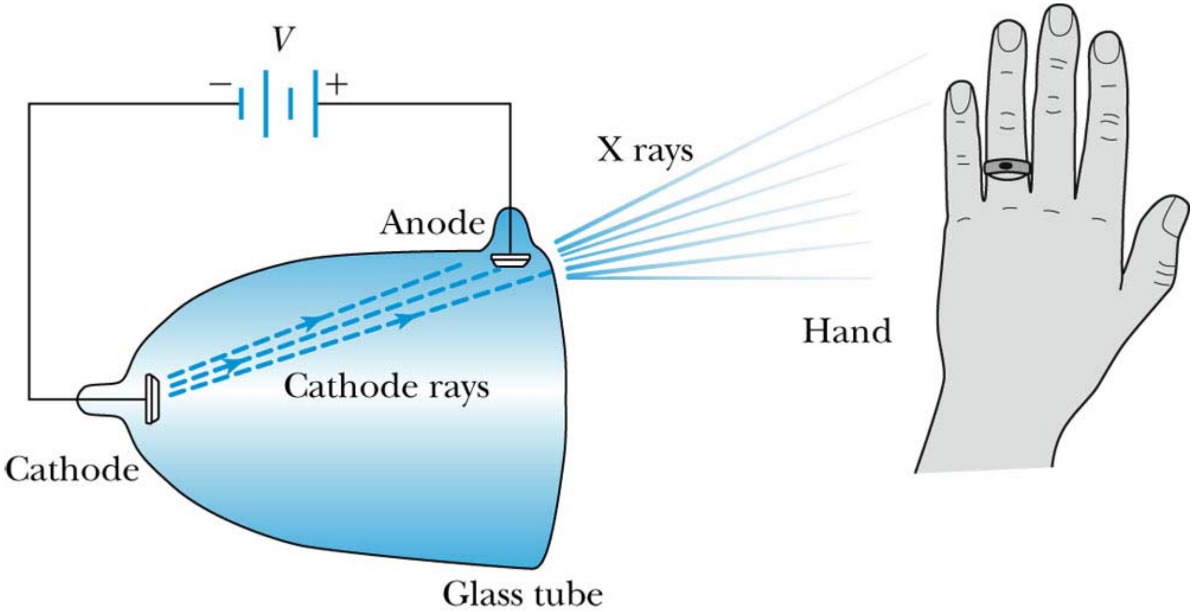
Experimental Basis for QM – Ch3

- This chapter describes the early evidence for quantization – including
  - Blackbody radiation
  - Photoelectric effect
  - Compton scattering
  - X-rays and their spectra
- We'll see how early quantum mechanical ideas were successful in explaining these and led to the development of a full-fledged theory called Quantum Mechanics

