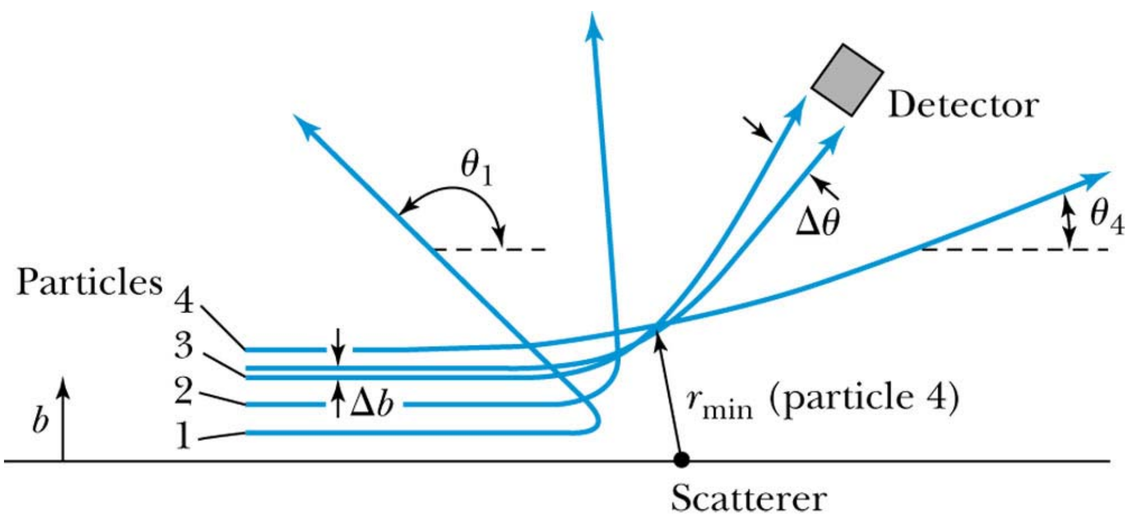
Atomic Models – the Nucleus

Rutherford (read his bio on pp 134-5), who had already won a Nobel for his work on radioactivity – had also named alpha, beta, gamma radiation, developed a scattering technique to study the atom.

He, together with Marsden & Geiger, directed energetic alpha particles onto thin gold-leaf targets and studied their scattering. In 1911 he reported that the results were not consistent with Thomson's plum-pudding model in which the electrons were embedded in a uniform positive charge background sphere – like raisins in a plum pudding.

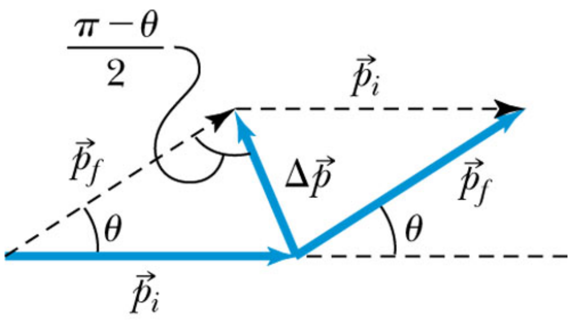


What they found was that as the impact parameter b got smaller there were large deviations, including backscattered alpha's – further they analyzed the results in terms of simple Coulomb scattering from a tiny central positive nucleus and found agreement with their data

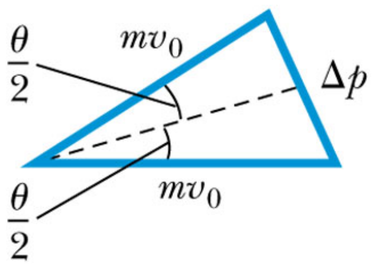
Rutherford scattering

- Rutherford made 4 assumptions in trying to understand their data:
  - The gold atom is so massive that it does not recoil – so the alpha does not loose KE
  - No multiple scattering exists (the target is thin)
  - Can treat everything as point particles
  - Only the Coulomb force is effective
- Derivation of Rutherford scattering equation
  - Start with expression for Δp for alpha (see diagram on right)

