# Introduction to Uncertainties

**Standard deviation.** When you have repeated the same measurement several times, common sense suggests that your "best" result is the average value of the measurements. We still need to know how "good" this average value is. One measure is called the standard deviation which indicates how much variation from the average exists in the data. For example if the average is 5.1 m/s, and the standard deviation is 0.3 m/s, then the number can be expressed as 5.1 $\pm$ 0.3 m/s.

**Rule 0. Numerical, fractional, and percent uncertainties.** The uncertainty in a quantity can be expressed in numerical, fractional, or percent forms. Thus in the above example, 0.3 m/s is a numerical uncertainty, but we could also express it as a fractional uncertainty as 0.3/5.1 $\simeq$ 0.0588 or a percent uncertainty as 0.3/5.1 x 100 % $\simeq$ 5.88 %.

**Rule 1. Addition and subtraction.** If you are adding or subtracting two uncertain numbers, then the numerical uncertainty of the sum or difference is the sum of the numerical uncertainties of the two numbers. For example, if A = 3.4 $\pm$ 0.5 m and B = 6.3 $\pm$ 0.2 m, then A + B = 9.7 $\pm$ 0.7 m , and A - B = - 2.9 $\pm$ 0.7 m. Notice that the numerical uncertainty is the same in these two cases, but the fractional uncertainty is very different.

**Rule2. Multiplication and division.** If you are multiplying or dividing two uncertain numbers, then the fractional uncertainty of the product or quotient is the sum of the fractional uncertainties of the two numbers. For example, if A = 3.4 $\pm$ 0.5 m and B = 0.334 $\pm$ 0.006 s, then the fractional uncertainty of the product or quotient is 0.5/3.4 + 0.006/0.334 $\simeq$ 0.165. Thus, AB = 1.1356 $\pm$ 0.1874 m·s and A/B = 10. 180 $\pm$ 1.710 m/s. Notice that the fractional uncertainties are the same, but the numerical uncertainties are different (even different units).

**Rule 3. Raising to a power.** If you are raising an uncertain number to a power n, (squaring it, or taking the square root, for example), then the fractional uncertainty in the resulting number has a fractional uncertainty n times the fractional uncertainty in the original number. Thus if you are calculating a number y = ½ g $t^2$ , where t = 2.36 $\pm$ 0.04 s, then the fractional uncertainty in $t^2$ is 0.0339 and the numerical uncertainty is 0.08 $s^2$.

**Rule 4. Complicated expressions.** In cases other than the above, you can do a calculation to find the numerical uncertainty. We could call this the "max/min" method. For example, if you are calculating $F_x$ = F cos $\theta$, where $\theta$ = 34.6 $\pm$ 0.2 °, and F = 13.2 $\pm$ 0.7 N, then we do a two-step calculation:

Step 1: Calculate the uncertainty in cos $\theta$ by comparing cos (34.6°) to cos (34.6°+ 0.2°). The uncertainty in cos $\theta$ is taken as the difference between these numbers. Thus, cos (34.6°) - cos (34.8°) = 0.00199, then cos (34.6 $\pm$ 0.2) = 0.82314 ± 0.00199.

Step 2: Use rule 2 above to calculate the uncertainty in F cos $\theta$. The result is F cos $\theta$ = 10.865 $\pm$ 0.602 N.

**Rule 5. Significant figures.** Once you get to a final result (no more computations are to be made with the quantity), then we usually keep one significant figure in the value of the uncertainty, and round off the quantity to the same decimal place. Thus the final result in the rule 4 example is 10.9 $\pm$ 0.6 N, and the final result in the examples of rule 2 are 10 $\pm$ 2 m/s and 1.1 $\pm$ 0.2 m·s.

Note: These simple rules are not quite "correct", but they will serve pretty well in the first course in physics.

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