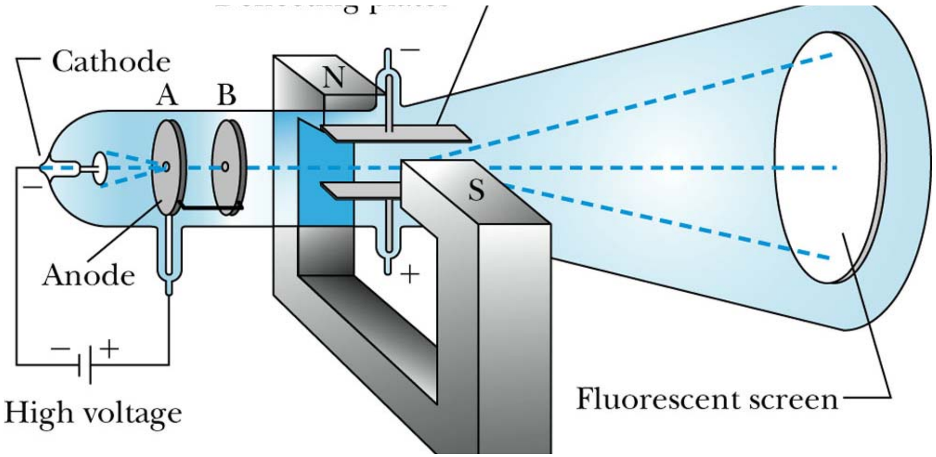
X-rays

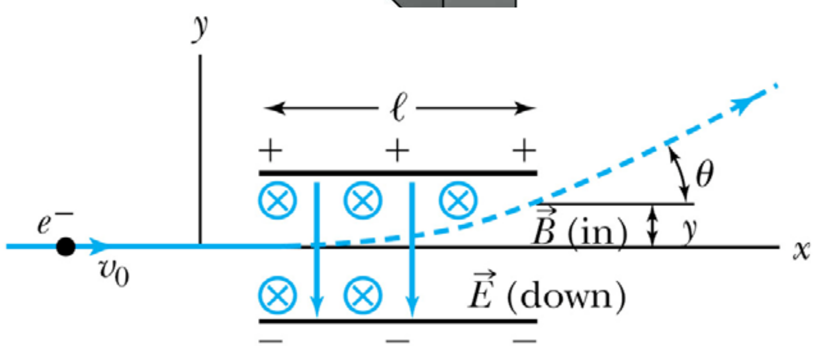


Discovered by Roentgen in 1895 who produced the first “medical” x-ray image of his wife’s hand a few days later. X-rays are high-energy photons and we’ll come back to study them a bit later in this chapter and again in the following chapter.

Electron discovery

Around the same time, J.J. Thomson discovered the electron (1897) to be a charged particle emitted from a heated cathode. He was able to determine its charge/mass ratio in a classic experiment.





Thomson found a value of e/m about 35% too low – but this was about 1000 x larger than expected at the time

In the applied E field across the plates

$F_{vert} = eE = ma_{vert}$

The time to cross the plates of length L is $t = L/v_o$, where v_o can be determined by adding a B field and balancing the 2 forces so there is no deflection:

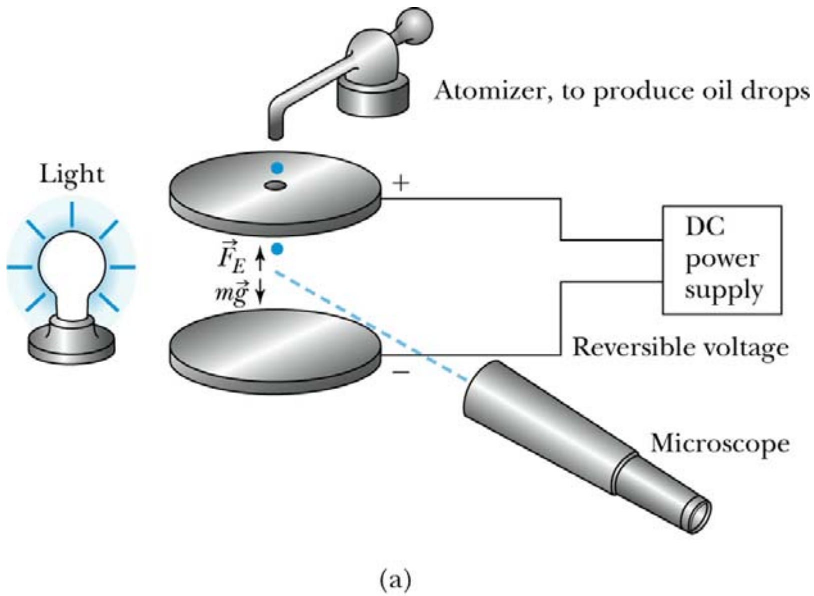
$eE = ev_oB$ so $v_o = E/B$

Then the deflection angle is

$\tan\theta = v_y/v_x = a_{vert}\, t/v_o = (eE/m)(L/E/B)/(E/B) = (e/m)(LB^2/E)$

So $(e/m) = \tan\theta(E/B^2L)$

Millikan Oil Drop Expt (1911; to find e)



(a)

Balancing mg by qE where E = V/d gives

q = mgd/V,

So if m can be determined e can be found.

Turning off E, the drops rapidly reach a measured terminal v, which depends on the drop radius – so then $m = \rho(4/3)\pi r^3$, where ρ is the mass density.

