North-South Trade, Unemployment and Growth: What's the Role of Labor Unions?

Mathematica Appendix A: The Main Model with Labor Unions

1. The Model's Building Blocks and Steady-State Equilibrium Equations

Main features

-Unions in both the North and the South.

-Endogenous imitation in South.

-Reference wages can be positive in the South

-Decreasing returns to both imitative and innnovative R&D following Dinopoulos (1994)

To simplify the set up for Mathematics, we did the following transformations

1. Discount rate is transformed such that dr = $\rho - $ n,

2. We use Ai = $\frac{\text{ai sN } \delta}{\text{n } \gamma}$ and Aμ = $\frac{\text{a}\mu\text{ sN } \delta}{\text{n } \gamma}$.

We first clear the parameters, variables, and functions.

```
In[1]:= Clear[i, u, wL, wS, wH, cN, cS, nN];
        Clear[LABS, VN, FEIN, FEIM];
        Clear[s, ηS, α, β, μ, ai, aμ, δ, γ, n, dr, τN, τS, λ, ϵ, wSM, wNM, WELN, WELS];
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We note first the normalization

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In[4]:= cS = 1;
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This is the share of industries and the BOT condition. Note that ηS = NS/NN

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In[5]:= nN = i/(i + μ); cN = cS * (ηS * i (1 + τN))/(μ * (1 + τS));
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$$nN = \frac{i}{i+\mu};\ cN = cS * \frac{\eta S * i\,(1+\tau N)}{\mu * (1+\tau S)};$$

These are the bargained wage rates, where we define K as below to simplify the entries:

In[6]:= $K = \frac{i + \mu\,(1 + \tau S)}{i\,(1 + \tau N) + \mu};$

$wS = \frac{\beta * K * (1 - \alpha)\,wNM + (1 - \beta) * wSM}{(1 - \beta * \alpha * \lambda)};\; wL = \frac{(1 - \alpha)\,wNM + (1 - \beta) * \alpha * \lambda * wSM\left(\frac{1}{K}\right)}{(1 - \beta * \alpha * \lambda)};$

We can now see, the wage levels and the relative North-South wage:

In[8]:= $\{wL, wS, (wL / wS)\}$

Out[8]= $\left\{\frac{wNM\,(1-\alpha) + \frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu + i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}{1 - \alpha\,\beta\,\lambda}, \frac{wSM\,(1-\beta) + \frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu + i\,(1+\tau N)}}{1 - \alpha\,\beta\,\lambda}, \frac{wNM\,(1-\alpha) + \frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu + i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}{wSM\,(1-\beta) + \frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu + i\,(1+\tau N)}}\right\}$

We have the value of a Northern produced divided by NN and then followed by the FEIN condition:

In[9]:= $VN = \frac{\left(cN\left(1 - \frac{wL}{\lambda * wS * (1+\tau N)}\right)\right) + \left(cS * \eta S * \left(\frac{1}{1+\tau S} - \frac{wL}{\lambda * wS}\right)\right)}{dr + (i + \mu)\,(1 + \epsilon)};$

In[10]:= $FEIN = VN == wL * Ai * i^{\frac{1-\epsilon}{\epsilon}};$

We have the value of a Southern firms divided by NN followed by the FEIM condition:

In[11]:= $VS = \frac{\left(cN\left(\frac{1}{(1+\tau N)} - \frac{wS}{wL}\right)\right) + \left(cS * \eta S * \left(1 - \frac{wS}{wL\,(1+\tau S)}\right)\right)}{dr + i};$

