Momentum

- An object of mass *m* traveling at velocity $\vec{\mathrm{v}}$ has a linear **momentum** (or just momentum), $\vec{p}$, given by $\vec{p} = m\vec{\mathrm{v}}$
- Units have no special name: kg-m/s
- With no net force on a particle, its momentum is constant (Newton #1), but any force will change its momentum
- How does p differ from v? Depends on m also – e.g. truck and car at same v
- How does p differ from K? It's a vector and K = $p^2/2m$. K is produced by W, while p is produced by F

Newton's Second Law

- We can re-write $\vec{F}_{net} = m\vec{a}$ as $\vec{F}_{net} = \dfrac{d\vec{p}}{dt}$
(this is the form Newton actually used)
- Check: $$\frac{d\vec{p}}{dt} = \frac{d\left(m\vec{\mathrm{v}}\right)}{dt} = m\frac{d\vec{\mathrm{v}}}{dt} = m\vec{a}$$

  so long as m = constant
- It turns out that this form is correct even if m changes; in that case using the product rule $$\frac{d\vec{p}}{dt} = m\vec{a} + \frac{dm}{dt}\vec{\mathrm{v}}$$

Conservation of momentum

- Suppose we consider a system of two interacting particles – isolated from their environment. Then

$$\vec{F}_{2on1} = \frac{\Delta \vec{p}_1}{\Delta t} \quad and \quad \vec{F}_{1on2} = \frac{\Delta \vec{p}_2}{\Delta t}$$

- But, from Newton's #3 we have

$$\vec{F}_{2on1} = -\vec{F}_{1on2}$$

- so that

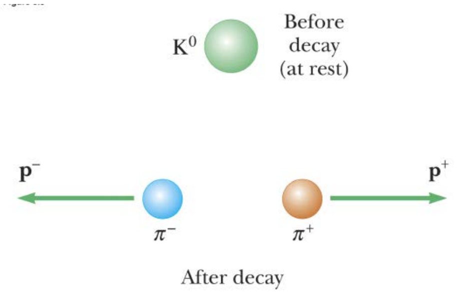
$$\frac{\Delta \vec{p}_1}{\Delta t} + \frac{\Delta \vec{p}_2}{\Delta t} = 0 \quad or \quad \frac{\Delta(\vec{p}_1 + \vec{p}_2)}{\Delta t} = 0 \quad or \quad \Delta(\vec{p}_1 + \vec{p}_2) = 0$$

- or $\vec{P}_{total} = constant$ - short for 3 component eqns
- For a system we have $\vec{F}_{net,external} = \dfrac{d\vec{p}_{total}}{dt}$ so that if there are no external forces then