Millikan made thousands of measurements that each gave a q that was a multiple of a fundamental unit of electric charge, $e = 1.602 \times 10^{-19}$ C

Line Spectra

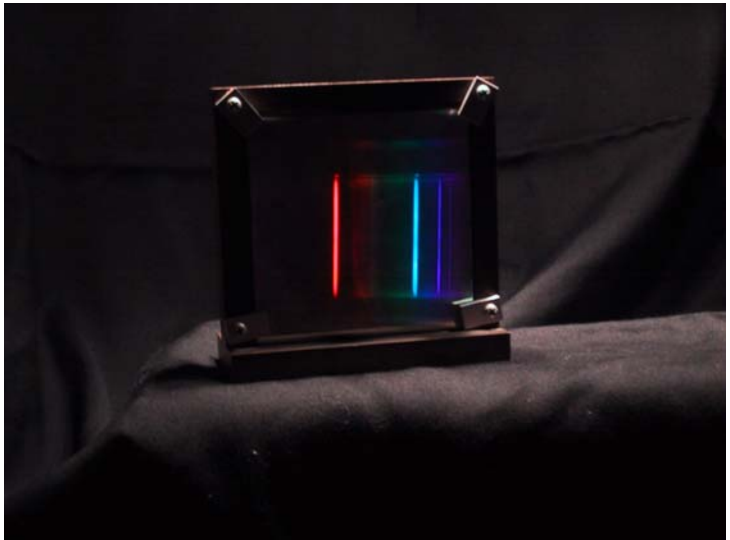
Line spectra = pattern of colored bands dispersed by a prism or diffraction grating – DEMO

- For Hydrogen, Balmer found that

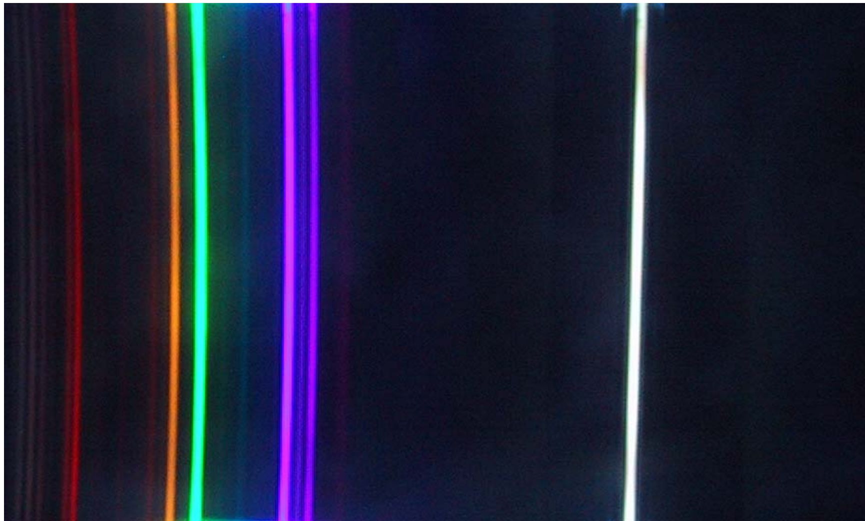
$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \quad where \quad R_H = 1.097x10^7\,m^{-1}$$

- For n = 3, 4, 5, the wavelengths are visible (n = 6 is 410 nm T&R call it visible too)

- Lab on H spectrum



Mercury line spectrum

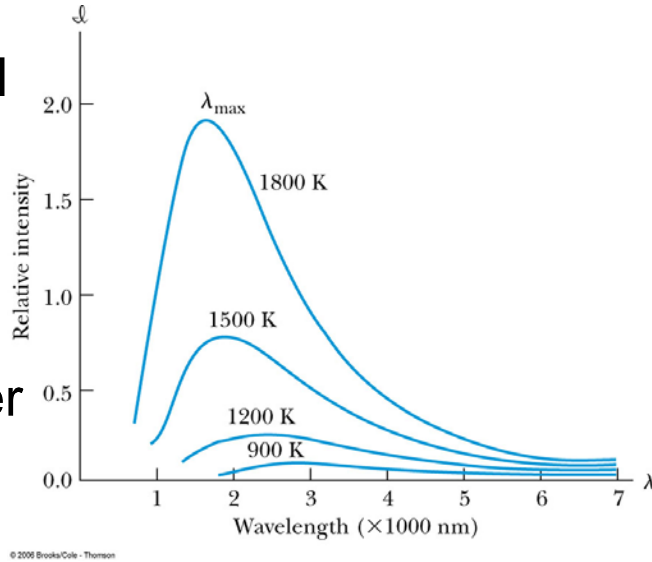


Blackbody Radiation

Thermal radiation – glows dark red at ~550° C; bright red ~700° C – DEMO

When an object is in thermal equilibrium, it emits and absorbs equal amounts of thermal energy – like the Earth (at least averaged over short years)

