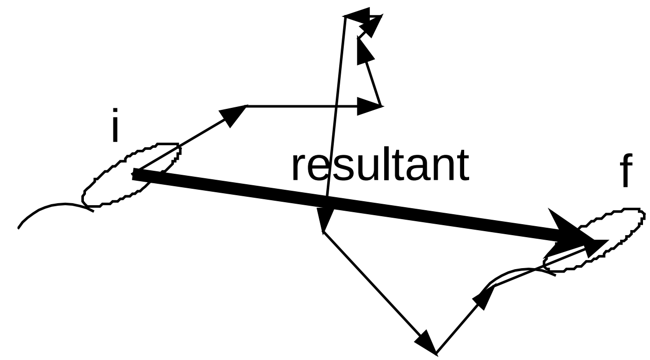
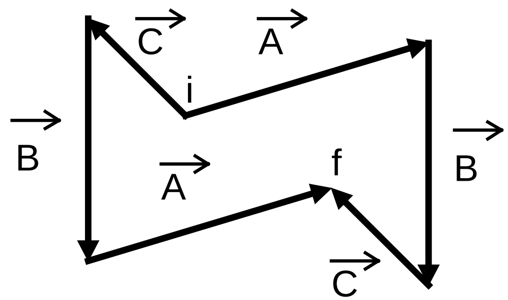
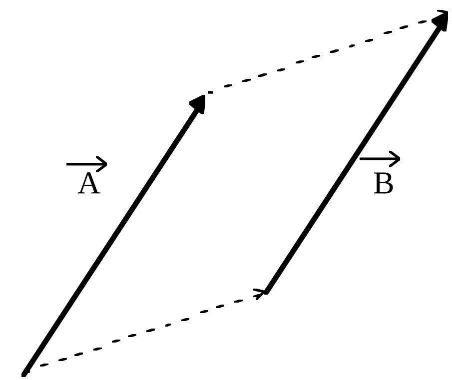
Vectors

- Scalars & vectors
- Adding displacement vectors



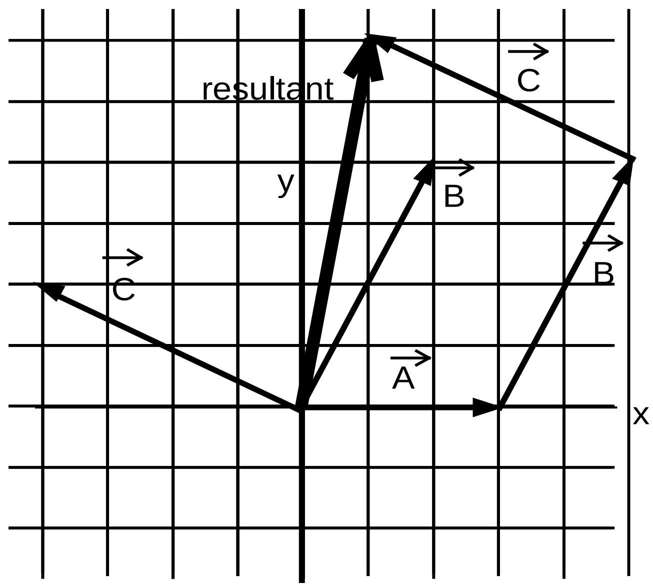
- What about adding other vectors - Vector equality
- Order does not matter:





Vector addition I

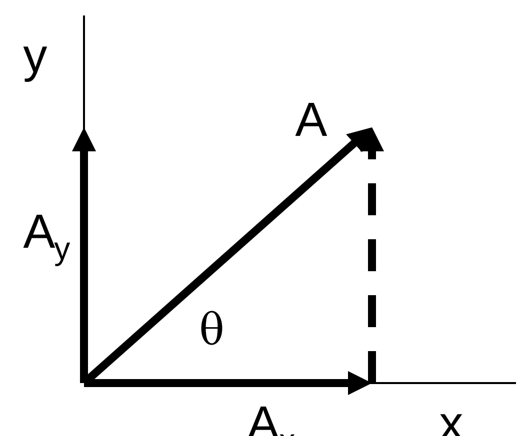
- Graphical vector addition



- Vector decomposition

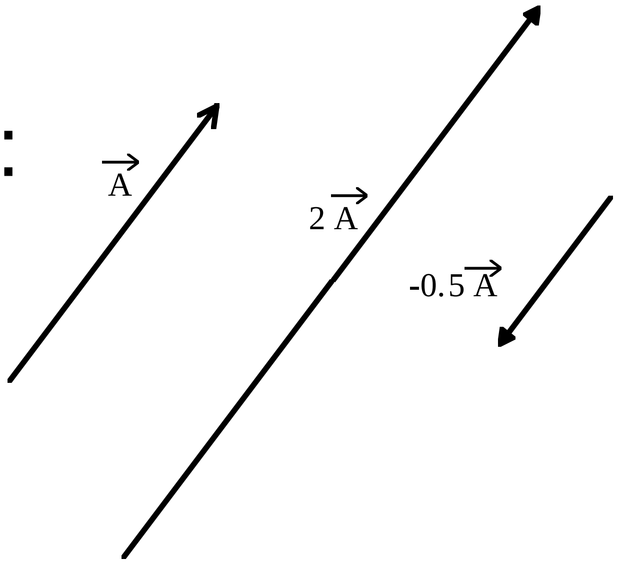
A_x=A cos θ

A_y=A sin θ



Vector Addition II

- Scalar multiplied by a vector:

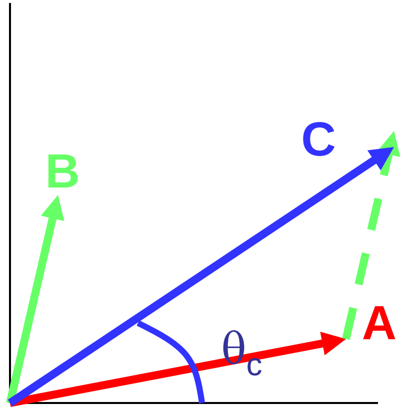


- Analytical vector addition:

$$\vec{A} = \left(A_x, A_y\right)$$

$$\vec{C} = \vec{A} + \vec{B} = ([A_x + B_x],[A_y + B_y])$$

$$\left|\vec{C}\right| = \sqrt{\left(C_x\right)^2 + \left(C_y\right)^2} \qquad \theta_c = \tan^{-1}\left(\frac{C_y}{C_x}\right)$$



Steps in Component Method of Vector Addition/Subtraction

1. Make a rough sketch of the vectors, if not given
2. Find the x-, y- (and z-) components of each vector, if not given order pair notation
3. Perform the algebraic +/-/or multiplication by a scalar separately to each component, finding the x-, y- (and z) components of the resultant
4. If needed, combine the components of the resultant, using the Pythagorean theorem and trigonometry, to find the magnitude and direction of the resultant

Example calculation: Given A = (5, 2) and B = (-3, - 5), express C = A + B in terms of a) unit vector notation, and b) magnitude and direction.

Kinematics in 2 (or 3) dimensions I

- Position vector: $\vec{r} = (x, y, z)$
- Velocity vector: $\vec{\mathrm{v}} = (\mathrm{v}_x, \mathrm{v}_y, \mathrm{v}_z) = \dfrac{d\vec{r}}{dt}$

  where $\mathrm{v}_x = \dfrac{dx}{dt}$ $\quad$ $\mathrm{v}_y = \dfrac{dy}{dt}$

- Acceleration vector: $\vec{a} = (a_x, a_y, a_z) = \dfrac{d\vec{\mathrm{v}}}{dt} = \dfrac{d^2\vec{r}}{dt^2}$

  where $a_x = \dfrac{d\mathrm{v}_x}{dt}$ $\quad$ $a_y = \dfrac{d\mathrm{v}_y}{dt}$

Kinematics in 2 (or 3) dimensions II

- When the object we are interested in is confined to move along a line, position, velocity, and acceleration are all along the same line.
- When the object is free to move in space, position, velocity, and acceleration can all point in different directions. This fact makes two- or three-dimensional motion more subtle to deal with.
- But, the preceding equations point out a very useful simplification: the x- (respectively, y-, z-) component of velocity only changes due to the x- (respectively, y-, z-) component of acceleration, and the x- (respectively, y-, z-) component of position only changes due to the x- (respectively, y-, z-) component of velocity.

Special Case I – Free fall

- If the acceleration = constant, as for example in free fall, then we can generalize two of our equations of motion from the 1-D case:

$$\vec{r} = \vec{r}_o + \vec{\mathrm{v}}_o t + \tfrac{1}{2}\vec{a}t^2$$

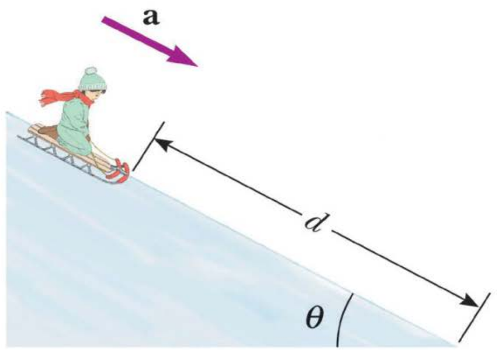
$$\vec{\mathrm{v}} = \vec{\mathrm{v}}_o + \vec{a}t$$

- Example problem: A lacrosse goalie clears the ball by throwing it downfield at a speed of 10 m/s at a 35º angle above the ground.
- How long will it be in the air? (assume the ball leaves the goalie's stick from ground level)
- How far will it go before hitting the ground, assuming no one is there to catch it?

