HW #2 Solutions

1. From diffraction, the laser beam will spread out with a half-angle spread of

$$\Delta\theta = 1.22\ \lambda/D \text{ (where D is the aperature size).}$$

So, $\Delta\theta = 1.22\ (500 \times 10^{-9}\ \text{m})/(0.01\ \text{m}) = 6.1 \times 10^{-5}$ radians or 0.0035^o.
Then the beam radius at the moon is $r = \Delta\theta\ (3.82 \times 10^8\ \text{m}) = 2.33 \times 10^4$ m, giving a beam area of $A = \pi r^2 = 1.71 \times 10^9\ \text{m}^2$. Given the moon's mean radius of 1.74×10^6 m = R, the ratio wanted is

$$\text{Beam area/Moon cross-sectional area} = \pi r^2/\pi R^2 = 0.0002,$$

so that 0.02% of the moon's projected area will be covered by the beam.

2. a. Since the energy of a photon is $E = hf$, and since $c = f\lambda$ holds as well, we can substitute $f = c/\lambda$ to find that

$$E = hc/\lambda.$$

Then substituting $h = 6.63 \times 10^{-34}$ Js, $c = 3\times10^8$ m/s, and $\lambda = 632.8 \times 10^{-9}$ m, we find

$$E = 3.14 \times 10^{-19}\ \text{J}.$$

Since 1 eV = 1.6×10^{-19} J, we can convert to eV, finding E = 1.96 eV.

b. The power, P, of a laser beam is defined as P = energy/time. This can also be written as

$$P = NE/t,$$

where E is the photon energy and N/t is the number of photons per second in the beam. Then

$$P = 10^{-3}\ \text{J/s} = (N/t)(3.14 \times 10^{-19}\ \text{J}), \text{ or, } (N/t) = 3.2 \times 10^{15} \text{ photons/sec.}$$

c. The heat needed to raise the water temperature by 1^oC is

$$\Delta Q = (100\text{g})(4.18\ \text{J/g}^o\text{C})(1^o\text{C}) = 418\ \text{J}.$$

If the laser beam heats the water uniformly and all the energy of each photon goes into heating the water, then

$$P(\Delta t) = (\text{photon energy absorbed by water in time } \Delta t) = \Delta Q,$$

so that $(10^{-3}\ \text{J/s})(\Delta t) = 418$ J, and we find that

$\Delta t = 4.18 \times 10^5$ s = 116 hours = 4.83 days – or really never since the heat will dissipate over these long times.

d. Repeating part c for a 100 W argon laser, gives 10^5 times more power so heating will occur 10^5 times faster or $\Delta t = 4.18$s, now a real effect.

e. Threshold damage occurs at 0.5 mJ/cm^2 = 5×10^{-4} J/cm^2.
With a 1 mW = 10^{-3} W He-Ne beam focused to a 50 μm radius, we can write the energy/area of the beam at threshold as

$$5 \times 10^{-4}\ \text{J/cm}^2 = (10^{-3}\ \text{J/s})(\Delta t)/[(\pi)(50 \times 10^{-4}\ \text{cm})^2]$$

Solving for Δt we find $\Delta t = 39\ \mu s\ (39 \times 10^{-6}\ \text{s})$