Simplest case is ideal blackbody (BB)– e.g. a small hole in a hollow container that traps all incident radiation



Two general results of BB radiation:

1. Maximum in intensity distribution shifts to shorter λ with increasing T

   $\lambda_{max}T = 2.898\text{x}10^{-3}$ m K - Wien's displacement law

2. Total power radiated increases rapidly with T: $R(T) = \varepsilon\sigma T^4$, where R(t) is the power per unit area integrated over all λ, ε ~1 is the emissivity ($\varepsilon = 1$ for ideal BB), and $\sigma = 5.67\text{x}10^{-8}$ W/m^{2}K^4

Stefan-Boltzmann Law

BB Radiation (2)

- There was no theory that could fit all the data just shown for BB radiation. There was a particularly difficult problem at short wavelength where all theories seemed to diverge (giving ∞ values)
- In 1900, Max Planck developed a theory that explained the results using the idea that the cavity walls had oscillators (at frequency f) that radiated energy E at values given by $E_n = nhf$, where $h = 6.626\times10^{-34}$ J s, and n is an integer
- Planck found that

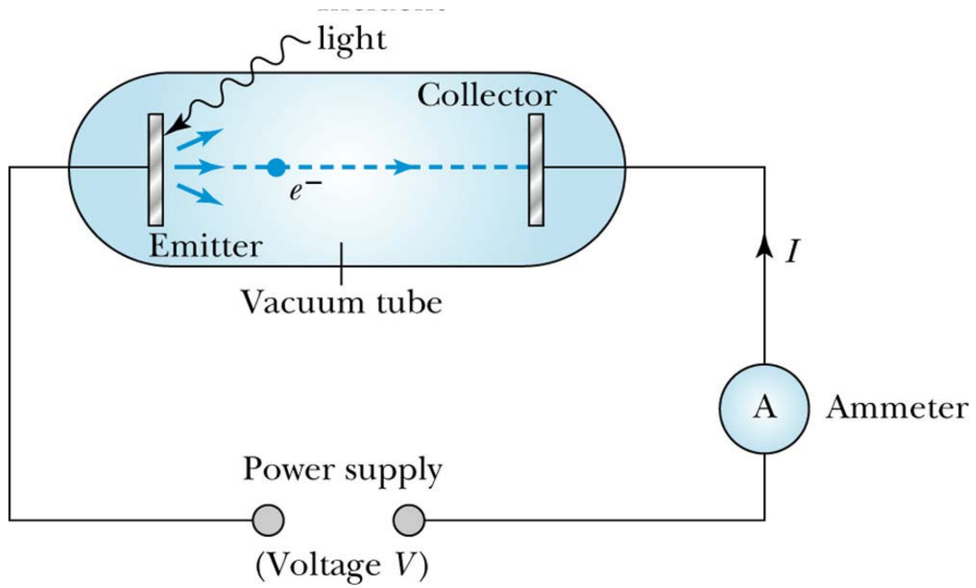
$$I(\lambda,T) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

where $hc/\lambda = hf = E$, so $e^{-E/kT}$

Photoelectric Effect

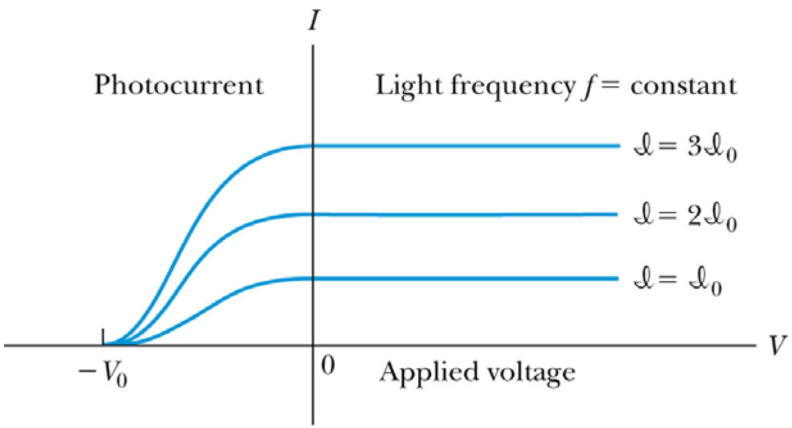
•This is the basis for most photon detection. It had no classical explanation for many years until Einstein (1905) gave an explanation based on the quantization of light in photons. (Lab on this)

