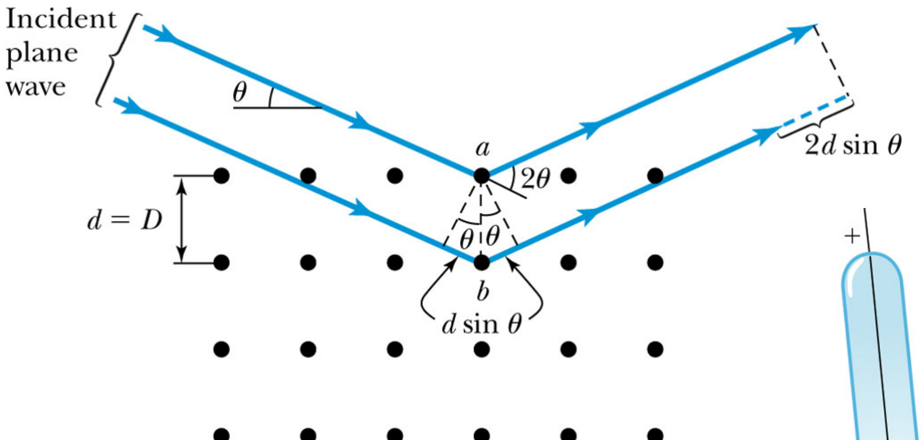
Evidence that x-rays are wave-like

- After their discovery in 1895 by Roentgen, their spectrum (including characteristic x-rays) was probed and their penetrating ability was exploited, but it was difficult to diffract x-rays (lack of good optics)
- Von Laue suggested in 1912 that atomic crystals could be used to diffract x-rays because their λ was on the order of atomic spacings
- Father-son team of the Braggs provided first evidence for x-ray diffraction from crystals – Bragg diffraction

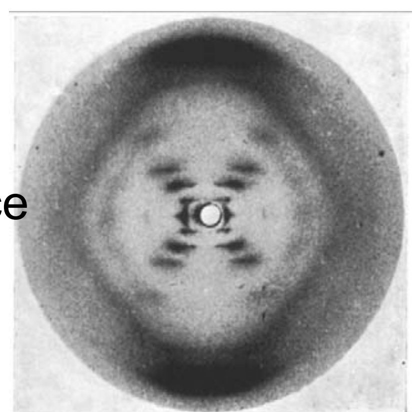
Bragg diffraction

- Crystal models – introduce Bragg planes
- Interference conditions – path difference considerations

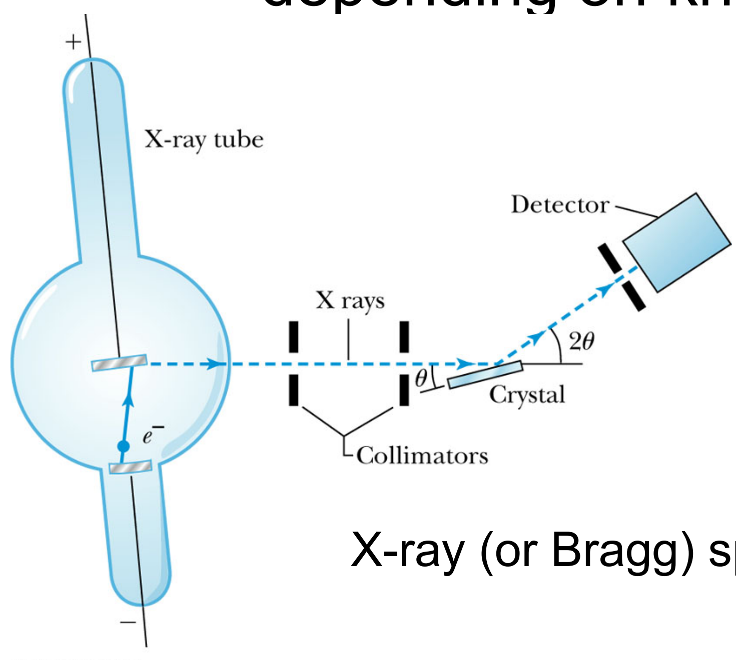


$n\lambda = 2d\sin\theta$

Can be used to find λ or d – depending on knowns



Interference demos



X-ray (or Bragg) spectrometer

De Broglie Waves

- By 1920, physicists realized that photons (incl x-rays) had both particle-like (photo-electric effect, Compton, etc.) and wave-like (interference/diffraction, polarization) natures.
- de Broglie suggested, in 1924 that all matter also has both wave and particle properties and he connected these by introducing a wavelength for particles with momentum p

