Eco 242 Eshragh Motahar

MACROECONOMIC THEORY AND POLICY

The Solow Growth Model

- ***Version 1: No population growth, no technological progress***

Consider the following production function in which Y is real GDP, K is the capital stock, and L is the labor force:

$$Y = F(K, L). \tag{1.1}$$

Assume that the above is a constant-returns-to-scale production function. We can, therefore, write (1.1) as:

$$Y / L = F(K / L, 1). \tag{1.2}$$

Let $y \equiv Y / L$, and $k \equiv K / L$. We can then re-write (1.2) as:

$$y = f(k). \tag{1.3}$$

Assume that this is a closed economy with no government. Now, demand for goods and services in this economy can be written as:

$$y = c + i. \tag{1.4}$$

Where c and i are consumption per worker and investment per worker, respectively. In the Solow model consumers save a fraction s of their income. Therefore,

$$c = (1 - s)y. \tag{1.5}$$

Consequently, from (1.5), (1.4) can be written as

$$y = (1 - s)y + i. \tag{1.6}$$

Now, by a simple re-arrangement of terms in (1.6), we get:

$$i = sy. \tag{1.7}$$

Now, assume that the depreciation rate of capital is δ. Keeping in mind that sy in (1.7) can be written equivalently as *sf(k)*, the change in the capital stock (per worker) can then be written as:

$$\Delta k = sf(k) - \delta k. \tag{1.8}$$

The steady-state value of *k*, denoted by *k**, is given by (1.8), when Δk is set to zero.

The Golden Rule level of capital

The steady-state value of *k* that maximizes *c* is called the Golden Rule level of capital. Policymakers may be able to choose *s* such that, in steady state, *c* is maximized. Now, note that in steady state

$$c^* = f(k^*) - \delta k^*. \tag{1.9}$$

A necessary condition for maximum *c** is

$$f'(k^*) - \delta = 0 \tag{1.10}$$

So, to find the Golden Rule saving rate, the following two equations, derived from (1.8) and (1.10), must be solved:

$$sf(k^*) = \delta k \quad and \quad f'(k^*) = \delta. \tag{1.11}$$

Some highlights of the model:

1. At the steady state, the (per capita GDP) growth rate is zero.
2. When $k<k^*$ positive growth takes place (and vice versa).
3. An increase in *s* will lead to short-term positive growth. The steady-state growth will be zero but at a higher *y*.