**Appendix. Formal Derivation of the Rawls and Gauthier Solutions**

Formally, the Rawlsian solution to the bargaining problem is defined as selecting the point in the bargaining set that first, respects the basic liberties of all players, and second, makes the amount of primary goods received by the least fortunate group as large as possible. In this case, we translate “primary goods” as “payoff”, and “basic liberties” as the right to cast a meaningful vote on the division of the social return. Let F be the payoff of the fortunate group and U be the payoff of the unfortunate group. Then the bargaining set is characterized by the three inequalities F +U ≤ 48, F +1.33U ≥ 38, and 1.33F +U ≥ 46 (plus the requirement that payoffs be weakly positive, which we omit for simplicity). The requirement that each player have a meaningful vote implies that any proposal which is worse than the disagreement point for half of the players cannot be selected, because it cannot command a (strict) majority in voting if each player votes for his/her preferred outcome. This translates into the constraints U ≥ 6 and F ≥ 30. The Rawlsian problem is then formulated as

Maximize min(F,U) subject to F +U ≤ 48, F + 1.33U ≥ 38, 1.33F +U ≥ 46, U ≥ 6, F ≥ 30

Only the first and last constraints turn out to be binding, and so the solution works out to be (30,18).

Formally, the Gauthier (or Kalai-Smorodinsky) solution starts by calculating the maximum amount that each group can get given that each other group gets at least as much utility as the other group gets at the disagreement point. Let Di be the utility each group gets at the disagreement point, and Mi be the maximum utility that group i can get in the bargaining set; then we have

Mi = max(i) subject toj ≥ Dj ji

Then calculate the relative concession of each group at each point in the bargaining set, defined as the ratio (Mi -i)/( Mi - Di). The solution to the bargaining problem is the one that minimizes the maximum relative concession:

Minimize max((MF -F)/( MF – DF), (MU-U)/( MU – DU)) subject to   
 F +U ≤ 48, F + 1.33U ≥ 38, 1.33F +U ≥ 46

We have DF=30 and DU=6; it is straightforward to show that MF=42 and MU=18, that being the utility obtained if all tokens are contributed and the other group gets the utility it gets at the disagreement point (j = Dj). We then have to solve

Minimize max((42-F)/12, (18-U)/12) subject to F +U ≤ 48, F + 1.33U ≥ 38, 1.33F +U ≥ 46

Only the first constraint binds, and the solution is (36, 12), at which each group’s relative concession is 50%, since (42-36)/12 = (18-12)/12 = 0.5. Any other feasible point involves a greater relative concession by one group or the other.