Applications of Newton's Laws

- Generalization to 2 or 3 dimensions – no modification for 1st and 3rd laws
- Second Law: $\vec{F} = m\vec{a}$
- Use vector nature of equations to help solve

Example Problem 1

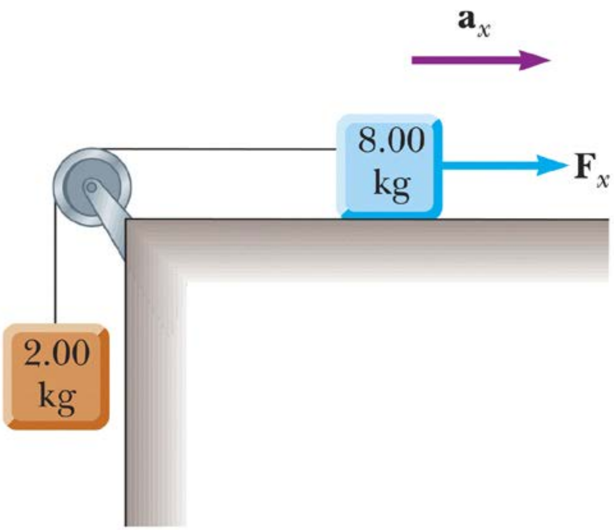


Find acceleration
Find speed at bottom

Problem Solving Strategy

1. *Make a rough sketch of the problem*, if there is not already one supplied as part of the problem, and *identify the object(s) whose motion is to be studied*, if that is not clear.
2. *Identify all the forces acting on the object* *(and only on that object)* *by constructing a carefully labeled external force diagram (such a diagram is sometimes known as a free-body diagram)*- a crucial step in solving the problem.
3. From the external force diagram, with a set of chosen coordinate axes, *write down the equations of motion*, the component Newton's Second Law equations, being very careful to use appropriate labeling and to write down the *x* and *y* components in separate equations.
4. Once the equations of motion are obtained, *solve for the unknowns of the problem*, by performing the required algebra.
5. Whenever possible, *check your results* in limiting cases or in simplified circumstances.

Example Problem 2



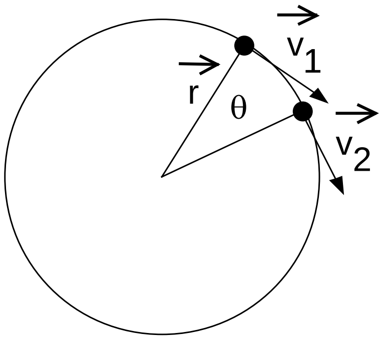
No friction present

a) For what values of F_x does the 2 kg mass rise?

b) For what values of F_x is the tension in the cord = 0?

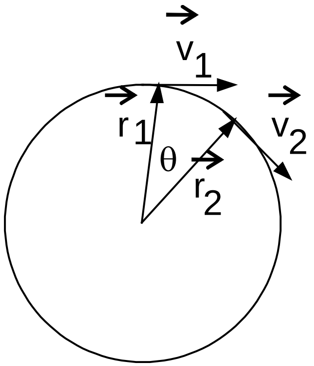
Special Case II- Circular Motion

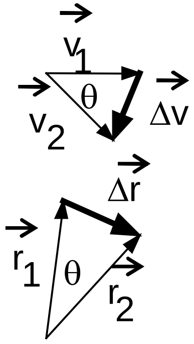
- A particle traveling in a circle is accelerating even if its speed is constant
- Uniform circular motion
- What is the acceleration? Magnitude and direction.



