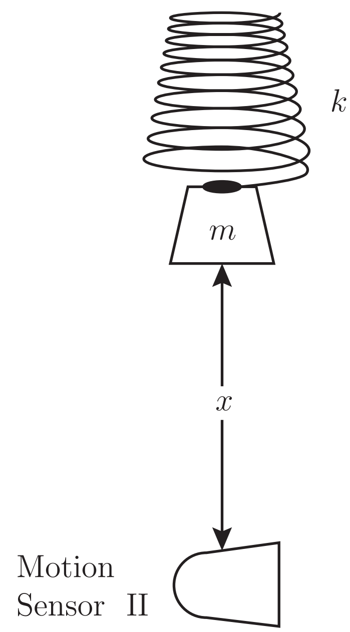
Hooke's Law and Oscillations

Physics 110 Laboratory

In this experiment we will investigate motion under the influence of a variable force. In particular, we first investigate the force that a spring exerts as it stretches. We then study the oscillating motion that a hanging mass experiences once it is suspended from the spring.



Introduction

Springs to a large extent obey **Hooke's law**, which says the force that an elastic object exerts when it is stretched or compressed is directly proportional to the distance over which it is compressed/stretched, namely $F = -kx$, where k is called the spring constant or stiffness, measured in N/m, and x is measured from the unstretched length. Although this "law" is an idealization, it holds true for most elastic media, and can describe a multitude of physical properties, from the operation of digital clocks to the color that a given material reflects. By examining the motion under the influence of a spring that follows Hooke's criterion we can get a broad understanding of most oscillating systems.

Apparatus

The experimental arrangement that we use in this lab is very simple. It consists of a conical brass spring, an assortment of hanging masses, clamps, a ruler, and a force sensor connected to the Capstone software.

Procedure

Hooke's Law

1. Hang the spring vertically from a clamp with the large diameter coils down. Attach a 50 g mass to the end of the spring. Make sure the system is static (not oscillating). In this case, Newton's Second Law says
$$\sum F = kx - mg = 0 \Rightarrow x = \frac{g}{k}m.$$
where m is the mass of the weight alone, and $g = 9.8\,\mathrm{m/s^2}$. This assumes that the spring itself has negligible mass.

2. Measure the distance that the spring stretches from its initial point when the 50 g mass is hung. Record your value in the table below. Repeat for the other masses listed and put the results in the table. **Please do not attach more than 400 g, which may cause a permanent deformation of your spring.**

m (kg)	x (m)
0.050	
0.100	
0.150	
0.200	
0.250	
0.300	
0.350	
0.400	

3. Enter your data into an Excel spreadsheet and calculate the magnitude of the force F exerted by the spring on each mass. Create a graph of the force F versus the stretch x. Title and label appropriately the axes of the plot. Does the spring appear to obey Hooke's law? Explain in the space below the graph.

4. Fit the data to determine an equation that describes the relationship between F and x and display the equation on the graph. Print a copy of your graph and attach it.

Using the Excel Linear Regression Tool

(a) Click on the Data item in the Ribbon

(b) On the far right, Data Analysis should appear. Click that.

(c) Click on Regression and hit OK.

(d) Click in the Input Y Range box, click the icon to the right, and drag to select your vertical values. Repeat for X. Make sure "New Worksheet Ply" is selected, and hit OK.

(e) At the bottom of the new sheet should be a table that looks something like this:

	Coefficients	Std. Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	0.2509	0.0035	71.20	6.10E-06	0.2397	0.2621
X Variable 1	4.669	0.0212	219.7	2.07E-07	4.601	4.736

These show the parameter estimates for the Y-Intercept and the slope in the first column. The standard error is the uncertainty for our purposes. In the above example, the slope would be 4.67 ± 0.02 for 66% confidence.

5. Use the linear regression tool in Excel to determine the value and uncertainty in the spring constant. Record them below, and be sure to include units.

$$k = \_\_\_\_\_\_\_\_\_\_ \pm \_\_\_\_\_\_\_\_\_\_$$

Oscillation of a mass on a vertical spring

Discussion

Unlike all the forces we've looked at so far (gravity, contact forces, friction, tension, etc.), the spring force is not constant in the motion. This means that actually solving Newton's second law requires more sophisticated mathematics. The sum of forces in the vertical direction is

$$kx - mg = ma.$$

This is a differential equation (an equation of a function and its derivatives). While in general quite complicated, the solution here is merely a sine function

$$x(t) = A\sin(2\pi ft - \delta)$$

where A is called the amplitude, δ is the phase shift, and f is the **frequency**, given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

The frequency is how often the mass reaches the same phase each second. Its inverse, the **period**, $T = 1/f$, is the time it takes to reach the same phase. Being a time, T is measured in seconds, whereas f is measured in 1/s, also called a Hertz.

Measurement

6. Hang a 200 g mass from the spring and place the motion sensor on the ground facing the hanging mass from below. Try to place the resting position of the mass around 30 cm from the sensor. Open Capstone. Recording position versus time, pull the mass straight downward by a few centimeters and let go. Record the position of the mass as it oscillates. Record data for four or five cycles of the motion.

7. Fit the data with a sine curve. The curve fit will be of the form $A\sin(\omega t + \delta)$, so instead of frequency f the software reads out angular frequency, $2\pi f$ as its coefficient. Determine the period T of this oscillation via $T = 2\pi/\omega$. Record your value in the table below. Repeat for the masses given in the table below.

m (kg)	T (s)
0.050	
0.100	
0.150	
0.200	
0.250	
0.300	
0.350	
0.400	

8. Enter the data into an Excel spreadsheet and create a graph of T^2 along the vertical axis versus m along the horizontal axis.

9. Fit the data to a line. Print a copy of the graph and attach it to the lab. Answer the following questions in the space below:

(a) Notice that the equation we derived for T is

$$T^2 = \frac{(2\pi)^2}{k} m,$$

which means that the intercept of the curve should be 0. Use the Excel regression tool on T^2 versus m. Is your intercept zero within the uncertainty?

(b) One possible reason for this discrepancy is that the mass of the spring is non-negligible. If m becomes $m + m_s$,

$$T^2 = \frac{(2\pi)^2}{k} m + \frac{(2\pi)^2}{k} m_s.$$

Estimate m_s from your fit.

(c) What is the spring constant you get from this fit, along with its uncertainty. Do your values for this match with the earlier measurement, within the measurement uncertainty?