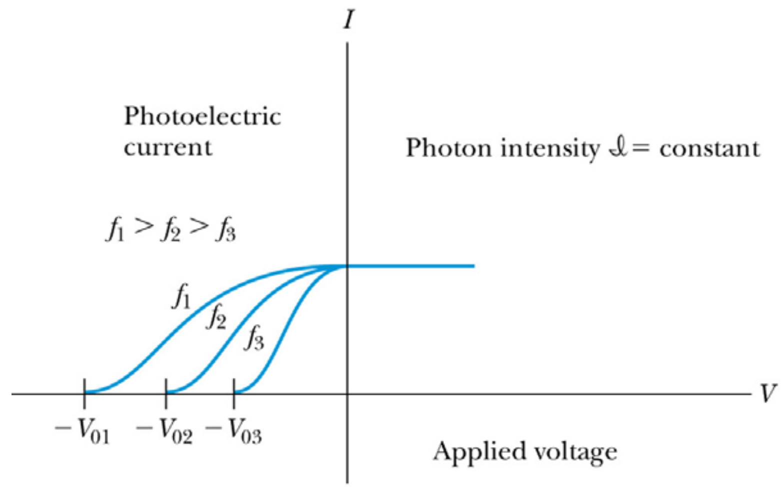
•How does it work?



•Light, interacting with the metal emitter (photocathode) electrode, releases electrons which are attracted to the collector (anode), making up an electric current (photocurrent) in the external circuit – Usually V is positive on collector

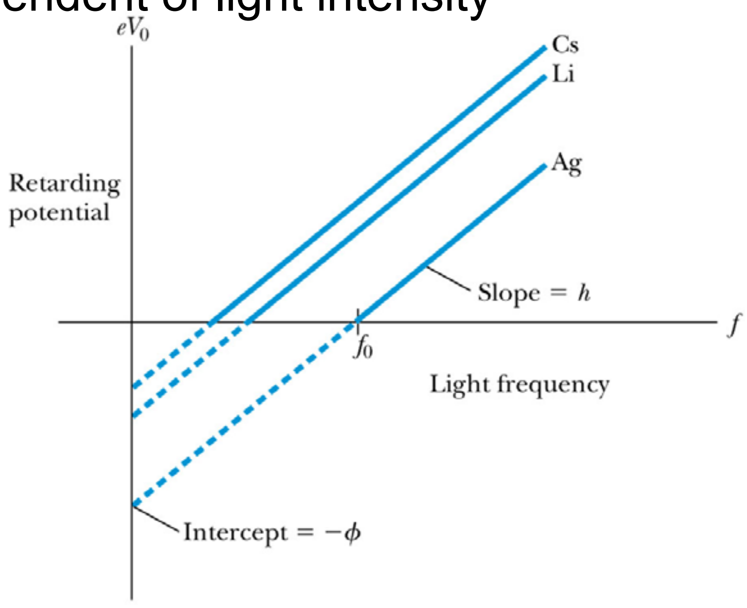
Photoelectric Effect (2)





A negative V (stopping potential) is sufficient to stop all photoelectrons, independent of light intensity

Different colors (f's) result in different stopping V's independent of intensity



Different metals, having different work functions, result in no photoelectrons below the corresponding stopping V

The number of photoelectrons produced is proportional to the light intensity (not shown) and the photoelectrons are always released nearly instantaneously

Photoelectric Effect (3)

- Einstein's Explanation:
  - Photon energy E = hf
  - Photons collide with electrons in metal and release all their energy to an electron:

    $hf = \phi + KE_{max}$ (electron) where ϕ is the work function = minimum energy to release an e^-
  - $hf = \phi + \frac{1}{2} mv^2_{max}$ - non-relativistic energies
  - But $eV_o = \frac{1}{2} mv^2_{max} = hf - \phi = hf - hf_o$
  - Explains all data – different I means different numbers of photons

Problem

- A potassium metal ($\phi = 2.1$ eV) plate is placed 1 m from a light source ($\lambda = 589$ nm) whose power is 1 W. Assume the light spreads out spherically uniformly.
  - Classically, how long would it take an electron to absorb the work function energy? Take the "area" of an electron to be a circular plate with 0.1 nm radius.
  - At what rate per unit area do photons strike the metal plate?
  - If the efficiency for photoelectric emission is 10%, what will be the photocurrent collected at the anode, if the plate has an area of 1 mm^2?
  - Repeat these if the wavelength is 633 nm.