In[12]:= $FEIM = VS == A\mu * wS * \mu^{\frac{1-\epsilon}{\epsilon}};$

We have the Southern and Northern labor market conditions:

In[13]:= $LABS = \left(\frac{1}{wL}\right)\left(\frac{cN}{\eta S} + \frac{cS}{(1 + \tau S)}\right)\left(\frac{\mu}{i + \mu}\right) + \left(nN * A\mu * \mu^{\frac{1}{\epsilon}} * \frac{1}{\eta S}\right) == 1 - uS;$

In[14]:= $LABN = \frac{nN}{\lambda * wS} * \left(\frac{cN}{(1 + \tau N)} + \frac{cS * \eta S}{1}\right) + \left(\left(Ai * i^{\frac{1}{\epsilon}}\right)\right) == (1 - sN - uN);$

We can solve the labor market conditions to obtain expressions for uS and uN

In[15]:= **{Solve[LABS, uS], Solve[LABN, uN]}**

Out[15]=

$$\left\{\left\{\left\{\mathrm{uS}\to 1-\frac{\mathrm{A}\mu\,\mathrm{i}\,\mu^{\frac{1}{\epsilon}}}{\eta\mathrm{S}\,(\mathrm{i}+\mu)}-\frac{(1-\alpha\,\beta\,\lambda)\,\mu\,\left(\frac{1}{1+\tau\mathrm{S}}+\frac{\mathrm{i}\,(1+\tau\mathrm{N})}{\mu\,(1+\tau\mathrm{S})}\right)}{(\mathrm{i}+\mu)\,\left(\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}\right)}\right\}\right\},\right.$$

$$\left.\left\{\left\{\mathrm{uN}\to 1-\mathrm{Ai}\,\mathrm{i}^{\frac{1}{\epsilon}}-\mathrm{sN}-\frac{\mathrm{i}\,(1-\alpha\,\beta\,\lambda)\,\left(\eta\mathrm{S}+\frac{\mathrm{i}\,\eta\mathrm{S}}{\mu\,(1+\tau\mathrm{S})}\right)}{\lambda\,(\mathrm{i}+\mu)\,\left(\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}\right)}\right\}\right\}\right\}$$

We can now see the long forms of the main steady-state equations FEIN, FEIM and RP explicitly:

In[16]:= **FEIN**

Out[16]=

$$\frac{\eta\mathrm{S}\,\left(\frac{1}{1+\tau\mathrm{S}}-\frac{\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}}{\lambda\,\left(\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}\right)}\right)+\frac{\mathrm{i}\,\eta\mathrm{S}\,(1+\tau\mathrm{N})\,\left(1-\frac{\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}}{\lambda\,(1+\tau\mathrm{N})\,\left(\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}\right)}\right)}{\mu\,(1+\tau\mathrm{S})}}{\mathrm{dr}+(1+\epsilon)\,(\mathrm{i}+\mu)}==\frac{\mathrm{Ai}\,\mathrm{i}^{\frac{1-\epsilon}{\epsilon}}\,\left(\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}\right)}{1-\alpha\,\beta\,\lambda}$$

In[17]:= **FEIM**

Out[17]=

$$\frac{\frac{\mathrm{i}\,\eta\mathrm{S}\,(1+\tau\mathrm{N})\,\left(\frac{1}{1+\tau\mathrm{N}}-\frac{\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}}{\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}}\right)}{\mu\,(1+\tau\mathrm{S})}+\eta\mathrm{S}\,\left(1-\frac{\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}}{(1+\tau\mathrm{S})\,\left(\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}\right)}\right)}{\mathrm{dr}+\mathrm{i}}==\frac{\mathrm{A}\mu\,\mu^{\frac{1-\epsilon}{\epsilon}}\,\left(\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}\right)}{1-\alpha\,\beta\,\lambda}$$

In[18]:= **RP = FEIN / FEIM**

Out[18]=