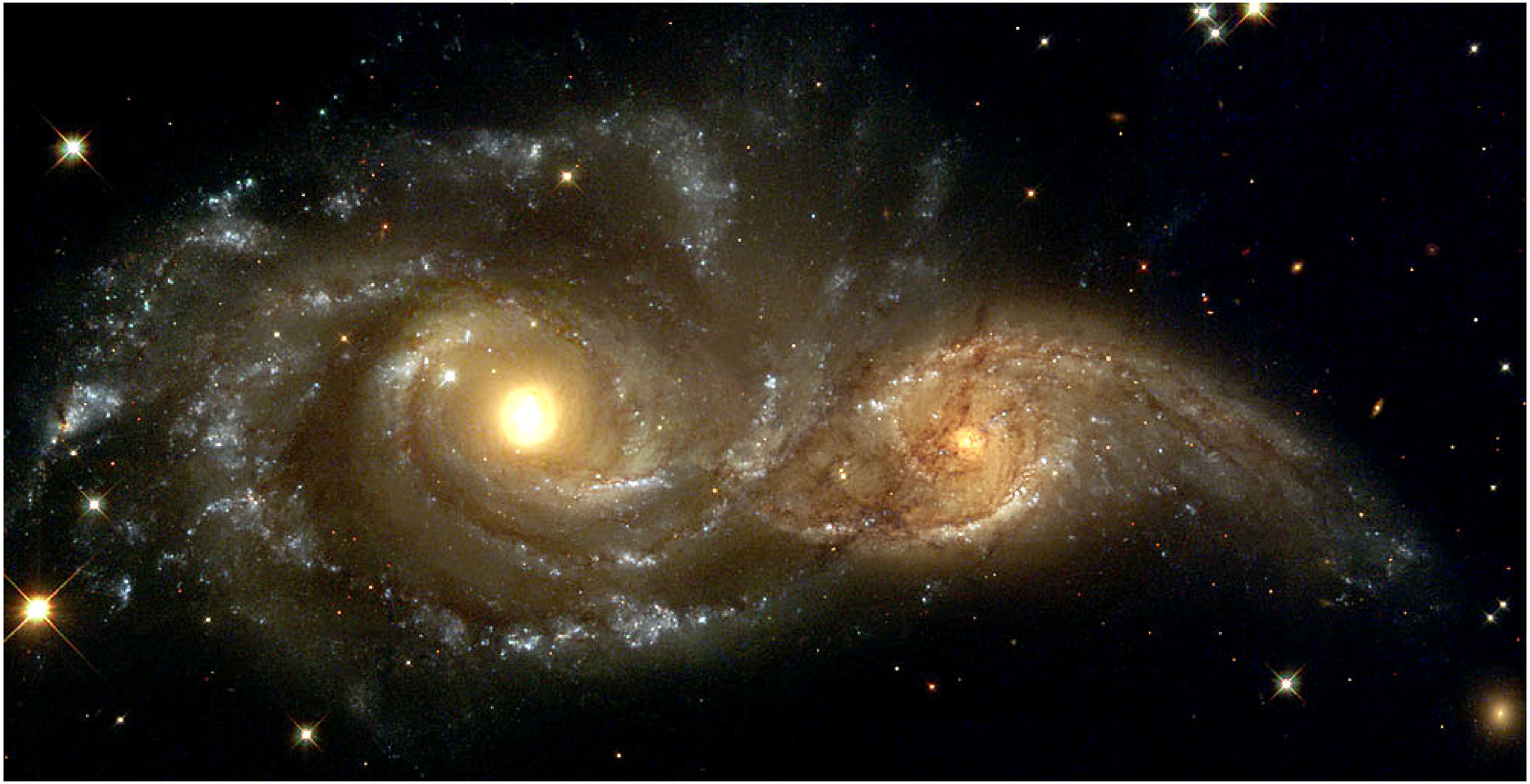
$$\vec{P}_{total} = constant$$

The power of Conservation of P

- A 60 kg boy dives horizontally with a speed of 2 m/s from a 100 kg rowboat at rest in a lake. Ignoring the frictional forces of the water, what is the recoil velocity of the boat?
- A neutral kaon decays into a pair of pions of opposite charge, but equal mass. If the kaon was at rest, show that the pions have equal and opposite momenta.
- Note application to PET- a medical imaging method



Collisions



Collisions

EJECTED = DEATH
and the corollary equation:
SEATBELTS = NOT EJECTED = LIFE

Impulse

- Collisions are characterized by large forces lasting short times
- From Newton's #2 for a system we have

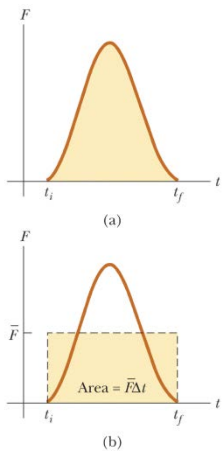
$$d\vec{p}_{total} = \vec{F}_{net\ external}dt \quad or \quad \Delta\vec{p}_{total} = \int_{t_i}^{t_f} \vec{F}_{net\ external}dt = I$$

where I is the impulse – red expression is the impulse momentum theorem

- We can replace the integral by

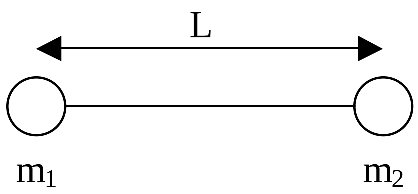
$$I = \left(\bar{\vec{F}}\right)\Delta t \qquad \text{where } \left(\bar{\vec{F}}\right) \text{ is the average}$$

force acting over the collision time Δt



Center of Mass 1

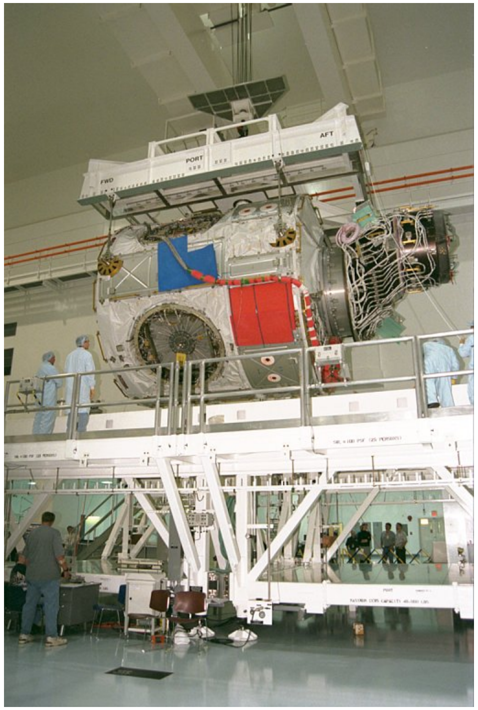
- Start consideration of real objects – no longer only points with no shape –
- Analogy with population center in the U.S.
- Center of mass = point where mass can be imagined to be concentrated to completely explain translational motion. It is also the balance point (so long as gravity is uniform)