$$\lambda = \frac{h}{p}$$

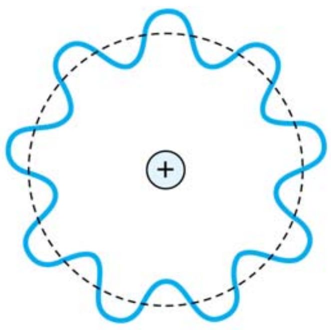
- This already works for photons since E = hf = pc implies that c/f = λ = h/p.
- These “particle” or matter waves have a wavelength called the “de Broglie wavelength”

Problems

1. Chapter 5 Problem 7
2. Problem 11
3. Problem 18

Explaining Bohr's Quantization of L

- Electron waves traveling around circular orbits (self) interfere so that the only "allowed" wavelengths are those that "resonate" or constructively interfere.
- These must fit in the orbit in such a way that $n\lambda = 2\pi r$



- But since $\lambda = h/p$, we have $2\pi r = n(h/p)$ or $L = pr = n(h/2\pi) = n\hbar$ – Bohr's condition

Davisson-Germer proof of wave-like electrons

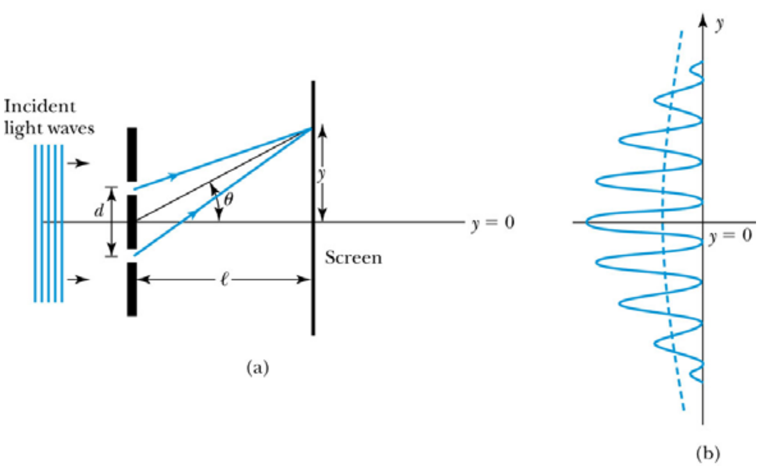
- Electrons diffract from a crystalline target in a very similar way to x-rays

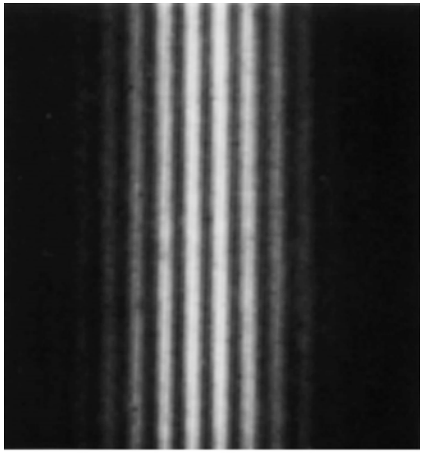
(a)

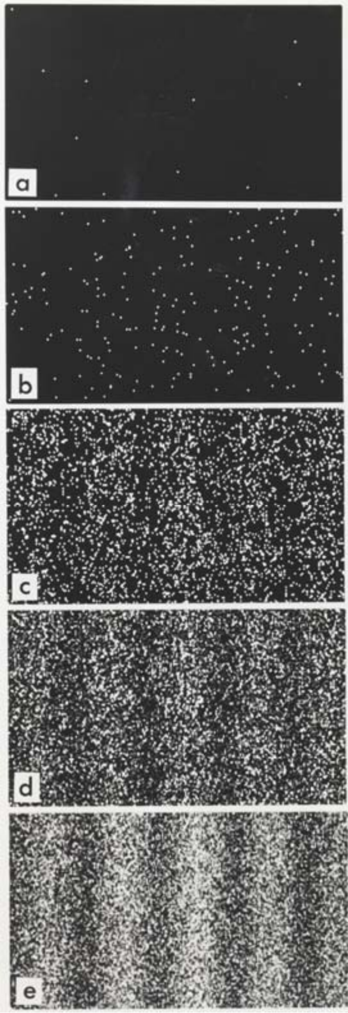
Demo of electron diffraction??

