Work

- In one dimension, work, a scalar, is defined for a constant force as

$$W = F_x \Delta x.$$

- Units of 1 N – m = 1 joule (J)
- A hiker does no work in supporting a backpack – Physics definition

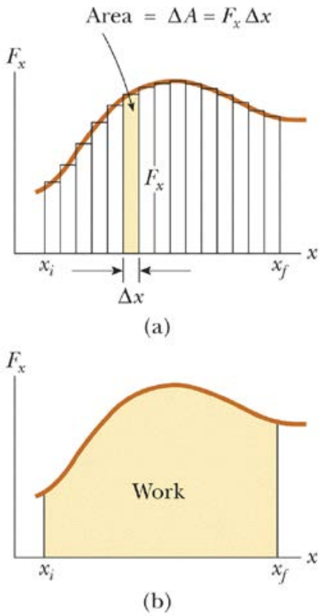
  is very specific – must have Δx

Example problem

- P.1 In mowing a lawn, a boy pushes a lawn mower a total distance of 350 m over the grass with a force of 90 N directed along the horizontal. How much work is done by the boy? If this work were the only expenditure of energy by the boy, how many such lawns would he have to mow to use the energy of a 200 Calorie candy bar? (use 1 Calorie = 4200 J)

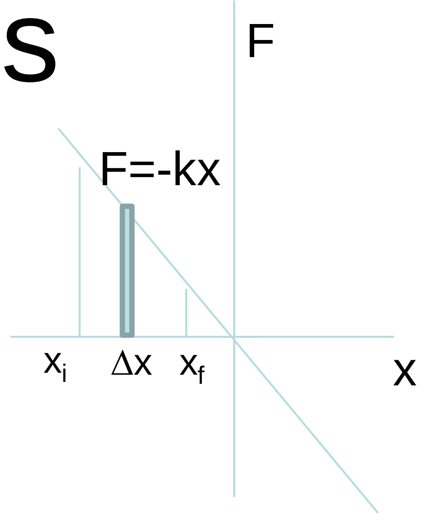
Suppose the force is not constant – still in one dimension

- Suppose we have a force that is not a constant. How can we define work?
- Graph F_x vs x – break up the interval into Δx's – note that the sum of products $\Sigma F_x \Delta x$ is approx. the area under the graph
- Then



$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

Work done by Springs



- If we have a linear spring that obeys Hooke's Law: F = -kx, then using the divide and conquer strategy we have

$$\sum F_x \Delta x = \sum area\ of\ rectangles$$

better approx as $\Delta x \rightarrow 0\ and\ \#\ rect. \rightarrow \infty$

resulting in area under the line – Showing all the details:

- $W = \lim_{\Delta x \rightarrow 0} \sum F_x \Delta x = \int_{x_i}^{x_f} F_x\, dx$ = area under line