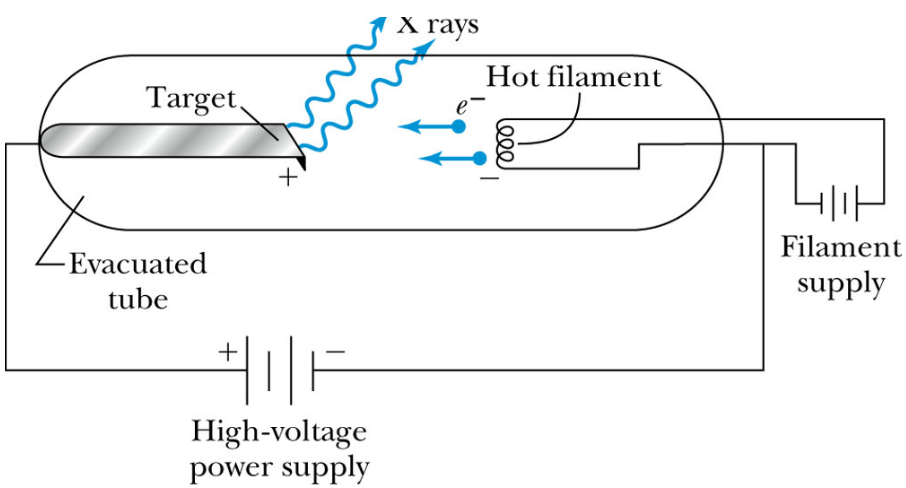
X-rays

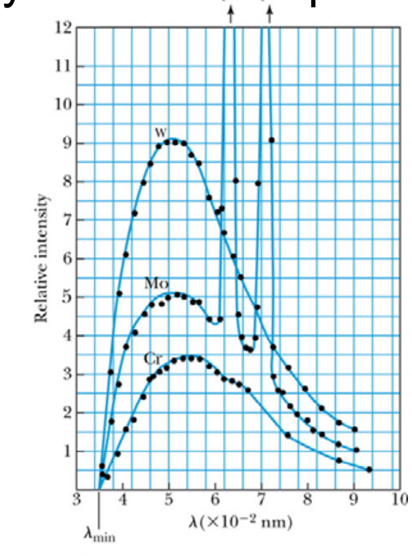
The inverse process to the photoelectric effect – electrons collide with atoms to cause the emission of photons (high-energy = x-rays)

However electrons can give up some of their energy at a time, while for photons it is an all-or-nothing process – for electrons, x-rays are released as electrons accelerate in a process called bremsstrahlung (braking radiation) –

The radiation (x-rays) emitted from this process using an x-ray tube (below) have a wide range of wavelengths and are used in science/engineering and medicine – here the energy balance is $eV_o = hf_{max} = hc/\lambda_{min}$ or $\lambda_{min} = hc/eV_o$ (Duane-Hunt rule)

X-ray spectra look like the one below, with a continuous background due to bremsstrahlung and narrow peaks – called characteristic x-rays – we will explain later





Compton Scattering (1922)