Waves or Particles? Double Slit

- Double slit experiment with light (demo)
- Double slit expt at low light levels
- Electron double slit expt
- Electron double slit at low intensities

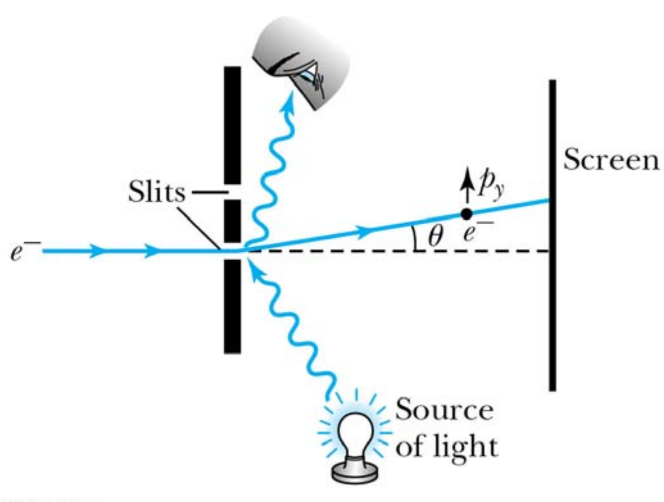






Where's the electron going?

- We think of electrons as particles – which slit does it go through?
- If we look to see, then each photon doing the looking interacts with each electron and perturbs it greatly, so there is no interference.
- If we don't try to look, then there is interference and we're forced to conclude that each electron goes through both slits and behaves as a wave



Photon must have $\lambda \sim d \rightarrow p = h/d$

But electron must also have $p \sim h/d$

So a collision of a photon with an electron will produce major momentum changes to the electron and eliminate the diffraction pattern

Wave-Particle Duality

- How are we to think about the dual nature of photons and matter – wave- and particle-like?
- Bohr: the Principle of Complementarity –

  it is not possible to describe any physically observable quantity simultaneously in terms of both particles and waves
- Perhaps not the most satisfying idea – matter and radiation appears to propagate via waves, but interact (creation/annihilation or detection) via particles

Math digression on Waves

- De Broglie matter waves suggest a further description. The displacement of a wave is

$$\Psi(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

- This is a solution to the wave equation

$$\frac{\partial^2\Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}$$

- Define the wave number *k* and the angular frequency *ω* as:

$$k \equiv \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

- The wave function is now: $\Psi(x, t) = A \sin (kx - \omega t)$.

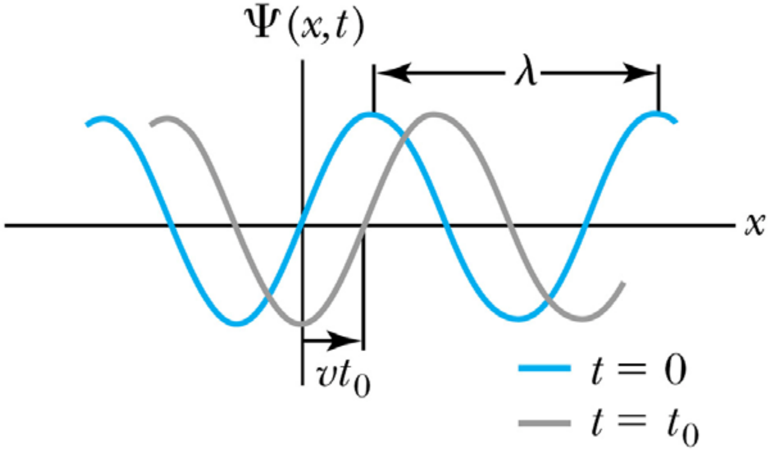
Wave Math (2)