$= \bar{F}\Delta x = ½\ (F_f + F_i)(x_f - x_i) =$

$\frac{1}{2}(-kx_f - kx_i)(x_f - x_i) =$

$-\frac{1}{2}k(x_f{}^2 - x_i{}^2) = \frac{1}{2}kx_i{}^2 - \frac{1}{2}kx_f{}^2$

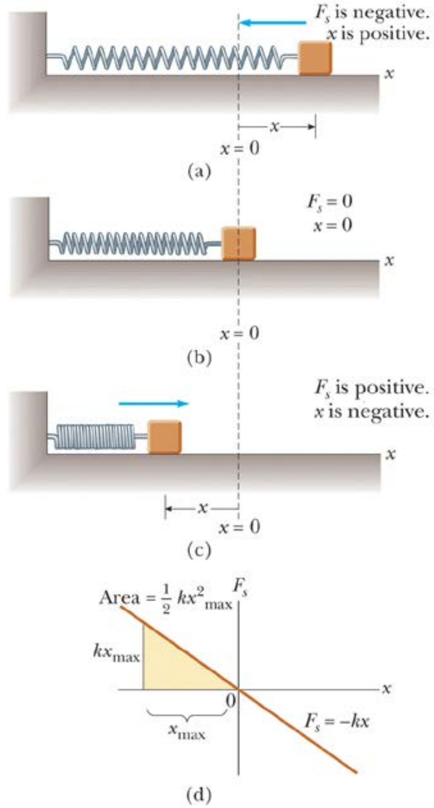
Springs

- Alternatively, using calculus: Work done by the spring force =

$$W_s = \int_{x_i}^{x_f} (-kx)dx = -\tfrac{1}{2}kx^2 \Big|_{x_i}^{x_f}$$

$$= \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2$$

- Same result in quick line; shortly see the significance of this



Kinetic Energy and Work- KE Theorem

- How much work is done on a particle of mass m by an external force F as the particle moves from x_i to x_f?

$$W = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} m \frac{d\mathrm{v}}{dt} dx = \int_{\mathrm{v}_i}^{\mathrm{v}_f} m\mathrm{v} d\mathrm{v} = \tfrac{1}{2} m\mathrm{v}_f^2 - \tfrac{1}{2} m\mathrm{v}_i^2$$

- Interpret this as $W_{net} = KE_f - KE_i = \Delta KE$, where

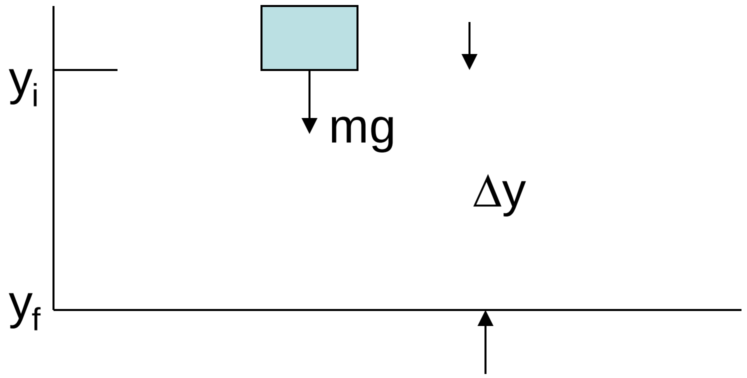
$KE = ½ mv^2$

Problem

- ***Example 4.4***

  Using the work-KE theorem, estimate the height to which a person can jump from rest. Make some reasonable assumptions as needed.

Gravitational Potential Energy



$$W = -mg\Delta y = -mg(y_f - y_i) = -(mgy_f - mgy_i).$$

- This is similar to the Work – KE theorem ($W_{net} = KE_f - KE_i$) in that *W* = the difference in a quantity at the two end-points of the motion.
- Let's call this $PE_{grav} = mgy$ = grav. potential energy; so that $W = -\Delta PE_{grav} = -(PE_{grav\text{-}f} - PE_{grav\text{-}i})$ Note the negative sign.
- *this is only valid near the Earth's surface where g = constant*

Conservation of Mechanical Energy

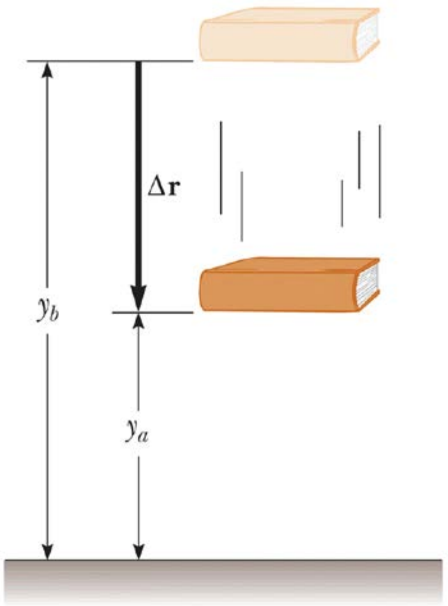
- Suppose we apply the work – kinetic energy theorem to an isolated system where the only source of work is gravity. For the book example

  $W_{on\ book} = (-mg)\ (y_a - y_b) = -\Delta PE_{grav}$

- Then $\Delta KE = -\Delta PE_{grav}$ or

  $KE_i + PE_i = KE_f + PE_f = E_{mech} =$ constant

- We call KE + PE = mechanical E – other types of PE.