    $\Delta p = 2mv_o \sin\theta/2$
  - Using Newton's second law $\Delta p = F_{\Delta p}\Delta t$, where we take the component of the Coulomb force along the net change in momentum direction



(a)



(b)

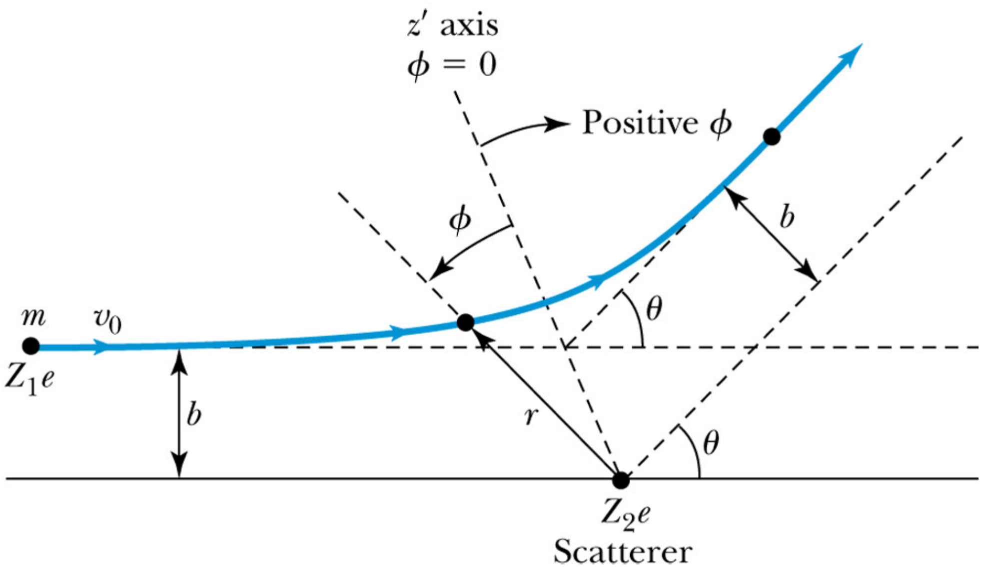
Rutherford scattering (2)

Using the diagram below, with z' along the Δp direction, we have

$$\Delta p = 2mv_o \sin\frac{\theta}{2} = \frac{1}{4\pi\varepsilon_o}\int\left[\frac{Z_1 Z_2 e^2}{r^2}\cos\phi\, dt\right]$$

But conservation of angular momentum gives $L = mv_o b = (mr^2)d\phi/dt$, so that $1/r^2 = (1/v_o b)d\phi/dt$ so our integral becomes:

$$2mv_o \sin\frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o v_o b}\int \cos\phi \frac{d\phi}{dt} dt = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o v_o b}\int \cos\phi\, d\phi$$



Rutherford (3)

The integral ranges from ϕ = -(π–θ)/2 to +(π–θ)/2, so

$$\int \cos\phi\, d\phi = \sin\phi \Big|_{-(\pi-\theta)/2}^{(\pi-\theta)/2} = 2\cos(\theta/2)$$

and we have, solving for b:

$$b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_o m v_o^2}\cot\frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o K}\cot\frac{\theta}{2}$$

- Now, we introduce the scattering cross-section, σ, $\sigma = \pi b^2$, as the cross-sectional area of interaction for scattering through angle θ or larger
- With a thin foil target with n atoms/volume and thickness t, the fraction of incident particles scattered by the target, f, is the number of target atoms per unit area (nt) multiplied by the scattering cross section or

$$f = nt\sigma = nt\pi b^2 = \pi nt\left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o K}\right)^2 \cot^2\frac{\theta}{2}$$

- One last complication is that the detector spans an angular range of $\Delta\theta$ so by differentiating the previous expression we find

$$df = -\pi nt\left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o K}\right)^2 \cot\frac{\theta}{2}\csc^2\frac{\theta}{2}\, d\theta$$

Rutherford (4)