- The phase velocity is the velocity of a point on the wave that has a given phase (for example, the crest) and is given by

$$v_{\text{ph}} = \frac{\lambda}{T} = \frac{\omega}{k}$$

- A phase constant *Φ* shifts the wave:

$$\Psi(x,t) = A\sin(kx - \omega t + \phi)\ .$$



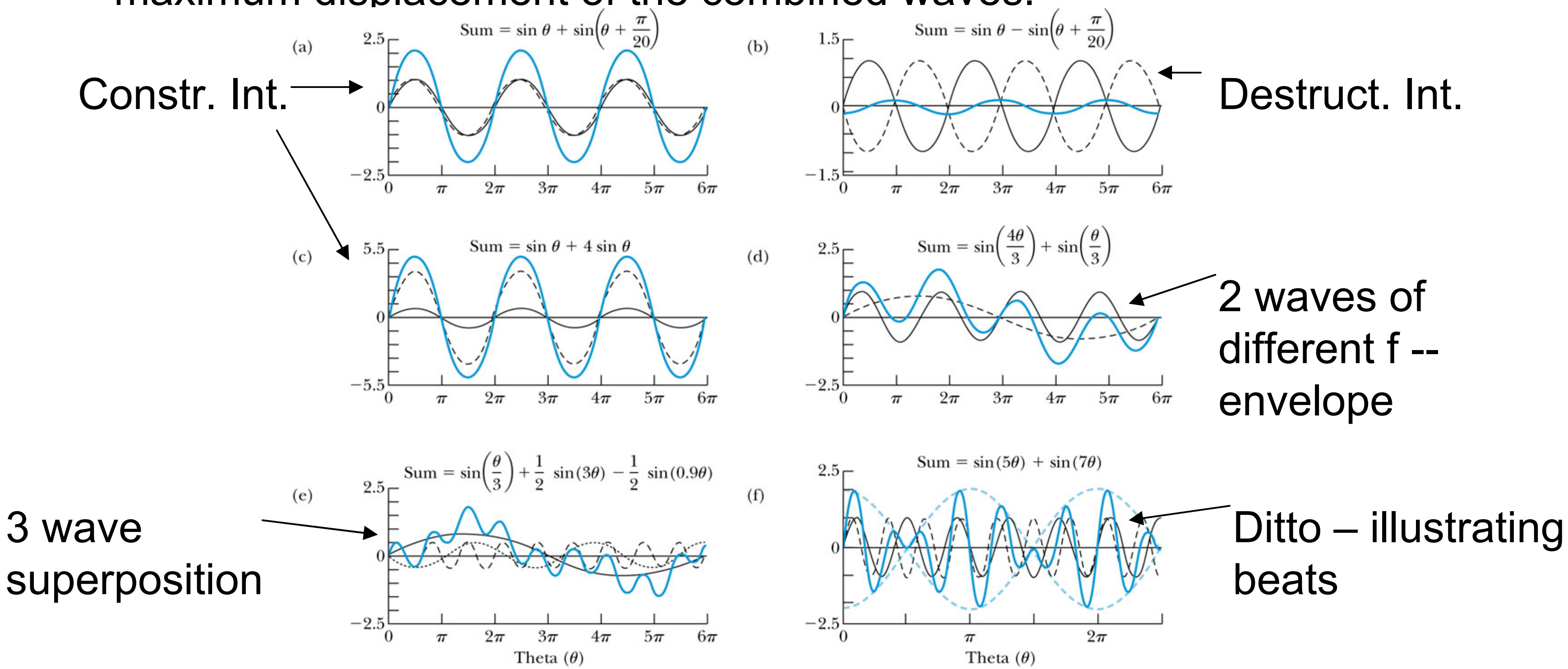
Wave Math (3)

- When two or more waves traverse the same region, they act independently of each other.
- Combining two waves yields: (details worked out)

$$\Psi(x,t) = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

$$= 2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\cos\left(k_{av}x - \omega_{av}t\right)$$

- The combined wave oscillates within an envelope that denotes the maximum displacement of the combined waves.

