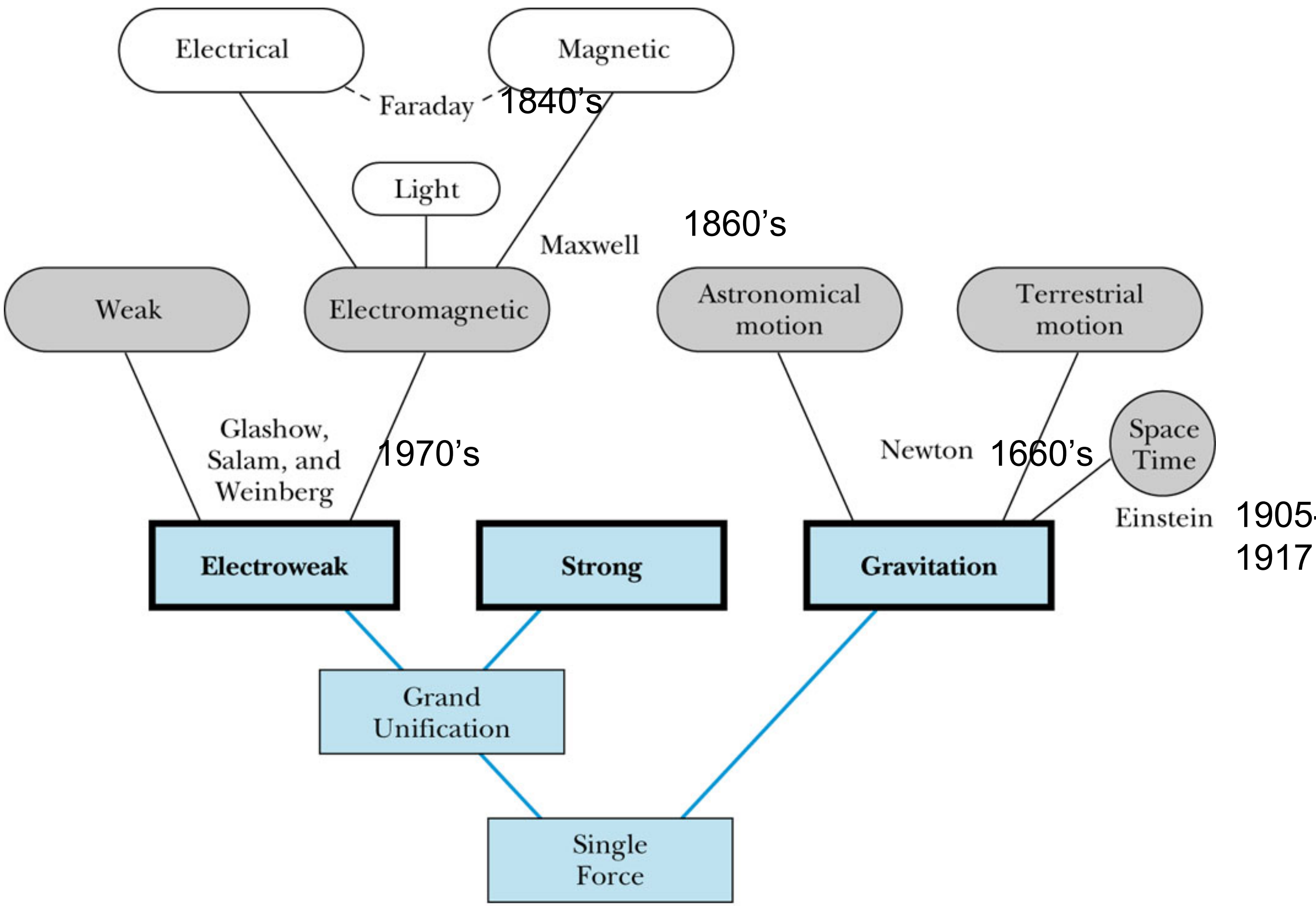
Introduction

- Classical vs Modern Physics
  - High speeds
  - Small (or very large) distances
- Classical Physics:
  - Conservation laws: energy, momentum (linear & angular), charge
  - Mechanics – Newton’s laws
  - Electromagnetism – Maxwell’s equations
  - Thermodynamics – basic laws
  - Forces of Nature

Fundamental Theories

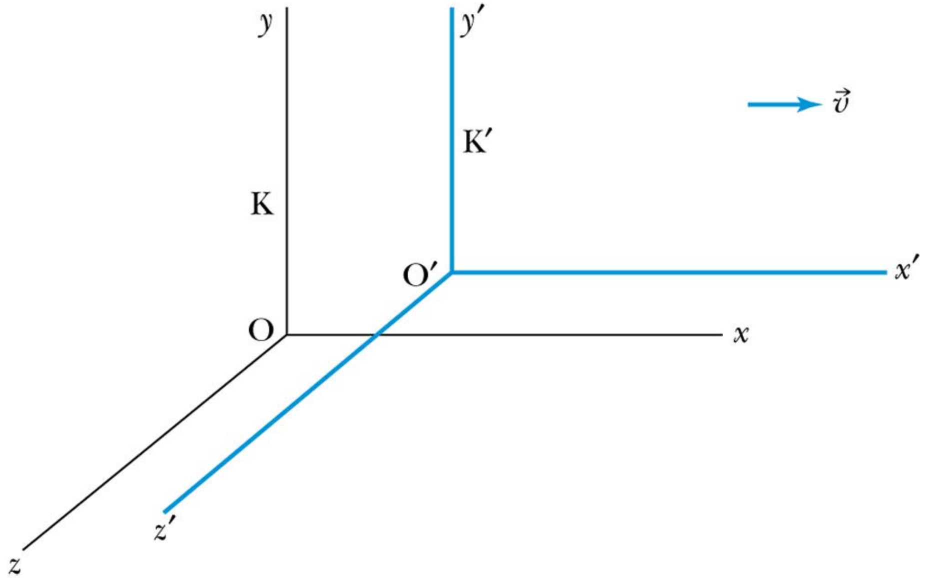


Pre-1905 Unexplained Issues

1. Ether – medium of the vacuum
2. Maxwell equations and Galilean invariance
3. Blackbody radiation
4. Discovery of X-rays (1895) and radioactivity (1896)
5. Discovery of electron (1897)
6. Zeeman effect (1896)

Special Relativity

- 1905 year for Einstein – photoelectric effect, Brownian motion, special relativity – at age 26!
- Relativity = Special + General (1917)
- Classical mechanics obeys Galilean transformation:

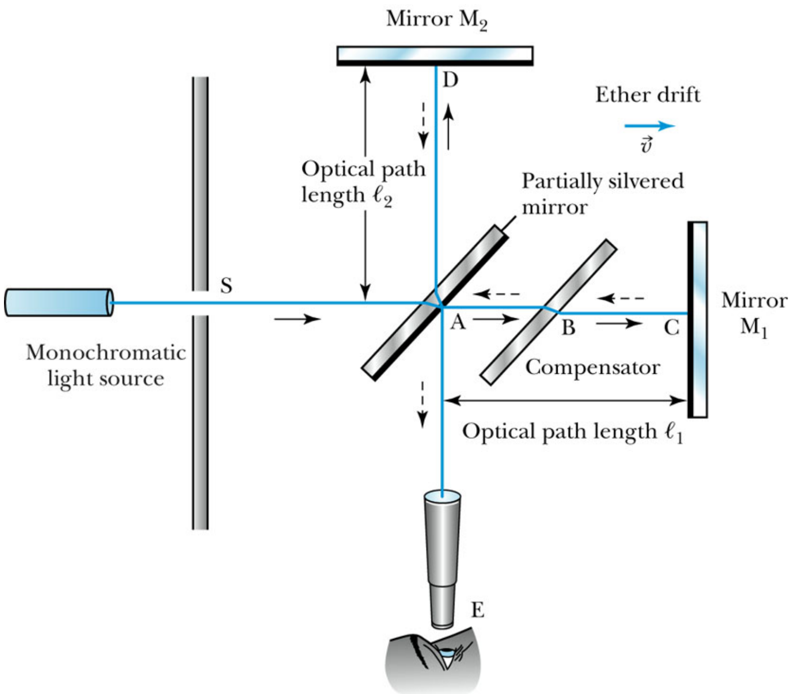


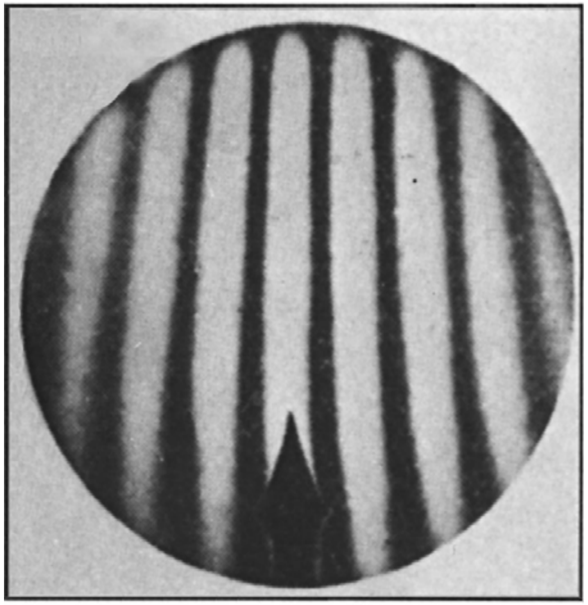
$x' = x - vt$
$y' = y$
$z' = z$
$t' = t$

Michelson- Morley Experiment

(first US Nobel prize to Michelson 1907)

- Search for ether via use of interferometer





Homework on this – null result even after several years of improvements in sensitivity of experiment – concluded there was no ether

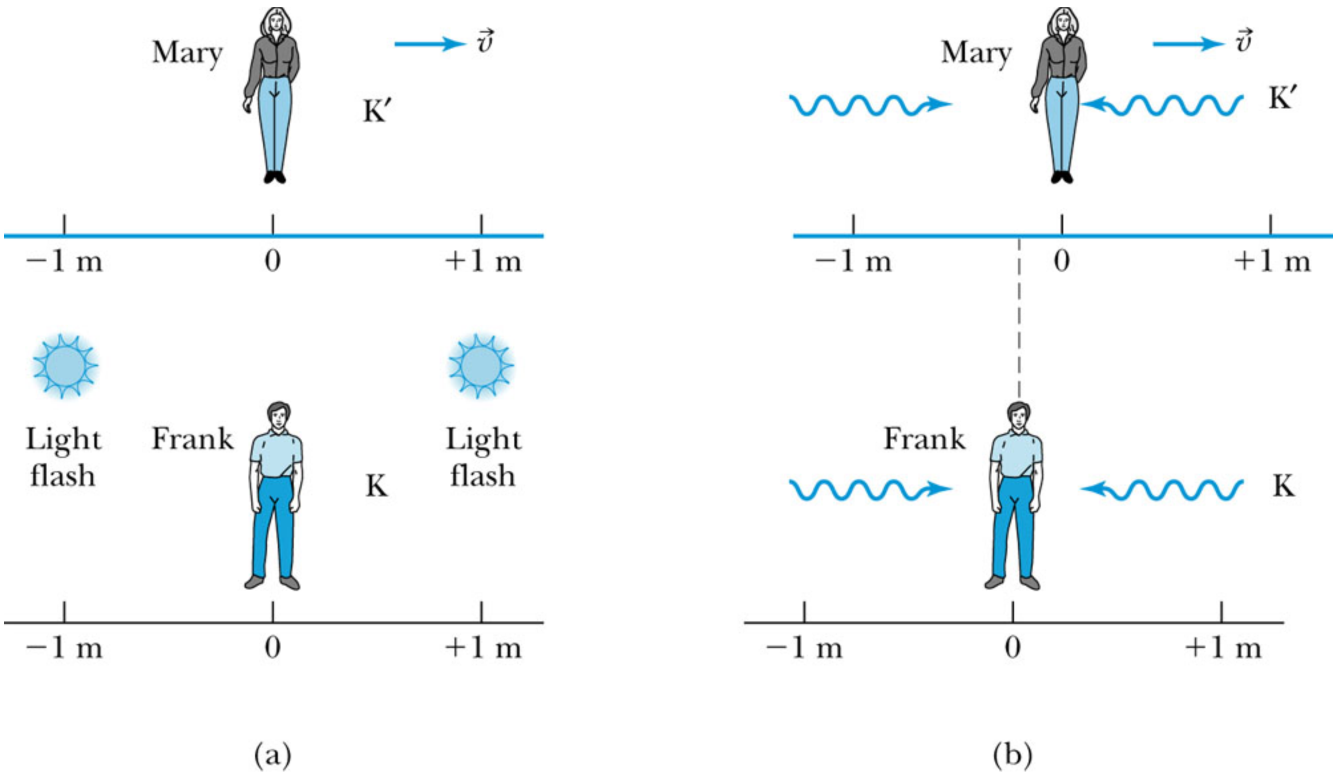
Einstein Postulates

Postulates:

1. Principle of relativity – all laws of physics are the same in all inertial systems – (generalization of Newton's relativity principle for mechanics)
2. Speed of light is a universal constant (this actually follows from (1))

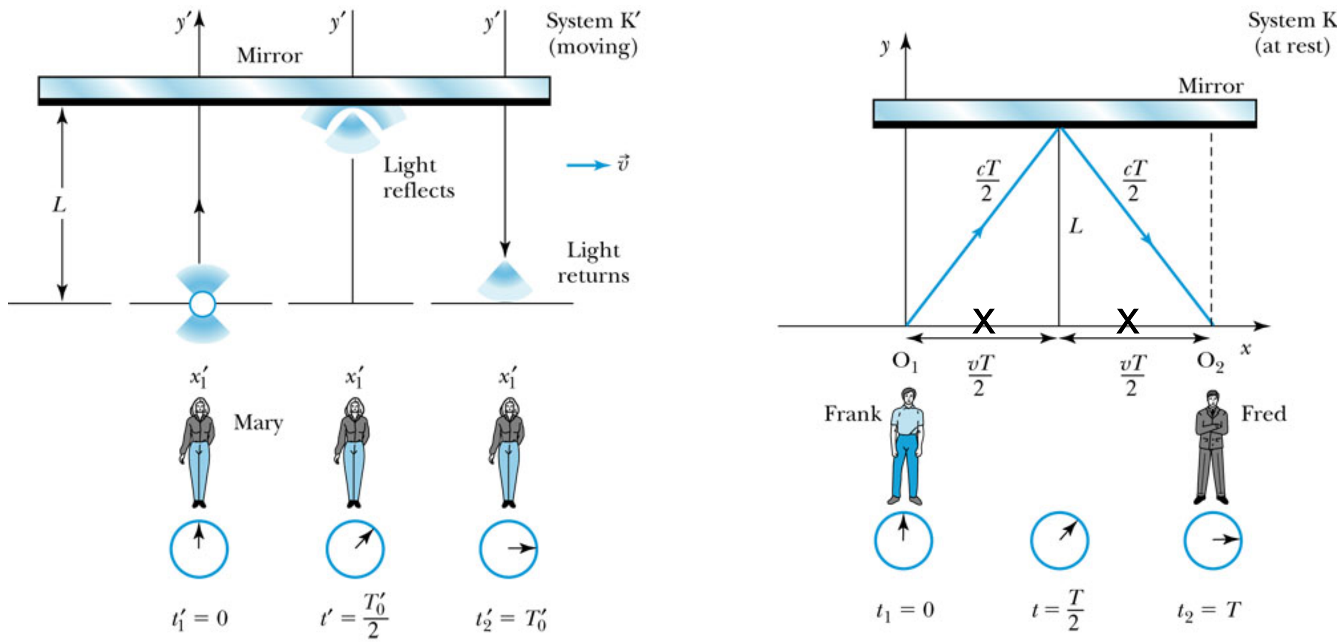
Einstein was most influenced by fact that Maxwell's equations are not Galilean invariant but satisfy (2) – he might not even have been aware of Michelson-Morley expt at the time

Simultaneity – Gedanken #1



Frank (F = fixed inertial frame) sees the light flashes to be simultaneous, while Mary (M= moving, inertial) sees them at different times. Simultaneity is not universal – not absolute.

Time Dilation – Gedanken #2



For Mary (in rest frame of mirror) Δt’ = 2L/c = T_o = proper time

For Frank (who sees two events at different spatial points) $\Delta t = T = \dfrac{2\sqrt{x^2 + L^2}}{c}$

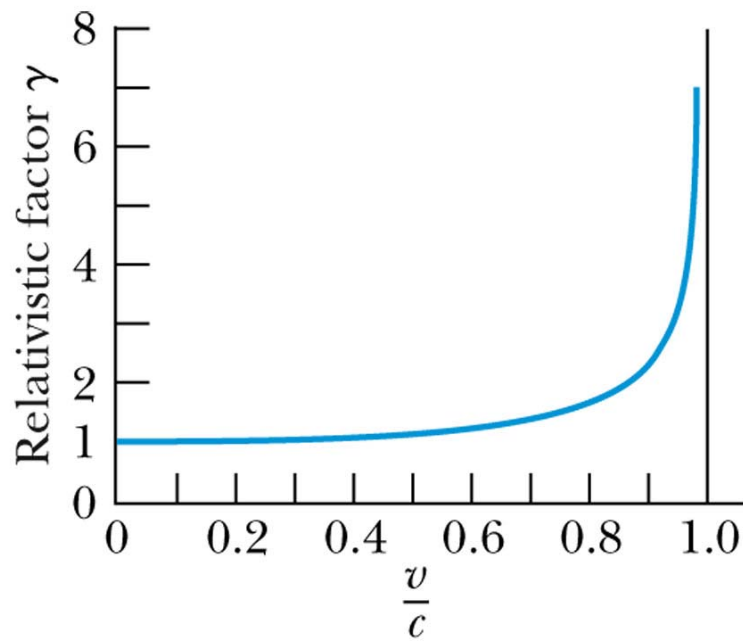
But $x = vT/2, so \quad T^2 = \dfrac{4}{c^2}\left[L^2 + \dfrac{v^2T^2}{4}\right] \quad or$

Solving for T $\quad T = \dfrac{2L/c}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{T_o}{\sqrt{1-\dfrac{v^2}{c^2}}} = \gamma T_o$

Time Dilation (2)