Example

- P.6 Free-Fall: A boy throws a 0.1 kg ball from a height of 1.2 m to land on the roof of a building 8 m high.

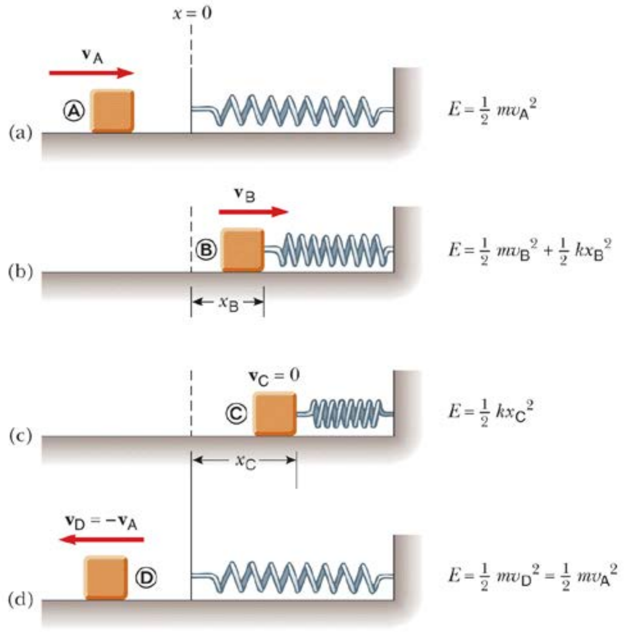
  a) What is the potential energy of the ball on the roof relative to its starting point? relative to the ground?
- b) What is the minimum kinetic energy the ball had to be given to reach the roof?
- c) If the ball falls off the roof, find its kinetic energy just before hitting the ground.

Elastic Potential Energy

- We've seen that the work done by a spring is given by $W_s = -\frac{1}{2} kx^2$. We can identify $PE_s = \frac{1}{2} kx^2$ so that $W_s = -\Delta PE_s$ and then

  $K + PE_{grav} + PE_s = E_{mech} =$ constant for an isolated system
- Example:

Given v_A, k and m, find the maximum compression of the spring

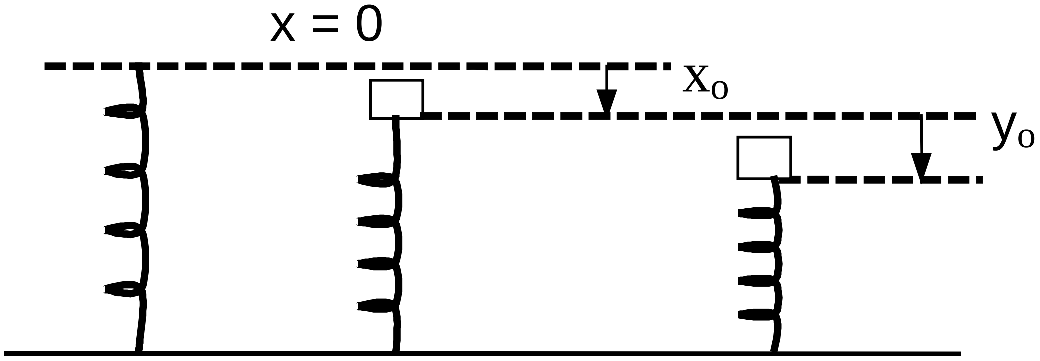


Second Spring Problem

- P.13 A 2 kg block slides back and forth on a frictionless horizontal surface bouncing between two identical springs with k = 5 N/m. If the maximum compression of a spring is 0.15 m, find the gliding velocity of the block between collisions with the springs.

Vertical Spring Example

- Ex. 4.6 A spring is held vertically and a 0.1 kg mass is placed on it, compressing it by 4 cm. The mass is then pulled down a further 5 cm and released giving it an initial velocity of 1 m/s downward. Find the maximum compression of the spring relative to its unstretched length.