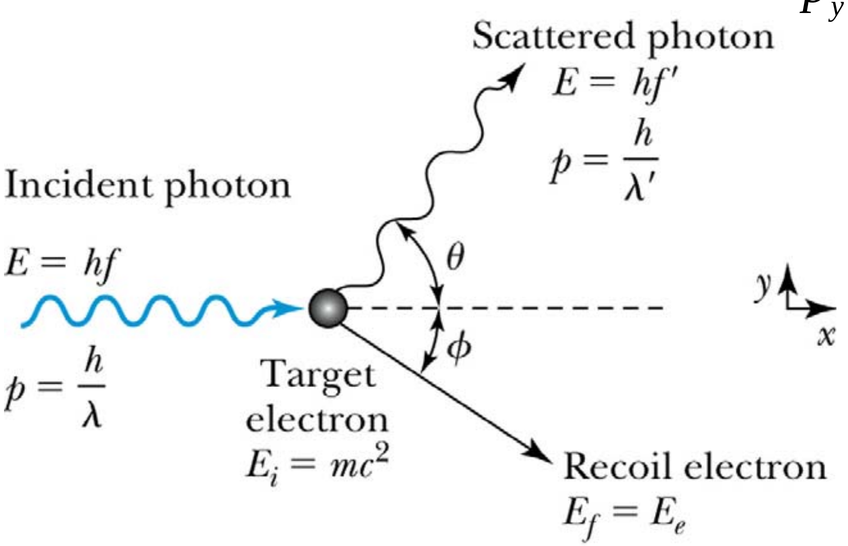
- Scattering of x-rays from a target (electron) – a game of billiards – Lab

•Treat x-ray photon and electron as particles with momentum & energy conserved in the collision. Relativity is needed since the electron can have high speeds.

•Conservation of energy: $mc^2 + \frac{hc}{\lambda} = \frac{hc}{\lambda'} + E \quad where \quad E^2 = m^2c^4 + p^2c^2$

•Conservation of momentum: $p_x: \quad \frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p\cos\phi$

$$p_y: \quad \frac{h}{\lambda'}\sin\theta = p\sin\phi$$



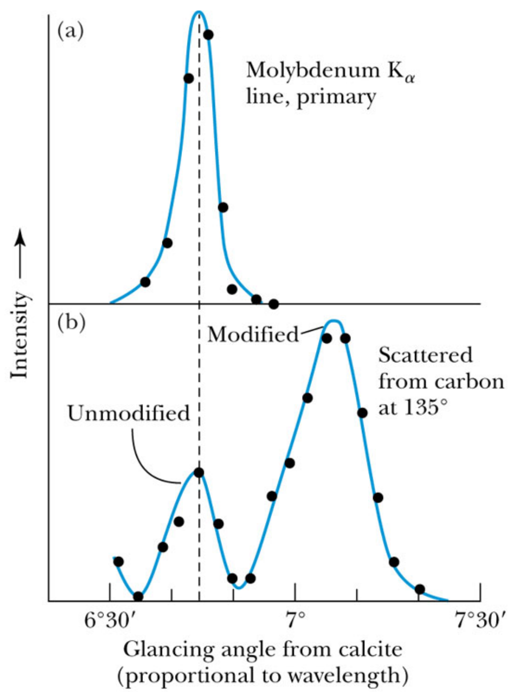
Algebra (eliminate ϕ) results in

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

$$= 2.426x10^{-3}\,nm(1 - \cos\theta)$$

Compton Scattering (2)

- Because the Compton wavelength of the electron = h/mc = 2.426 x 10^{-3} nm is so small, only for very short wavelengths will the fractional shift $\Delta\lambda/\lambda$ be appreciable. For example for visible light (500 nm), $\Delta\lambda/\lambda$ is only 10^{-5} and also the probability of a collision is small. For x-rays, the ratio can be several % - easily observed.



Problem

- Consider an x-ray beam, with λ = 0.1 nm, and also a γ-beam with λ = 0.00188 nm. If the radiation scattered from free electrons is viewed at 90^o to the incident beam:

  (a) What is the Compton wavelength shift in each case?

  (b) What kinetic energy is given to a recoiling electron in each case?

  (c) What percentage of the incident photon energy is lost in the collision in each case?

Pair Production & Annihilation

Pair Production: A photon transforms into a particle-antiparticle pair (all conservation laws must be obeyed: energy, charge, momentum, etc.)
For example: $\gamma \rightarrow e^+ + e^-$
{note that charge and energy are conserved ($hf > 2mc^2 = 1.022$MeV), but momentum cannot be (see text), unless the reaction takes place in matter where a nearby nucleus can absorb some momentum}

Pair Annihilation: A particle and antiparticle collide and produce a pure energy photon For example: $e^+ + e^- \rightarrow \gamma + \gamma$
{note that in free space two photons are required to conserve momentum}

PET scan