- So, moving clocks tick slower, by a factor of gamma, γ

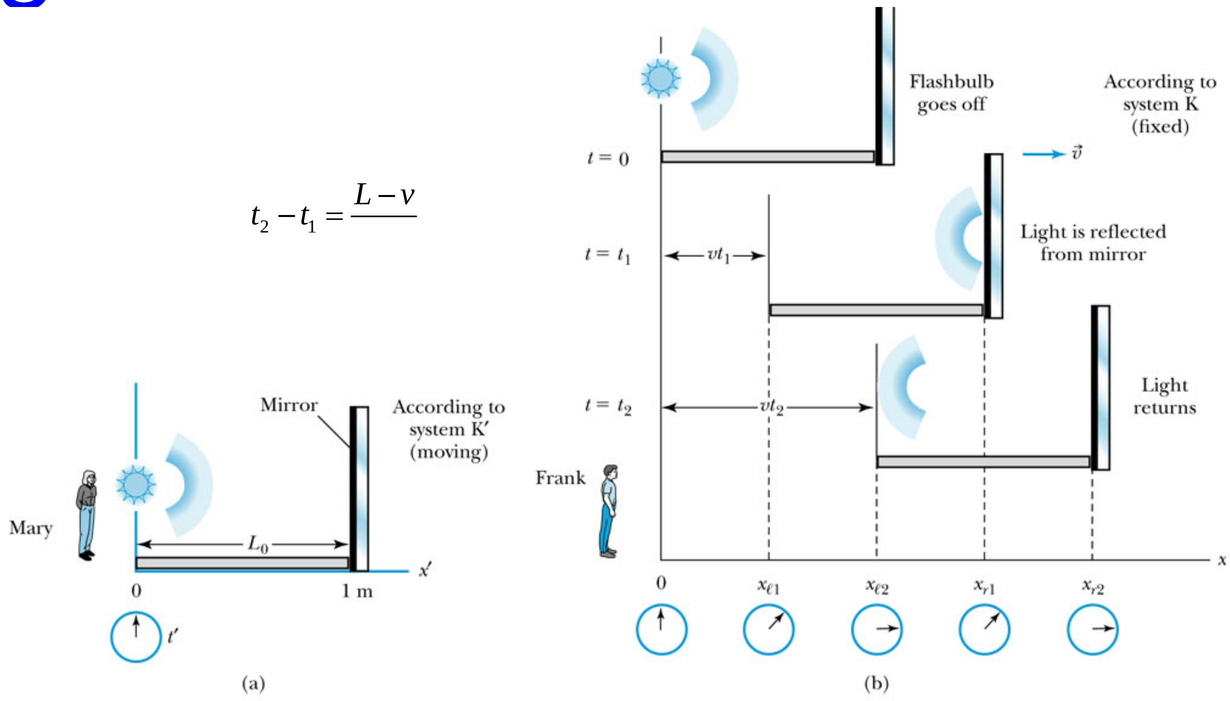
If $v/c > 0.1$ ($\gamma \sim 1.005$ – or 0.5% correction) then we should use relativity



- Proper time = T_o = time between two events at the same spatial point, ie., in the rest frame – it is always the shortest time interval between two events
- Muon decay story

Length Contraction – Gedanken #3

$$t_2 - t_1 = \frac{L - v}{}$$



Mary, in the rest frame, measures the proper length L_o (using 1 clock and $T_o = 2L_o/c$ or $L_o = cT_o/2$)

Frank sees the object moving and requires two clocks to measure the time interval for light to return to the left end. Derive that

$$L = L_o\sqrt{1-\frac{v^2}{c^2}} = \frac{L_o}{\gamma}$$

Length Contraction (2)

Frank sees t_1 = time to mirror = $$t_1 = \frac{L + vt_1}{c}$$

And t_2 = return time = $$t_2 = \frac{L - vt_2}{c}$$

So, total time = $$T = \frac{L}{c-v} + \frac{L}{c-v} = \frac{2L}{c}\frac{1}{1-\frac{v^2}{c^2}} = \gamma^2\frac{2L}{c}$$

But T = $\gamma T_o = \gamma$(2L_o/c) so

L = L_o/γ or $$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

- Moving objects contract along the direction of motion by the factor gamma. Rest length (or proper length) is longest measured length of object.

Problems

1. What is the apparent thickness of the Earth's atmosphere from the muon's perspective?

   We found that v = 0.999978c for the muon, with γ = 151.4. Therefore the muon will see the 100 km distance to the Earth's surface contracted to L_o/γ = 100km/151.4 = 0.66 km or 660 m. To the muon, with its decay time of 2.2 μs, this distance allows it to reach the Earth since the speed is 660m/2.2 μs = c.
2. Problem 20
3. Problem 25

Solutions

- P20:

The round-trip distance is $d = 40$ ly. Assume the same constant speed $v = \beta c$ for the entire round trip. In the rocket's reference frame the distance is only $d' = d\sqrt{1-\beta^2}$. Then in the rocket's frame of reference

$$v = \frac{\text{distance}}{\text{time}} = \frac{d'}{40\text{ y}} = \frac{40\,\text{ly}\,\sqrt{1-\beta^2}}{40\,y} = c\sqrt{1-\beta^2}$$

Rearranging

$$\beta = \frac{v}{c} = \sqrt{1-\beta^2}$$

Solving for β we find $\beta = \sqrt{0.5}$, or $v = \sqrt{0.5}\,c \approx 0.71c$. To find the elapsed time t on earth, we know $t' = 40$ y, so

$$t = \gamma t' = \frac{1}{\sqrt{1-\beta^2}}\,40\,\text{y} \;= 56.6\,\text{y}.$$

- P25

The clocks' rates differ by a factor of $\gamma = 1/\sqrt{1-v^2/c^2}$. Since β is very small we will use the binomial theorem approximation $\gamma \approx 1 + \beta^2/2$. Then the time difference is

$$\Delta t = t - t' = t - \gamma t = t\,(\gamma - 1)$$

Using $\gamma - 1 \approx \beta^2/2$ and the fact that the time for the trip equals distance divided by speed,

$$\begin{aligned}\Delta t &= t\,(\beta^2/2) = \frac{8\times10^6\text{ m}}{375\,\text{m/s}}\,\frac{\left(\frac{375\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2}{2}\\ &= 1.67\times10^{-8}\text{ s} \;= 16.7\text{ ns}\end{aligned}$$

Lorentz Transformation

- Coordinate transformation – must be linear and must keep c same in all frames and must reduce to Galilean at low v

$$with \quad \beta = v/c \quad and \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - vx/c^2}{\sqrt{1-\beta^2}} = \gamma(t - vx/c^2)$$

- Text derives time dilation and length contraction from these transformations

Velocity addition

- Must ensure no v>c is possible
- Using Lorentz transformation for primed to unprimed:

$$dx = \gamma(dx' + v\,dt')$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \gamma(dt' + (v/c^2)dx')$$

- Velocities are defined by $u_x = dx/dt$, etc.

$$u_x = dx/dt = \gamma(dx' + v\,dt')/\gamma(dt' + (v/c^2)dx') =$$

$$u_x = \frac{u_x' + v}{1 + (v/c^2)u_x'}$$

Velocity addition (2)

- Note that this form ensures that velocities cannot exceed c. Check that if $u_x' = c$, $u_x = c$ as well, independent of v.