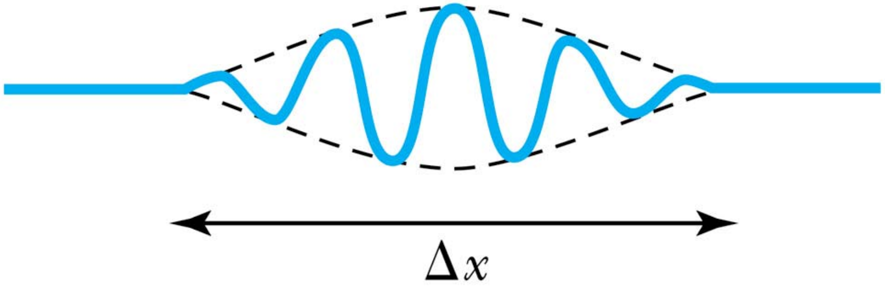


Wave Math (4)

- When combining many waves with different amplitudes and frequencies, a pulse, or **wave packet**, is formed which moves at a **group velocity**:

$$u_{gr} = \Delta\omega \,/\, \Delta k.$$

http://www.csupomona.edu/~ajm/materials/animations/packets.html



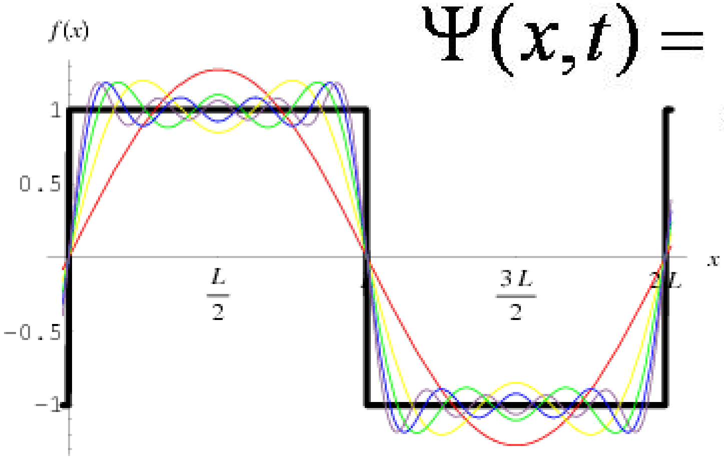
Wave Math (5)

- The sum of many waves that form a wave packet is called a **Fourier series**:

$$\Psi(x,t) = \sum_i A_i \cos(k_i x - \omega_i t)$$

- Summing an infinite number of waves yields the Fourier integral:

$$\Psi(x,t) = \int \tilde{A}(k)\cos(kx - \omega t)\, dk$$



- The localization of the wave packet over a small region to describe a particle requires a large range of wave numbers. Conversely, a small range of wave numbers cannot produce a wave packet localized within a small distance.

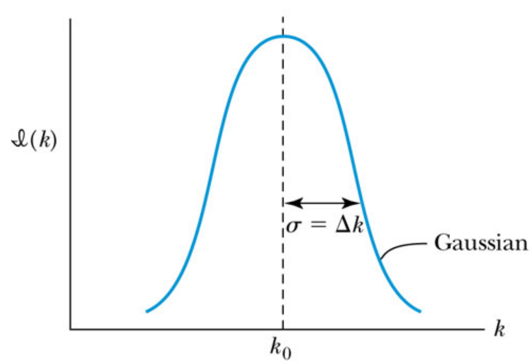
Wave Math (6)

- For a wave packet, the spatial Δx and temporal Δt extent of the packet are given by

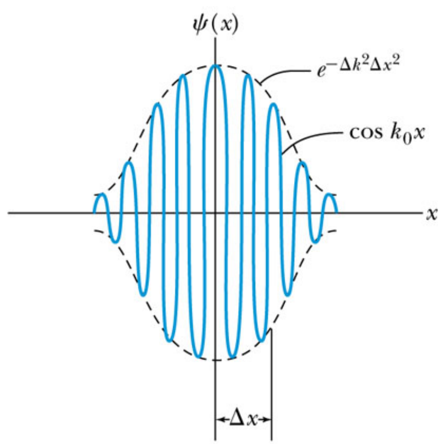
$$\Delta k \Delta x = 2\pi \quad and \quad \Delta\omega \Delta t = 2\pi$$

- A Gaussian wave packet describes the envelope of a pulse wave.