Uniform Circular Motion

- With $\vec{v}_1 = \vec{v}_2$ we have two sets of similar triangles so $\dfrac{\Delta v}{v} = \dfrac{\Delta r}{r}$





Or $\dfrac{\Delta v}{\Delta t} = \dfrac{v}{r}\dfrac{\Delta r}{\Delta t}$;but

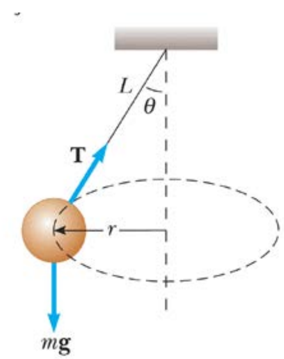
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v}{r} \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{v^2}{r}$$

- Direction is toward the center of the circle – centripetal acceleration – perpendicular to $\vec{v}$
- Centripetal force needed to produce this acceleration

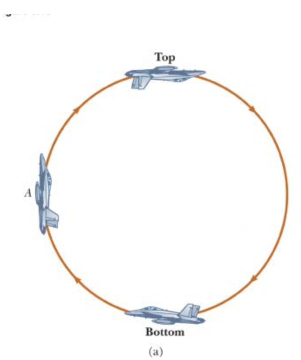
$$\vec{F}_{net} = m\vec{a}_{cent} = m\frac{v^2}{r}(-\hat{r})$$

Example Problems 3&4

- Conical pendulum



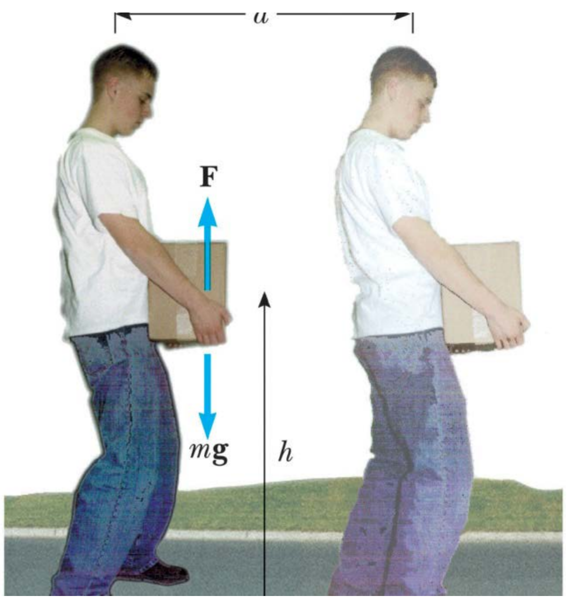
- Loop-the-loop



(a)

Work in 2 or 3 dimensions

- The general definition of work when the force is constant is defined as
  $$W = F\Delta x \cos\theta$$
  where θ is the angle between F and x
- Why? This picks out the component of F along the motion as that part, and only that part, that can do work.



If he picks up the box and walks horizontally a distance d at const. v how much work is done by him and by the gravitational force?

Example Problem

- Problem 27

A 100 kg sled is slid across a smooth ice field by a group of 4 dogs tied to the sled pulling with a 350 N force along a rope at an angle of 20^o above the horizontal.

a) If the sled travels at a constant speed, find the drag force on the sled.

b) Find the work done by the dogs after pulling the sled for 1 km.

Friction

- Up till now we have ignored ever-present friction forces
- Often these are thought of as causing complications, but friction is essential for most types of motion
- Without it we would not be able to walk, automobiles would not be able to move, and even machinery would not be able to function

Kinetic Friction 1

- Imagine two solid objects sliding relative to each other, such as a block sliding on a table surface.
- Friction is the contact force acting parallel to the surface of contact (as contrasted with the normal force which is also a contact force but is directed perpendicular to the contact surface).
- It is produced by electromagnetic interactions between the molecules at the contacting surfaces of the two objects.
- The frictional force on an object is always directed opposite to its velocity.
- It is found that while the frictional force depends on the nature of the material surfaces, surprisingly, it does not depend on the contact area (to a good approximation).

Microscopic surface of copper block

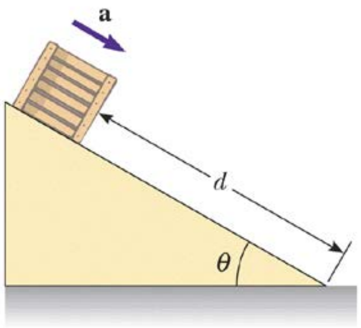
Kinetic Friction 2

- The kinetic friction is proportional to the normal force, *N*, and can be written as

  $$F_k = \mu_k N \ ,$$

  where μ_k is the coefficient of kinetic friction, which depends on the two material surfaces.
- Careful with directions here – this is not a vector equation
- Empirical, approximate equation; the coefficient of kinetic friction depends on the degree of smoothness of the surfaces, as well as on whether they are wet or lubricated

Example Problem



(a)

If θ = 30^o and a = g/3 what is the kinetic coefficient of friction?