- Note that even though y' = y and z' = z, the forms for u_y and u_z are not the same since Δt transforms with frame (see text)

Twin Paradox

- Statement of Paradox
- Analysis with Spacetime (Minkowski) diagrams

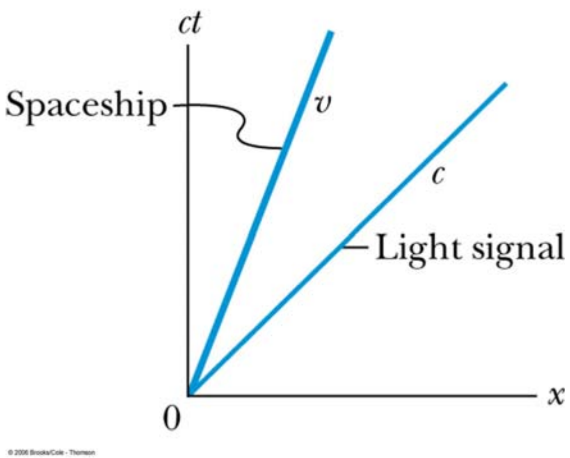
$$\Delta s^2 = \Delta x^2 - c^2 \Delta t^2$$

$$= spacetime\ interval$$

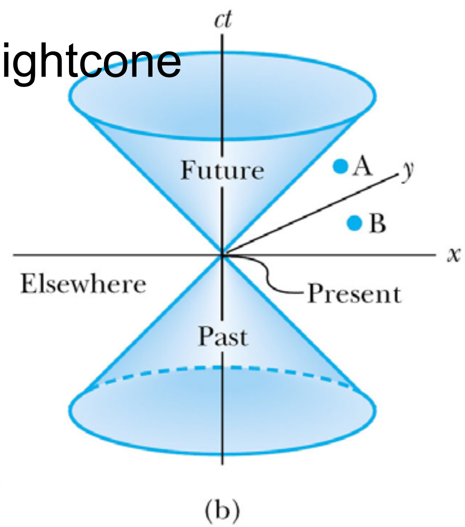
$$if\ = 0 \rightarrow lightlike$$

$$if > 0 \rightarrow spacelike$$

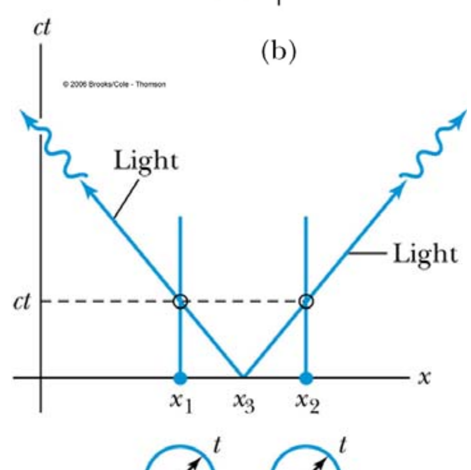
$$if < 0 \rightarrow timelike$$

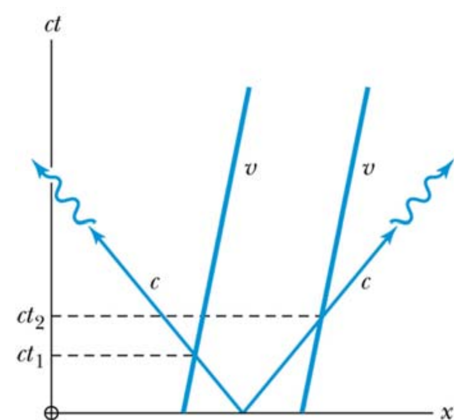


lightcone

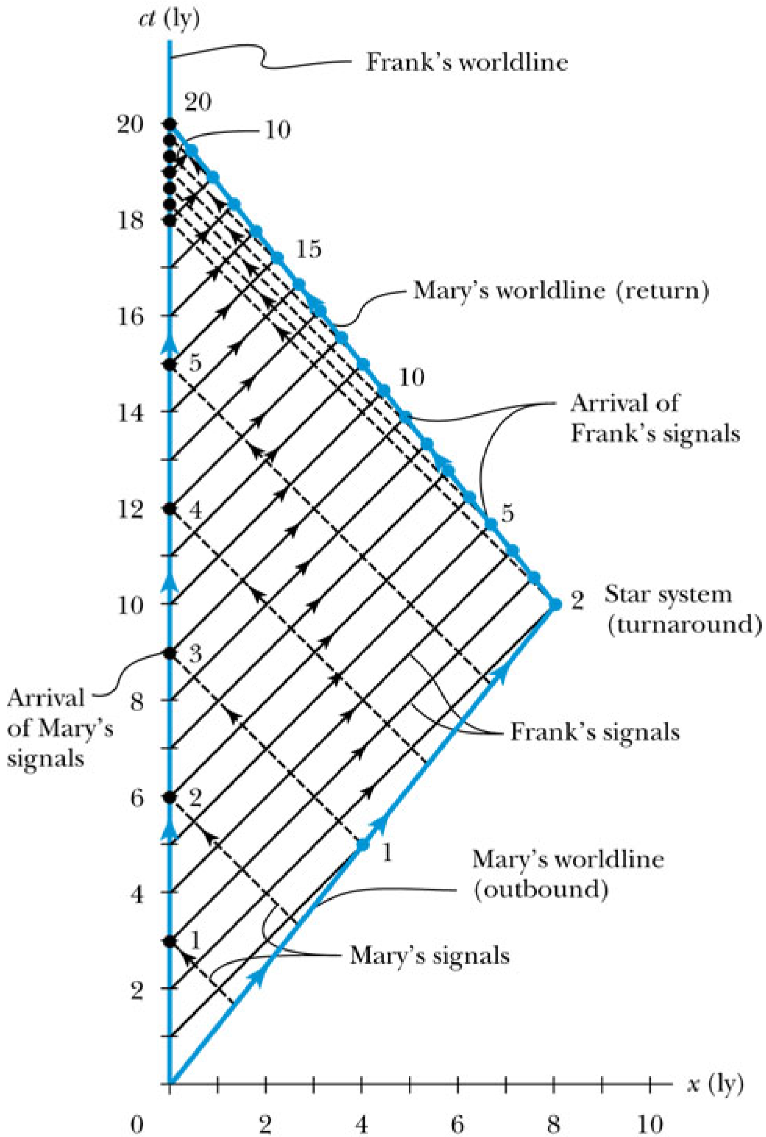


simultaneity





Twin Paradox (2)