If $m_1 = m_2$ it's at center

If not, then

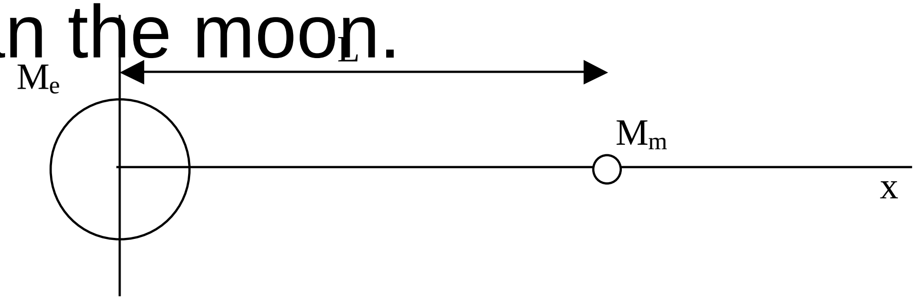
$$x_{cm} = \left(\frac{m_1}{m_1 + m_2}\right) x_1 + \left(\frac{m_2}{m_1 + m_2}\right) x_2,$$



Node I, a part of the International Space Station having its center of mass determined by suspending it from above

Center of Mass Example

- Example 6.5. Find the center of mass of the earth-moon system given that the mean radius of the earth is 6.37 x 10^6 m, the mean radius of the moon is 1.74 x 10^6 m, the earth-moon mean separation distance is 3.82 x 10^8 m, and that the earth is 81.5 times more massive than the moon.



$$x_{cm} = \frac{M_e(0) + M_m(L)}{M_e + M_m} = \frac{1}{1 + \frac{M_e}{M_m}} L = 0.012\,L = 4.63\,x10^6\ m.$$

Center of Mass generalizations

- In one-dimension with more point masses:

$$x_{cm} = \frac{\sum_i m_i x_i}{M_{total}}$$

- In 3-dimensions with point masses:

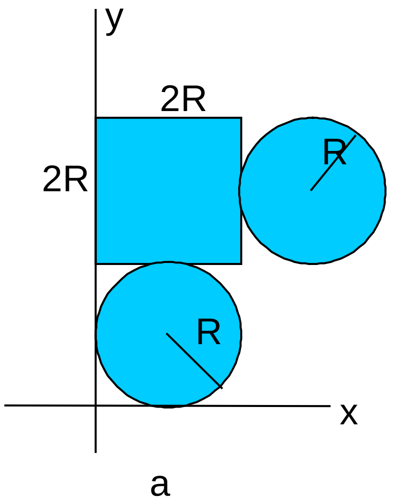
$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{M_{total}}$$

- With extended masses in 3-dim:

$$\vec{r}_{cm} = \frac{\int \vec{r}\, dm}{M_{total}}$$

Example

- Center of mass of an extended object
  - The solid object shown in the figure below is made from a uniform material and has a constant thickness. Find the center of mass using the coordinate system shown. Take R = 0.1 m.



High Jump

Can the center of mass of the person go under the bar??

Motion of a system of particles or an extended object

- Suppose we calculate the time derivative of the center of mass position:

$$\mathrm{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{M_{total}} \Sigma m_i \frac{d\vec{r}_i}{dt} = \frac{1}{M_{total}} \Sigma m_i \vec{\mathrm{v}}_i = \frac{1}{M_{total}} \Sigma \vec{p}_i = \frac{1}{M_{total}} \vec{p}_{total}$$

- What can we conclude? Total M · v_{cm} = total momentum.
- If we take another derivative, we'll find

$$\frac{d\vec{p}_{total}}{dt} = M_{total} \vec{a}_{cm} = \Sigma m_i \vec{a}_i = \Sigma \vec{F}_i$$

but the sum of all forces has two parts: internal and external forces and the sum of all internal forces = 0 (they cancel in pairs due to Newton's #3) so we are left with

$$M_{total} \vec{a}_{cm} = \Sigma \vec{F}_{external} = \frac{d\vec{p}_{total}}{dt}$$

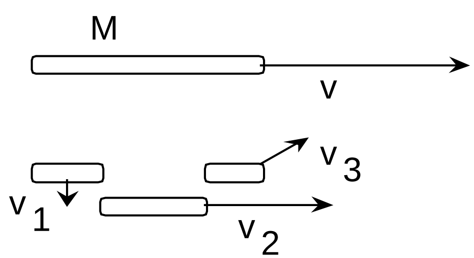
General Principle of Conservation of Momentum

$$M_{total}\vec{a}_{cm} = \Sigma\vec{F}_{external} = \frac{d\vec{p}_{total}}{dt}$$

- So, if the net external force = 0 then $\vec{p}_{total}$ is constant. Therefore quite in general, an isolated system conserves momentum.

Example Problem:

Example 6.8. A rocket of mass *M* explodes into 3 pieces at the top of its trajectory where it had been traveling horizontally at a speed v = 10 m/s at the moment of the explosion. If one fragment of mass 0.25 *M* falls vertically at a speed of $v_1 = 1.2$ m/s, a second fragment of mass 0.5 *M* continues in the original direction, and the third fragment exits in the forward direction at a 45^o angle above the horizontal, find the final velocities of the second and third fragments. Also compare the initial and final kinetic energies to see how much was lost or gained.



An unmanned Titan rocket explodes shortly after takeoff in 1998. Despite fragmenting into many pieces the center of mass continues in a well-defined trajectory