Conservative Forces

- Conservative forces do work which only depends on the end points of the motion and not the path taken.
- Examples include gravity and spring forces – an important counter-example is friction
- Three other equivalent definitions for a conservative force:
  - work done in moving a closed path = 0
  - no loss of mechanical energy to internal energy
  - Work can be written as $W = -\Delta PE$

Potential Energy and Force Connection

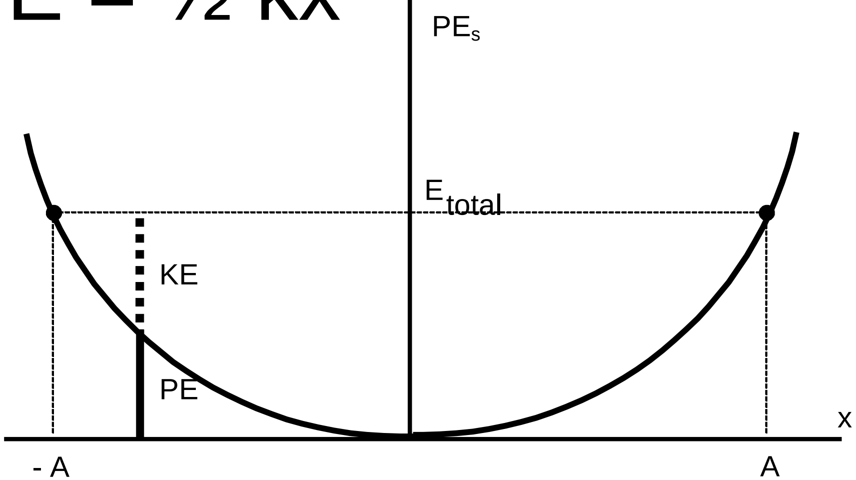
- Since

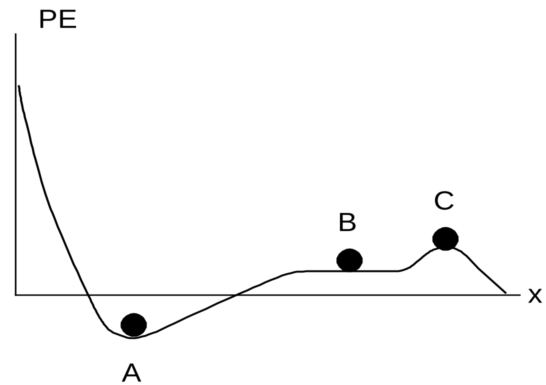
$$W = \int F_x dx = -\Delta PE \quad \textit{we have in } 1\text{-dim} \quad PE_f = -\int_{x_i}^{x_f} F_x dx + PE_i$$

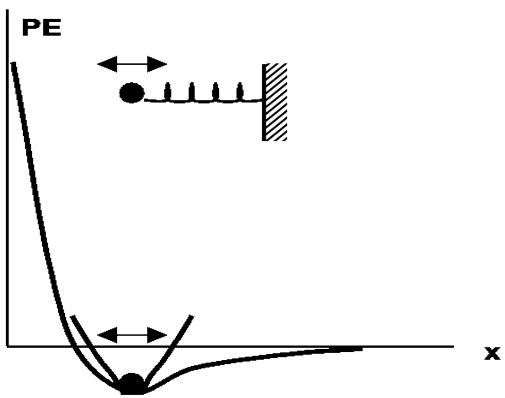
- We can also write $F_x = -\dfrac{d(PE)}{dx}$
- When we come to motion in more than one dimension, we will see that it is much easier to find PE, a scalar, than Force, a vector
- Examples:
  - gravity: F_y = -d(mgy)/dy = -mg
  - Springs: F = -d(1/2 kx^2)/dx = -kx

Potential Energy Diagrams

- $PE = \frac{1}{2} kx^2$







POWER

- Power is the rate of energy transfer
- If some work W is done in a time interval Δt then $P = W/\Delta t$ – this is the average power usually written with a bar over it $\bar{P}$
- Instantaneous power is $P = dW/dt$; but since $dW = F dx$ this can also be written as $P = F \text{v}$
- Units for P are 1 J/s = 1 W (watt)

Example

- Ex. 4.8 Let's try to calculate the wind power possible to tap using high efficiency windmills. Assume a wind speed of 10 m/s (about 20 mph) and a windmill with rotor blades of 45 m diameter.