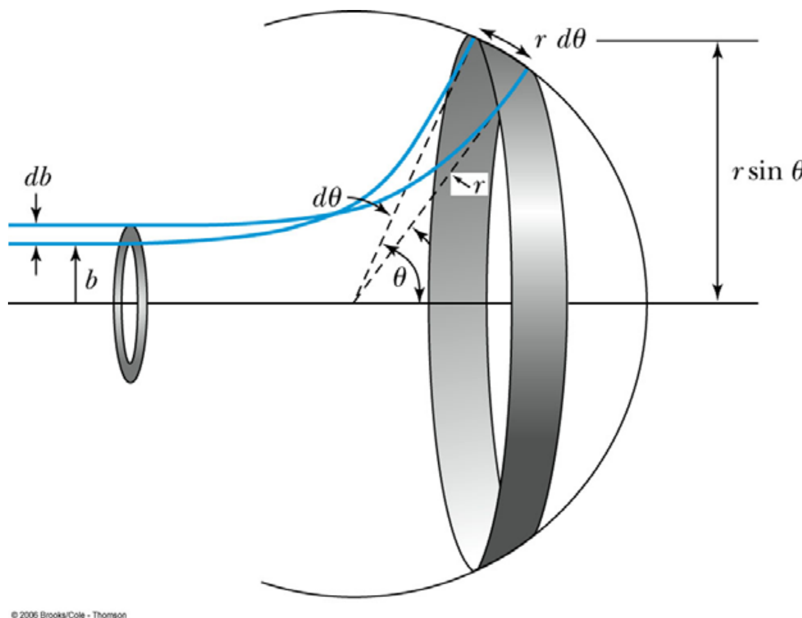
- Finally, if the number of incident particles is N_i, then the number scattered per unit area into the detector spanning a ring of annular width $d\theta$ is $N(\theta) = N_i|df|/dA$, where dA, from the figure below is given by

$$dA = (r\,d\theta)(2\pi r \sin\theta) = 2\pi r^2 \sin\theta\, d\theta$$

- Putting in df and dA, we find the final result for $N(\theta)$

$$N(\theta) = \frac{N_i|df|}{dA} = \frac{N_i \pi n t \left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_o K}\right)^2 \cot\frac{\theta}{2}\csc^2\frac{\theta}{2}\, d\theta}{2\pi r^2 \sin\theta\, d\theta}$$

$$N(\theta) = \frac{N_i n t}{16}\left(\frac{e^2}{4\pi\varepsilon_o}\right)^2 \frac{Z_1^{\,2} Z_2^{\,2}}{r^2 K^2 \sin^4\frac{\theta}{2}}$$



1. Z^2 dependence
2. K^{-2} dependence
3. $\sin^4(\theta/2)$
4. t dependence

All these were verified

Problems

- Problems 7 and 10 in chapter

7. The fraction f is proportional to nt and to Z^2 from Equation (4.12). The question states, however, the number of scattering nuclei per unit area is equal so nt is the same for either target. Therefore

$$\frac{N(\text{Au})}{N(\text{Al})} = \frac{n(\text{Au})t(79)^2}{n(\text{Al})t(13)^2} = \frac{79^2}{13^2} = 36.93$$

* 10. From the Rutherford scattering result, the number detected through a small angle is inversely proportional to $\sin^4\left(\frac{\theta}{2}\right)$. Thus

