Exam 2 Solutions

Part I
1. D
2. C
3. B
4. C
5. B
6. C
7. B
8. C
9. A
10. C

Part II
1. See text
2. a. $1=\int \Psi^2 dx=\int A^2 x^4 dx = A^2/5 \quad so \quad A=\sqrt{5} \quad and \quad \Psi=\sqrt{5}x^2$

b. $P=\int_{0.25}^{0.75}\Psi^2 dx = 5x^5/5\Big|_{.25}^{.75}=0.236$

c. $\langle x\rangle=\int_0^1 x\Psi^2 dx = 5x^6/6\Big|_0^1 = 0.83$

3. a. n = 3; ℓ = 1; m_ℓ = 1, 0, -1; L = $\sqrt{\ell(\ell+1)}\hbar=\sqrt{2}\hbar$; L_z = 0 or ±ħ; L_x and L_y are unrestricted except that $L_x^2+L_y^2=L^2-L_z^2=2\hbar^2-\left(either \quad 0 \quad or \quad \hbar^2\right)=2\hbar^2 \quad or \quad \hbar^2$

b.

$$\Delta E\Delta t \ge \frac{\hbar}{2} \quad so \quad \Delta t \ge \frac{\hbar}{2\Delta E}=\frac{\hbar}{2\left(\frac{1}{2}mv^2\cdot 10^{-3}\right)}=0.0012s$$

$$x=v\Delta t=1.2cm$$

c.

$$E=\frac{p^2}{2m}+\frac{1}{2}kx^2 \quad so \quad \langle E\rangle=\frac{\langle p^2\rangle}{2m}+\frac{1}{2}k\langle x^2\rangle$$

$but \quad \langle p^2\rangle=(\Delta p)^2$ since $\bar{p}=0$ and $\langle x^2\rangle=(x)^2$ since $\bar{x}=0$

$$so \quad \langle E\rangle=\frac{\Delta p^2}{2m}+\frac{1}{2}k(\Delta x)^2$$

$Now, \quad \Delta p \ge \frac{\hbar}{2\Delta x}, \quad so$ minimum $\Delta p=\frac{\hbar}{2\Delta x}$

$$Then \quad E=\frac{\hbar^2}{8m(\Delta x)^2}+\frac{1}{2}k(\Delta x)^2$$

$$Finally, \quad with \quad \Delta x = x \quad \frac{dE}{dx}=-\frac{\hbar^2}{4mx^3}+kx=0 \quad gives \quad x^4=\frac{\hbar^2}{4mk}$$

$$Giving \quad E=\frac{\hbar^2}{8m\left(\frac{\hbar}{2\sqrt{mk}}\right)}+\frac{1}{2}k\left(\frac{\hbar}{2\sqrt{mk}}\right)=\frac{\hbar}{2}\sqrt{\frac{k}{m}}=\frac{hf}{2}$$