Note: Mary travels 8 ly to a star at v = 0.8c

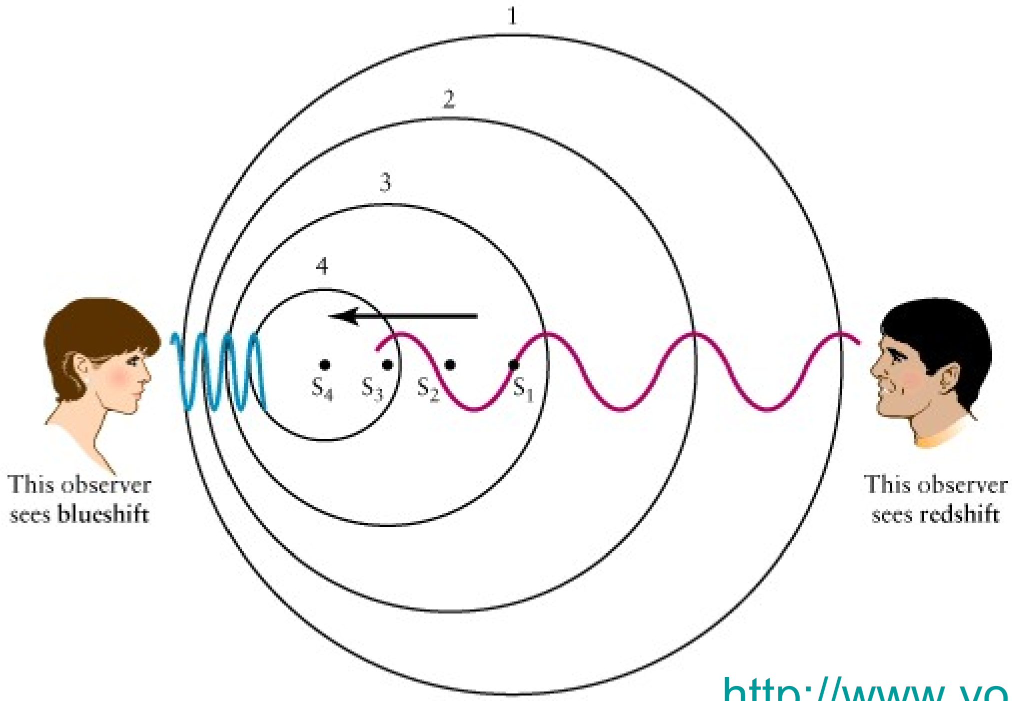
In spacetime diagram, Mary's wordline has a slope = c/0.8c = ±1.25

According to Frank, Mary takes 10 years each way for a total of 20 years, so Frank ages 20 years

Mary's clock ticks slower, so her travel time is 10/γ = 6 years one way and she ages 12 years

Frank is an inertial observer, while Mary is not

Doppler Effect



For sound the frequency shift depends on 3 variables: the source, observer and medium speeds

$$f' = f\,\frac{v \pm v_o}{v \mp v_s}$$

http://www.youtube.com/watch?v=Man9ulEYSgk

For light the equation must be different since there is no medium. Derivation for HW.

$$f = f_o\sqrt{\frac{1 \pm \beta}{1 \mp \beta}}$$

Doppler Effect (2)

Applications:

1. Radar – echo signal used to monitor speed of cars/planes/clouds-air masses
2. Astronomy – star light is typically red-shifted; correlates star recessional speed with distance away via Hubble's law; or use red/blue shifts to detect rotational motion of galaxies
3. Laser cooling – use laser tuned to be absorbed by faster moving atoms traveling toward the laser light- to slow the atoms down

Relativistic Dynamics

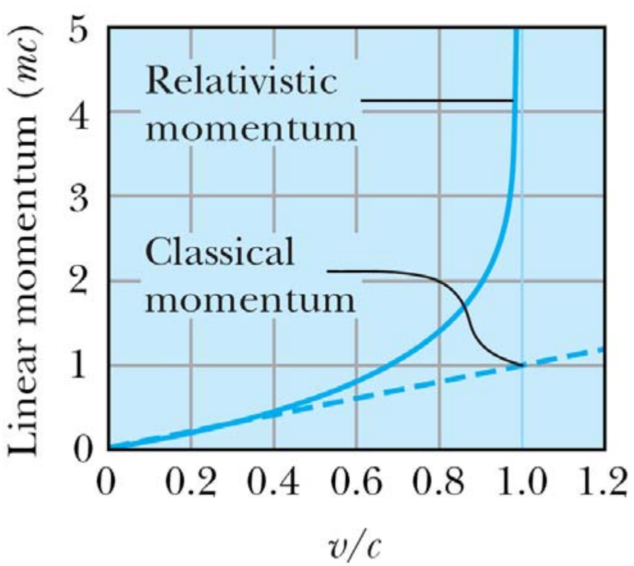
- Momentum (connected to Force) and Energy
- We need to generalize (or re-define) these two quantities so that
  - The conservation laws hold
  - They reduce to the classical expressions when $v \ll c$
- For momentum $p_{classical} = mdx/dt$ has ambiguity in the time and position variables and also is not conserved in high speed collisions
- Re-define momentum as $\vec{p} = \gamma m \vec{u}$ where u is the particle velocity and $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$
- Note that here u is the particle's velocity and not that of the reference frame

Momentum

- Sometimes the factor γ is grouped with m to form a velocity dependent mass called the relativistic mass – T&R keep m constant as the "rest mass"
- With this definition, we can write

$$\vec{F} = \frac{d\vec{p}}{dt}$$

  for Newton's second law, where F, p and t are all measured by the same observer (see HW – where you'll show that if F $\perp$ u, then F = mγa, while if F $\parallel$ u, then F = γ^3ma)



Energy

- Does the form of the Kinetic Energy change from its classical value? Remember that its expression comes from the Work-KE theorem : $W = \int_1^2 \vec{F} \cdot d\vec{s} = K_2 - K_1$

- So, we do the same calculation with the relativistic value for F to find

  $K = \gamma mc^2 - mc^2$

- Check that this reduces to ½ mu^2 for $u \ll c$

Energy (2)

- Interpret $E = mc^2 + K = \gamma mc^2$ = total relativistic energy,

  with $E_o = mc^2$ = rest energy
- Conservation of mass-energy (or just energy – meaning relativistic)
- Can show that $E^2 = p^2c^2 + m^2c^4$
- Massless particles, such as photons, have $E = pc$ – so that they carry momentum as well as energy