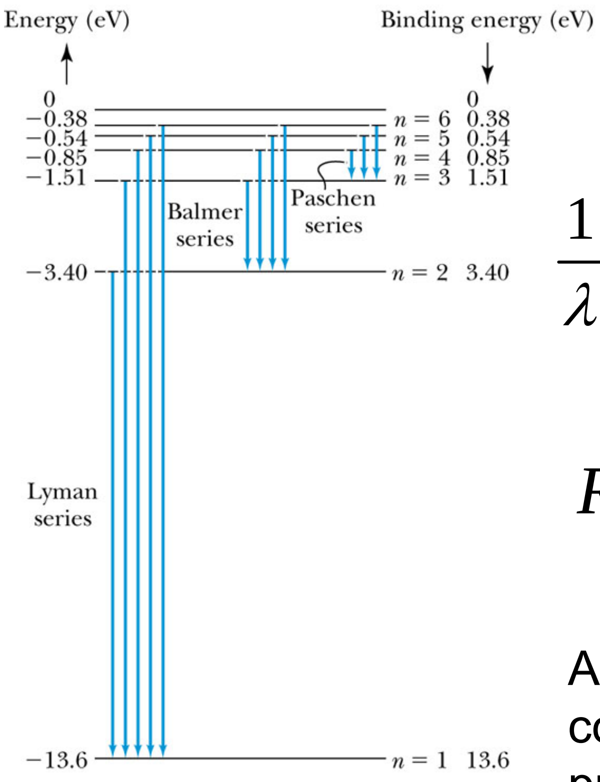
$$\frac{n(50^\circ)}{n(6^\circ)} = \frac{\sin^4(3^\circ)}{\sin^4(25^\circ)} = 2.35 \times 10^{-4}$$

and if they count 2000 at 6° the number counted at 50° is $(2000)\left(2.35 \times 10^{-4}\right) = 0.47$ which is insufficient.

Bohr Model of Atom

- Bohr's assumptions
  - Stationary states – definite E
  - Transitions between these give absorption or emission of photons: $\Delta E = hf$
  - Classical physics governs stationary states but not transitions between them
  - Angular momentum of atom is quantized to be a multiple of $h/2\pi = \hbar$
- Derivation of energy levels: $E_n = -E_o/n^2$
- Transitions between these gives spectrum

Bohr Model (2)



$$hf = E_u - E_l = -E_o\left[\frac{1}{n_u^2} - \frac{1}{n_l^2}\right] \quad ; \quad E_o = 13.6eV$$

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E_u - E_l}{hc} = \frac{-E_o}{hc}\left[\frac{1}{n_u^2} - \frac{1}{n_l^2}\right] = R_\infty\left[\frac{1}{n_l^2} - \frac{1}{n_u^2}\right]$$

$$R_\infty = \frac{E_o}{hc} = 1.097373x10^7 m^{-1} = Rydberg$$

Actually, the electron and proton revolve around their common center of mass – in mechanics, this 2-body problem can be reduced to a 1-body problem where the electron mass is replaced by the reduced mass μ, where $\mu = \frac{mM}{m+M} = \frac{m}{1+\frac{m}{M}} = 0.999456m$ so that R = $(\mu/m)R_\infty$

Problem

- Problems 22 and 34

Solutions

22. We've seen that $L = mvr = n\hbar$ results in $v = n\hbar/mr$ but also $r = n^2a_o$ so that

    $v = \hbar/nma_o$ or $v/c = \hbar/nmca_o = \alpha/n$, where $\alpha = 1/137$

    So for n = 1, 2, 3, we have v/c = 0.0073, 0.0036, and 0.0024.

34. Since $r_n = n^2a_o$, we have:

    (a) $r_2 – r_1 = 3a_o$; (b) $r_5 – r_2 = 21a_o$; (c) $r_6 – r_5 = 11a_o$; (d) $r_{11} – r_{10} = 21a_o$

    (note $r_m – r_n = (m+n)a_o$ for $m-n = 1$)

Correspondence Principle

- In the limits where classical and quantum physics must agree, the quantum theory must reduce to classical results
- For example, in the Bohr model for large n, where the energy level spacing is very small (almost continuous), for a transition from n+1 to n we should get the classical result for the frequency of emitted light. (Show this in class)