$$\frac{\frac{\eta\mathrm{S}\,\left(\frac{1}{1+\tau\mathrm{S}}-\frac{\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}}{\lambda\,\left(\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}\right)}\right)+\frac{\mathrm{i}\,\eta\mathrm{S}\,(1+\tau\mathrm{N})\,\left(1-\frac{\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}}{\lambda\,(1+\tau\mathrm{N})\,\left(\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}\right)}\right)}{\mu\,(1+\tau\mathrm{S})}}{\mathrm{dr}+(1+\epsilon)\,(\mathrm{i}+\mu)}==\frac{\mathrm{Ai}\,\mathrm{i}^{\frac{1-\epsilon}{\epsilon}}\,\left(\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}\right)}{1-\alpha\,\beta\,\lambda}}{\frac{\frac{\mathrm{i}\,\eta\mathrm{S}\,(1+\tau\mathrm{N})\,\left(\frac{1}{1+\tau\mathrm{N}}-\frac{\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}}{\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}}\right)}{\mu\,(1+\tau\mathrm{S})}+\eta\mathrm{S}\,\left(1-\frac{\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}}{(1+\tau\mathrm{S})\,\left(\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}\right)}\right)}{\mathrm{dr}+\mathrm{i}}==\frac{\mathrm{A}\mu\,\mu^{\frac{1-\epsilon}{\epsilon}}\,\left(\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}\right)}{1-\alpha\,\beta\,\lambda}}$$

In[19]:= $$\text{WELN} = \frac{1}{\text{dr} + 0.01}\left(\frac{\text{i} * \text{Log}[\lambda]}{\text{dr} + 0.01} + \frac{\mu}{\text{i} + \mu}\,\text{Log}\left[\frac{\text{cN}}{\text{wL}}\right] + \frac{\text{i}}{\text{i} + \mu} * \text{Log}\left[\frac{\text{cN}}{\lambda * \text{wS} * (1 + \tau\text{N})}\right]\right)$$

Out[19]= $$\frac{\frac{\text{i}\,\text{Log}[\lambda]}{0.01+\text{dr}} + \frac{\mu\,\text{Log}\left[\frac{\text{i}\,\eta\text{S}\,(1-\alpha\,\beta\,\lambda)\,(1+\tau\text{N})}{\mu\,(1+\tau\text{S})\left(\text{wNM}\,(1-\alpha)+\frac{\text{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\text{i}\,(1+\tau\text{N}))}{\text{i}+\mu\,(1+\tau\text{S})}\right)}\right]}{\text{i}+\mu} + \frac{\text{i}\,\text{Log}\left[\frac{\text{i}\,\eta\text{S}\,(1-\alpha\,\beta\,\lambda)}{\lambda\,\mu\,(1+\tau\text{S})\left(\text{wSM}\,(1-\beta)+\frac{\text{wNM}\,(1-\alpha)\,\beta\,(\text{i}+\mu\,(1+\tau\text{S}))}{\mu+\text{i}\,(1+\tau\text{N})}\right)}\right]}{\text{i}+\mu}}{0.01+\text{dr}}$$

In[20]:= $$\text{WELS} = \frac{1}{\text{dr} + 0.01}\left(\frac{\text{i} * \text{Log}[\lambda]}{\text{dr} + 0.01} + \frac{\mu}{\text{i} + \mu}\,\text{Log}\left[\frac{\text{cS}}{\text{wL}\,(1 + \tau\text{S})}\right] + \frac{\text{i}}{\text{i} + \mu} * \text{Log}\left[\frac{\text{cS}}{\lambda * \text{wS}}\right]\right)$$

Out[20]= $$\frac{\frac{\text{i}\,\text{Log}[\lambda]}{0.01+\text{dr}} + \frac{\mu\,\text{Log}\left[\frac{1-\alpha\,\beta\,\lambda}{(1+\tau\text{S})\left(\text{wNM}\,(1-\alpha)+\frac{\text{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\text{i}\,(1+\tau\text{N}))}{\text{i}+\mu\,(1+\tau\text{S})}\right)}\right]}{\text{i}+\mu} + \frac{\text{i}\,\text{Log}\left[\frac{1-\alpha\,\beta\,\lambda}{\lambda\left(\text{wSM}\,(1-\beta)+\frac{\text{wNM}\,(1-\alpha)\,\beta\,(\text{i}+\mu\,(1+\tau\text{S}))}{\mu+\text{i}\,(1+\tau\text{N})}\right)}\right]}{\text{i}+\mu}}{0.01+\text{dr}}$$