Energy Details – eV & Binding E

- For elementary particles, best energy units are eV, where 1 eV = 1e x 1V = 1.6×10^{-19} J (electron rest mass = $9.11 \times 10^{-31} c^2$ = 8.2×10^{-14} J = 0.511MeV)
- For mass units 1 amu = 1u = 1/12 M(^{12}C) =1.66×10^{-27}kg = 931.5 MeV/c^2
- For momentum use MeV/c
- Work through Example 2.13
- Binding energy $E_B = \sum m_i c^2 - M_{bound} c^2$
- Binding energy is work needed to dissociate bound system into separate constituents at rest

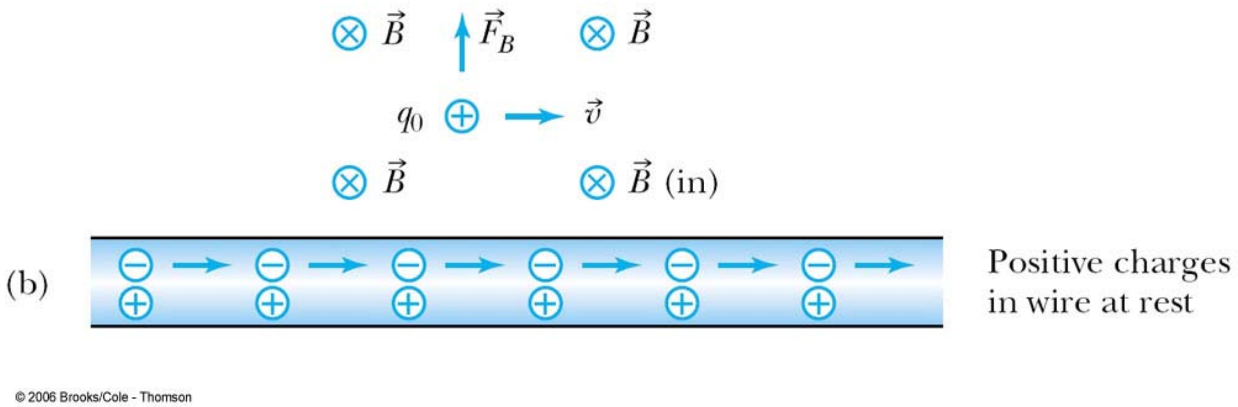
Example of Binding Energy

- Consider the capture of a neutron by a H atom to form an atom of deuterium or “heavy hydrogen”
- Energy is released, mostly in the form of gamma rays with a total energy of 2.23 MeV – where does this energy come from?
- $E_b = m(H)c^2 + m(n)c^2 - m(\text{deuterium})\,c^2$

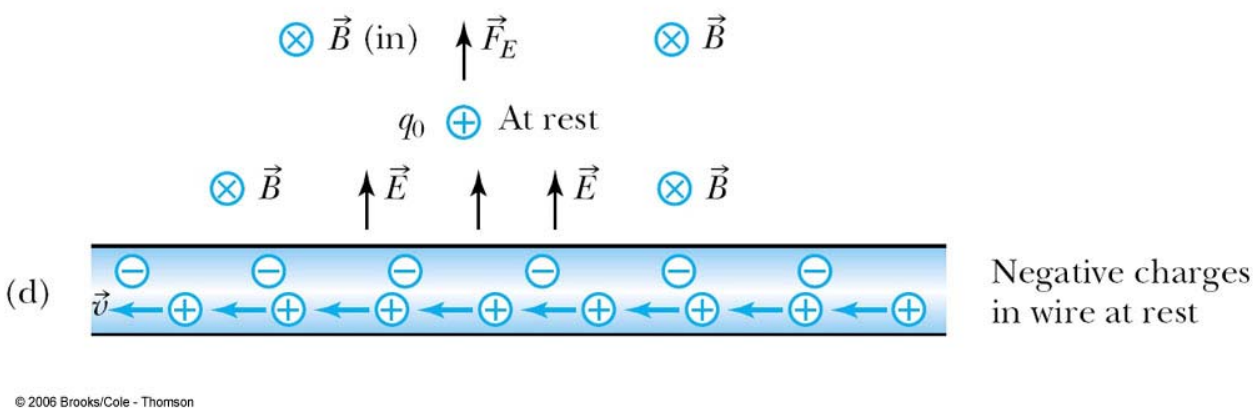
  $= (1.007276u + 1.008665u - 2.01355u)c^2$

  $= (0.002391u)c^2 = 2.23\text{ MeV}$

Relativity and E&M



In this frame, neutral wire with current produces only a B field, resulting in a magnetic force on the moving q



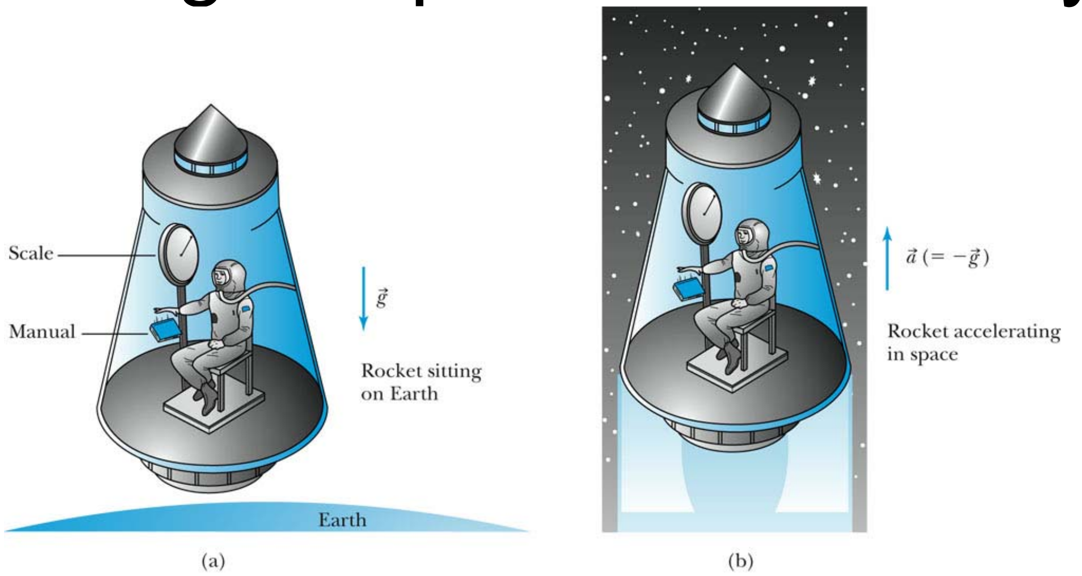
In this frame, q is at rest and there is only an electric force (of the same magnitude) resulting from a net charge on the wire due to length contraction