Limitations of Bohr Model

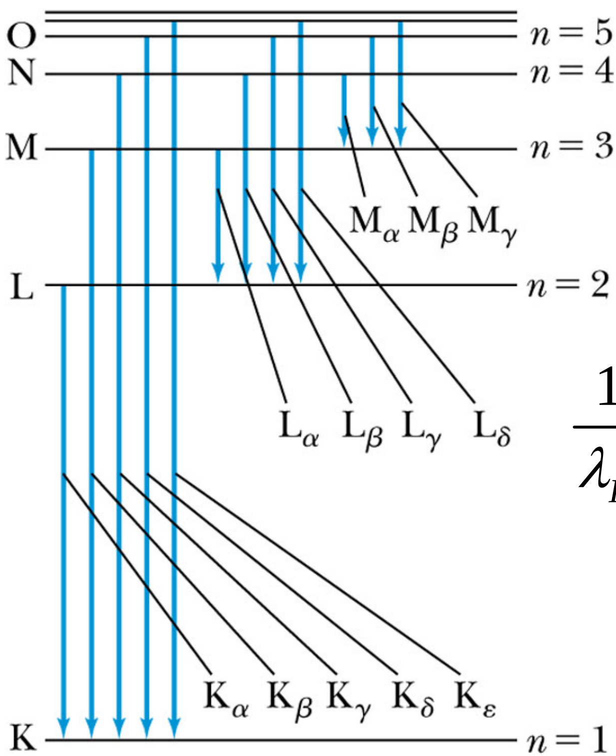
1. Why is L quantized?
2. Why are the “stationary orbits” stable?
3. How long do atoms stay in excited states?
4. What about atoms with 2 or more electrons?
5. Could not predict intensities of lines or the fine structure seen in the presence of fields.
6. Could not explain the binding of atoms to form molecules.

Bohr theory has an ad hoc nature that leaves lots of open questions

It can be generalized to other single-electron (hydrogen-like) atoms that have been ionized (see text)

Characteristic X-rays

- Characteristic x-rays arise from electron transitions from an upper energy level (outer shell) to a lower one (inner shell) that is vacated by a collision with a high energy electron

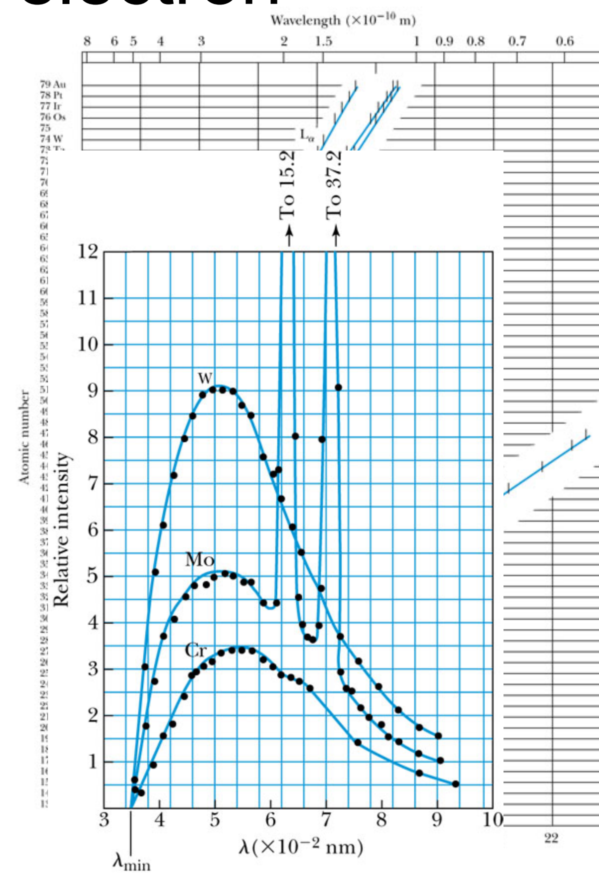


For K-shell vacancy, there still remains 1 K shell electron (two e^- are allowed in K shell) so, Moseley found that

$$\frac{1}{\lambda_K} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

so that a plot of Z vs $f^{1/2}$ should be linear

Showed that Z (not A) was the important factor in the Periodic Table



Franck Hertz Experiment

Experiment: (to be done in lab) accelerated electrons pass through Hg gas. As the energy of the electrons increases (according to increase in V), collisions can lead to excitation of Hg atoms (requires 4.88 eV for transition from ground to first excited state). At higher energies, each accelerated electron can make multiple collisions with different Hg atoms leading to the current vs V graph shown below.