$$\Psi(x,0) = \Psi(x) = Ae^{-\Delta k^2 x^2}\cos(k_0 x)$$



(a)



(b)

The group velocity is $u_{gr} = \dfrac{d\omega}{dk} = \dfrac{d(\hbar\omega)}{d(\hbar k)} = \dfrac{dE}{dp}$

Wave Math (7)

- Considering the group velocity of a de Broglie wave packet yields:

$$u_{\mathrm{gr}} = \frac{dE}{dp} = \frac{pc^2}{E}$$

- The relationship between the phase velocity and the group velocity is

$$u_{\mathrm{gr}} = \frac{d\omega}{dk} = \frac{d}{dk}(v_{\mathrm{ph}} k) = v_{\mathrm{ph}} + k\frac{dv_{\mathrm{ph}}}{dk}$$

- Hence the group velocity may be greater or less than the phase velocity. A medium is called nondispersive when the phase velocity is the same for all frequencies and equal to the group velocity.

Uncertainty Principle

- Heisenberg (1927) first proposed that it is impossible to simultaneously measure both the position and momentum of any particle. It follows from our discussion of the extent of wave packets:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(h/p)} = p\frac{2\pi}{h} = \frac{p}{\hbar} \qquad So \quad \Delta k = \frac{\Delta p}{\hbar}$$

$$But\ \ for\ a\ Gaussian\ packet: \quad \Delta k \Delta x = \frac{\Delta p}{\hbar}\Delta x = \frac{1}{2}$$

$$So\ \ for\ a\ GP, \quad \Delta p_x \Delta x = \frac{\hbar}{2}$$

$$In\ general, \quad \boxed{\Delta p_x \Delta x \geq \frac{\hbar}{2}}$$

http://www.youtube.com/watch?v=KT7xJ0tjB4A

Uncertainty (2)

- It turns out that every pair of so-called conjugate variables has an uncertainty relation. Conjugate pairs include (p_x, x) and the other two spatial components, (E,t), and (L,θ).
- For example, for Energy-time:

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Example Problems

1. The speed of a bullet (m = 50 g) and the speed of an electron are to be measured to be the same, namely 300 m/s, with an uncertainty of 0.01%. With what fundamental accuracy could we have located the position of each, if the position is measured simultaneously with the speed in the same experiment?
2. An atom can radiate at any time after it is excited. It is found that the typical average lifetime of an excited atom is about 10^{-8} s.
   a. What is the minimum uncertainty in the frequency of the emitted photon?
   b. Most photons from sodium atoms are in two spectral lines at about $\lambda = 589$ nm. What is the fractional width of either line, $\Delta f/f$?
   c. Calculate the uncertainty ΔE in the energy of the excited state, and therefore in the energy of the emitted photon.

Wave Functions and Probability

- For light, the intensity is proportional to the square of the E and B fields in the wave picture, or to Nhf in the particle picture.
- The wave function determines the likelihood (or probability) of finding a particle at a particular position in space at a given time.

$$P(y,t)dy = \left|\Psi(y,t)\right|^2 dy$$

- The total probability of finding the electron is 1. Forcing this condition on the wave function is called normalization.

$$\int_{-\infty}^{\infty} P(y,t)dy = \int_{-\infty}^{\infty} \left|\Psi(y,t)\right|^2 dy = 1$$

Example of Particle in a Box

- A particle of mass *m* is trapped in a one-dimensional box of width *L*.
- The particle is treated as a wave.
- The box puts boundary conditions on the wave. The wave function must be zero at the walls of the box and on the outside.
- In order for the probability to vanish at the walls, we must have an integral number of half wavelengths in the box.

$$\frac{n\lambda}{2} = \ell \quad \text{or} \quad \lambda_n = \frac{2\ell}{n} \quad (n = 1, 2, 3, \ldots)$$

- The energy of the particle is $E = \text{K.E.} = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

- The possible wavelengths are quantized which yields the energy:

$$E_n = \frac{h^2}{2m}\left(\frac{n}{2\ell}\right)^2 = n^2\frac{h^2}{8m\ell^2} \quad (n = 1, 2, 3, \ldots)$$

- The possible energies of the particle are quantized. Note E cannot = 0