Physics 100
Laser Module

Homework 4 Solutions

The equilibrium populations are given by the Boltzmann equation
$N_2/N_1 = \exp[-(E_2-E_1)/kT]$ and $N_3/N_1 = \exp[-(E_3-E_1)/kT]$, where $N_1 + N_2 + N_3 = 10^{20}$.

At room temperature $kT = (1.38 \times 10^{-23})(300) = 4.14 \times 10^{-21}$ J = 0.0259 eV
So that
$N_2/N_1 = \exp[-0.2/0.0259] = 4.4 \times 10^{-4}$
And
$N_3/N_1 = \exp[-0.6/0.0259] = 8.7 \times 10^{-11}$

So
$N_1 + N_2 + N_3 = 10^{20}$ or $1 + (N_2/N_1) + (N_3/N_1) = 10^{20}/N_1 = 1 + 4.4 \times 10^{-4} + 8.7 \times 10^{-11}$.
Solution is
$N_1 = 9.996 \times 10^{19}$
$N_2 = 4.4 \times 10^{16}$
$N_3 = 8.5 \times 10^{9}$

b. At 5000K kT = 0.432, so repeating the calculation, we find that

$N_2/N_1 = \exp[-0.2/0.432] = 0.629$
$N_3/N_1 = \exp[-0.6/0.432] = 0.249$

So
$1 + N_2/N_1 + N_3/N_1 = 10^{20}/N_1 = 1 + 0.629 + 0.249 = 1.879$
Solving:
$N_1 = 5.32 \times 10^{19}$
$N_2 = 3.35 \times 10^{19}$
$N_3 = 1.32 \times 10^{19}$

c. The laser transition is from E_2 to E_1, so $\Delta E = E_2 - E_1 = 0.2$ eV
Using $E = hc/\lambda$ we find $\lambda = 6.22 \times 10^{-6}$ m or 6220 nm or 6.22 μm in the IR