Static Friction 1

- When two objects are in contact, but at rest with respect to each other, there are also molecular bonds that form between contact points.
- Just sitting at rest does not result in any net force along a direction parallel to the contact surface; if there were, this force would spontaneously make the object accelerate.
- But if we try to push a block with a force directed along the table surface, the molecular bonds supply a frictional force in the opposite direction, opposing the impending motion.
- This type of friction is called static friction and arises in response to an applied force which would otherwise result in motion.
- As long as there is no motion, the static friction force is always as large as it has to be to cause a net balance of all forces on the block.

Static Friction 2

- Maximum static friction force depends solely on the nature of the two surface materials and the normal force but not on the surface area, and is given by

$$F_{s,\,max} = \mu_s N,$$

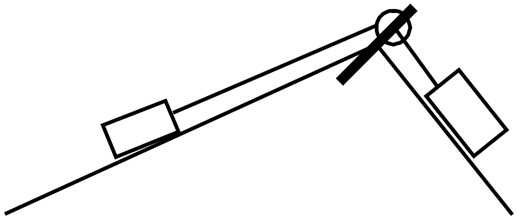
where μ_s is the coefficient of static friction

- In general F_s is as large as it needs to be to stop motion until the max is reached, so

$$F_s \leq \mu_s N$$

Problem 55

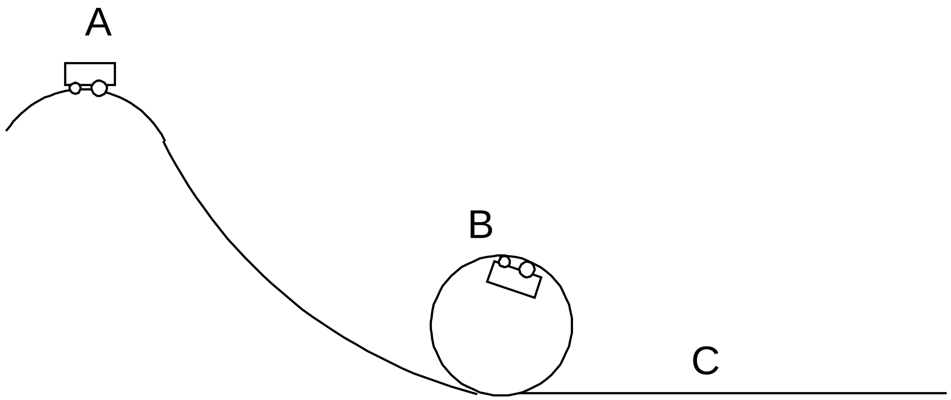
- Two blocks are attached by a light cord with each block sitting on a different inclined plane as shown.



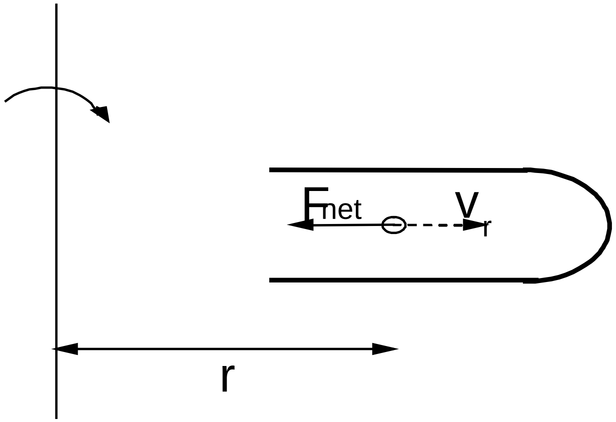
- If the angles of inclination are 30º and 60º, the respective masses are 10 kg and 6 kg, and the coefficients of sliding and static friction are 0.3 and 0.5, do the masses move, and if so in which direction and with what acceleration?

Example 2

- In a loop-the-loop roller coaster ride the car of mass m starts from rest at point A at a height *H*. The loop-the-loop has a height of H/3. Assuming no friction, find: a) the speed of the roller coaster car at point B at the top of the loop-the-loop and b) the speed of the car at point C.



Centrifugation



$$f\mathrm{v}_r + F_B = ma_{cent},$$

$$F_B = m_o a_{cent}$$

$$so \qquad f\mathrm{v}_\mathrm{r} = (m - m_o)a_{cent}$$

$$s = \frac{\mathrm{v}_r}{a_{cent}} = \frac{(m - m_o)}{f}$$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Keep your rotors balanced and spin below rated speed