E&M (2)

- Conclusion is that depending on reference frame, the interpretation of E/B can be different but the net physics (resulting force) must be the same.
- Therefore E and B must somehow be couple and transform from one frame to another – sort of like (x,y,z,t) and how this transforms. In fact the 3 E and 3 B components form a 16-component 2nd rank tensor (4x4 matrix) that transforms according to the rules of relativity

Quick General Relativity

- Einstein spent 12 years developing general relativity after 1905
- Inertial mass (F=ma) vs gravitational mass(F=mg)
- Thought experiment – usually an elevator

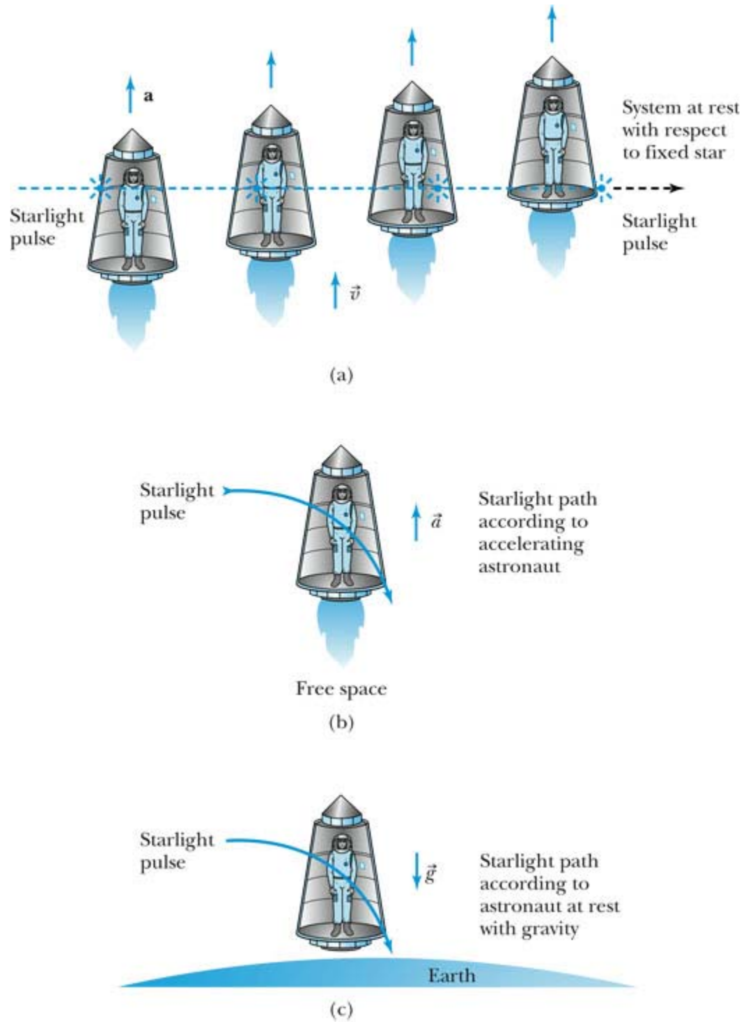


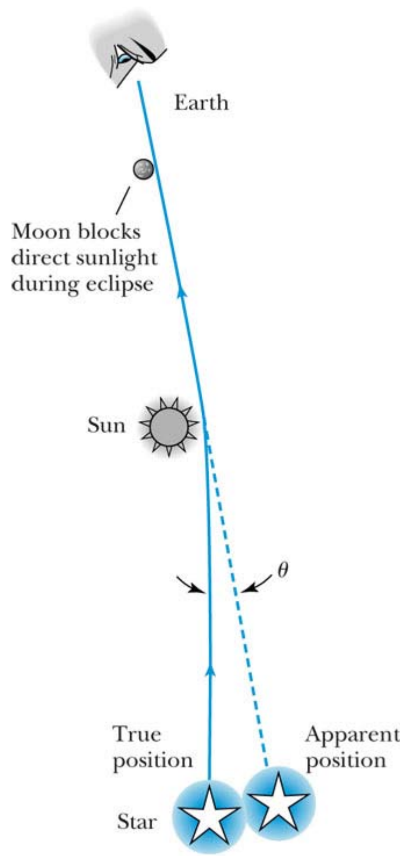
Equivalence principle:

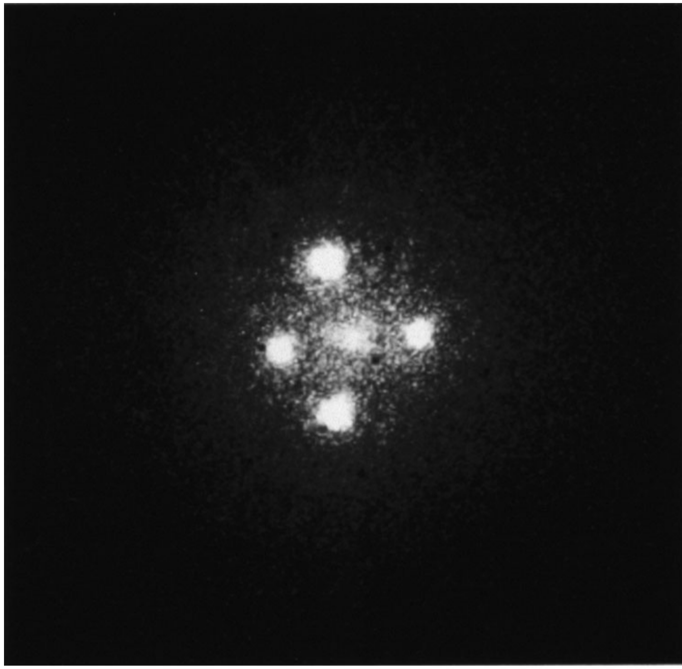
All physics must be the same, so cannot distinguish gravity from acceleration

GR (2)

- One conclusion is that light must bend in a gravitational field







GR (3)

- Einstein relates gravity, which is proportional to mass, to spatial curvature – described by a metric tensor
- While GR is mainly needed for an understanding of astronomical objects on large distance scales, surprisingly it is needed and used for GPS (Global Positioning Systems) where ultrahigh (ns/day) timing precision is needed for precise locationing. Without a GR correction to the timing, 39,000ns/day would be lost and GPS would not work well