2. Graphical Representation of the Model in (μ,ι) Space: Figure 1

Let’s now plot the FEIN and RP curves in (μ,i) space. We need to explicitly enter the above expressions when using ContourPlot.

In[21]:= **Manipulate[**

$$\text{ContourPlot}\Big[\Big\{\left(\eta S\left(\frac{1}{1+\tau S}-\frac{wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}{\lambda\left(wSM\,(1-\beta)+\frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}\right)}\right)+\frac{i\,\eta S\,(1+\tau N)\left(1-\frac{wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}{\lambda\,(1+\tau N)\left(wSM\,(1-\beta)+\frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}\right)}\right)}{\mu\,(1+\tau S)}\right)\Big/$$

$$(dr+(i+\mu)\,(1+\epsilon))\Big/\,1/(dr+i)\left(\frac{i\,\eta S\,(1+\tau N)\left(\frac{1}{1+\tau N}-\frac{wSM\,(1-\beta)+\frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}}{wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}\right)}{\mu\,(1+\tau S)}+\right.$$

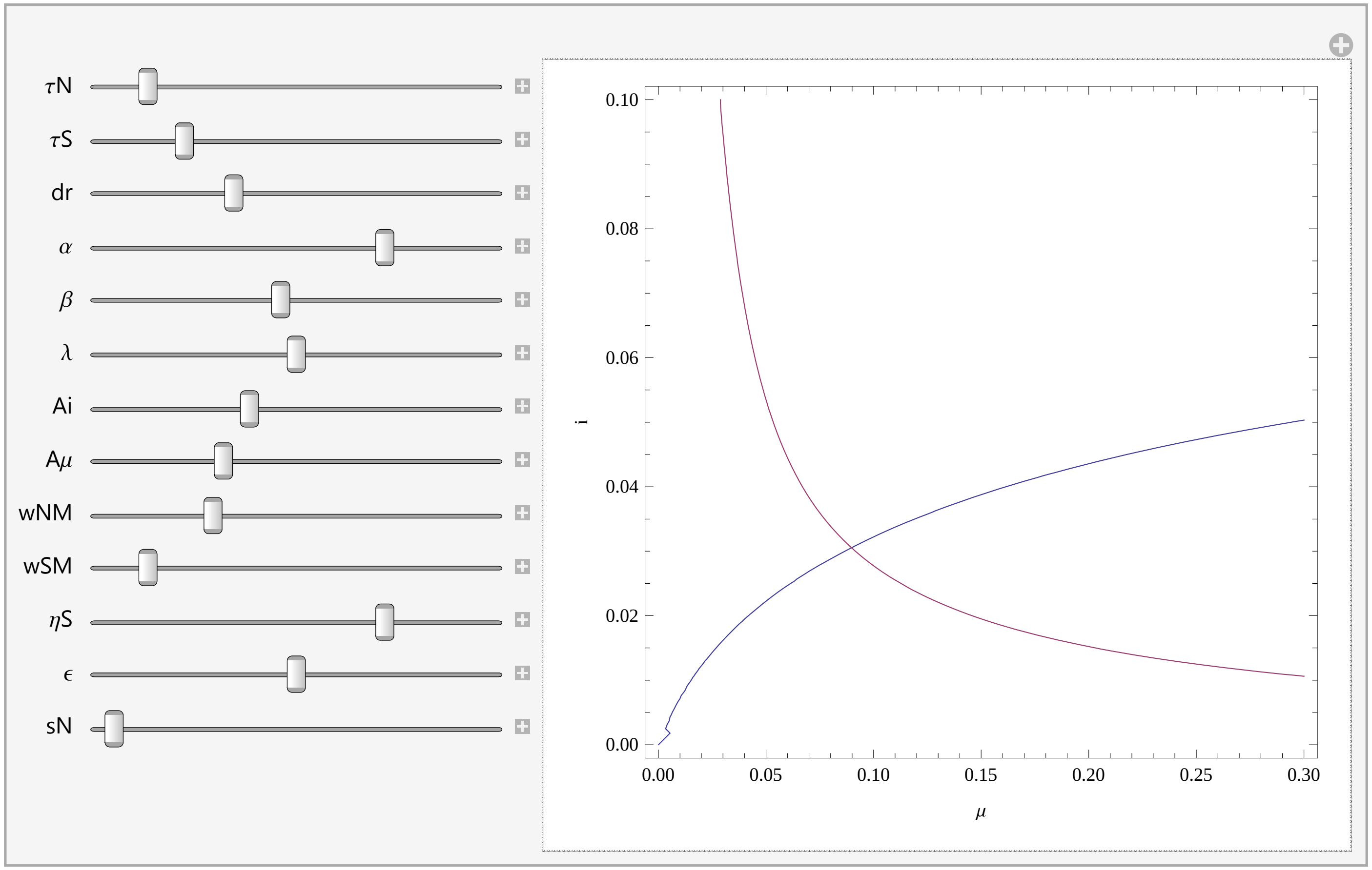
$$\left.\eta S\left(1-\frac{wSM\,(1-\beta)+\frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}}{(1+\tau S)\left(wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}\right)}\right)\right)==\frac{\frac{Ai\,i^{\frac{1-\epsilon}{\epsilon}}\left(wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}\right)}{1-\alpha\,\beta\,\lambda}}{\frac{A\mu\,\mu^{\frac{1-\epsilon}{\epsilon}}\left(wSM\,(1-\beta)+\frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}\right)}{1-\alpha\,\beta\,\lambda}},\ \frac{1}{dr+(1+\epsilon)\,(i+\mu)}$$

$$\left(\eta S\left(\frac{1}{1+\tau S}-\frac{wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}{\lambda\left(wSM\,(1-\beta)+\frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}\right)}\right)+\frac{i\,\eta S\,(1+\tau N)\left(1-\frac{wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}{\lambda\,(1+\tau N)\left(wSM\,(1-\beta)+\frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}\right)}\right)}{\mu\,(1+\tau S)}\right)==$$

$$\frac{Ai\,i^{\frac{1-\epsilon}{\epsilon}}\left(wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}\right)}{1-\alpha\,\beta\,\lambda}\Big\},\ \{\mu,0,0.3\},\ \{i,0,0.1\},\ \text{FrameLabel}\to\{\text{"}\mu\text{"},\text{"i"}\}\Big],$$

$\{\{\tau N, 0.1\}, 0, 1\}, \{\{\tau S, 0.2\}, 0, 1\}, \{\{dr, 0.06\}, 0.02, 0.14\}, \{\{\alpha, 0.76\}, 0.1, 1\}, \{\{\beta, 0.51\}, 0.1, 1\},$
$\{\{\lambda, 2\}, 1, 3\}, \{\{Ai, 75\}, 1, 200\}, \{\{A\mu, 335\}, 50, 1000\}, \{\{wNM, 0.55\}, 0.0, 2\},$
$\{\{wSM, 0.2\}, 0.0, 2\}, \{\{\eta S, 3.93\}, 1, 5\}, \{\{\epsilon, 0.5\}, 0, 1\}, \{\{sN, 0.01\}, 0, 1\}]$

Out[21]=



The above is Figure 1 of the paper. RP is upward sloping and FEIN is downward sloping. All shifts can be seen above by changing the parameters. Note that when wSM > 0, we also have shifts in the RP curve due to tariff changes. More specifically:

-A lower τN shifts RP to the left

-A lower τS shift RP to the right.

3. Numerical Representation of the Model

In[22]:=
```
Clear[FEIN, FEIM, wLF, wSF, cNF, WELNF, WELSF, R1F, R2F, R3F, R4F, R5F, R6F, R7F, uNF, uSF, i, μ, τN, τS,
  dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, WELN, WELS, R1, R2, R3, R4, R5, R6, R7]; cS = 1;
```

We enter the main steady-state equations of the model as functions. We also enter the restrictions that should hold at the steady-state for an interior equilibrium. All restrictions as entered below must be positive in equilibrium.

In[23]:= FEIN[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_] :=

$$\frac{1}{dr + (1+\epsilon)\,(i+\mu)}\left(\eta S\left(\frac{1}{1+\tau S} - \frac{wNM\,(1-\alpha) + \frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}{\lambda\left(wSM\,(1-\beta) + \frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}\right)}\right) + \frac{i\,\eta S\,(1+\tau N)\left(1 - \frac{wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}{\lambda\,(1+\tau N)\left(wSM\,(1-\beta)+\frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}\right)}\right)}{\mu\,(1+\tau S)}\right) == \frac{Ai\, i^{\frac{1-\epsilon}{\epsilon}}\left(wNM\,(1-\alpha) + \frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}\right)}{1-\alpha\,\beta\,\lambda};$$

FEIM[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_] :=

$$\frac{1}{dr + i}\left(\frac{i\,\eta S\,(1+\tau N)\left(\frac{1}{1+\tau N} - \frac{wSM\,(1-\beta)+\frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}}{wNM\,(1-\alpha)+\frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}\right)}{\mu\,(1+\tau S)} + \eta S\left(1 - \frac{wSM\,(1-\beta) + \frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}}{(1+\tau S)\left(wNM\,(1-\alpha) + \frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}\right)}\right)\right) ==$$

$$\frac{A\mu\,\mu^{\frac{1-\epsilon}{\epsilon}}\left(wSM\,(1-\beta) + \frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}\right)}{1-\alpha\,\beta\,\lambda};$$

In[25]:= uNF[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, uN_] :=

$$uN == 1 - Ai\, i^{\frac{1}{\epsilon}} - sN - \frac{i\,(1-\alpha\,\beta\,\lambda)\left(\eta S + \frac{i\,\eta S}{\mu\,(1+\tau S)}\right)}{\lambda\,(i+\mu)\left(wSM\,(1-\beta) + \frac{wNM\,(1-\alpha)\,\beta\,(i+\mu\,(1+\tau S))}{\mu+i\,(1+\tau N)}\right)};$$

uSF[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, uS_] :=

$$uS == 1 - \frac{A\mu\, i\,\mu^{\frac{1}{\epsilon}}}{\eta S\,(i+\mu)} - \frac{(1-\alpha\,\beta\,\lambda)\,\mu\left(\frac{1}{1+\tau S} + \frac{i\,(1+\tau N)}{\mu\,(1+\tau S)}\right)}{(i+\mu)\left(wNM\,(1-\alpha) + \frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}\right)};$$

In[27]:= wLF[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_] :=

$$wL == \frac{wNM\,(1-\alpha) + \frac{wSM\,\alpha\,(1-\beta)\,\lambda\,(\mu+i\,(1+\tau N))}{i+\mu\,(1+\tau S)}}{1-\alpha\,\beta\,\lambda};$$

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wSF[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_] :=
  wS == (wSM (1 - β) + (wNM (1-α) β (i+μ (1+τS)))/(μ+i (1+τN)))/(1 - α β λ);
wHF[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_] :=
  wH == (ϵ * (1 - α) * wNM * Ai * (i^(1/ϵ)))/((1 - (α * β * λ)) * sN);
cNF[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_] :=
  cN == (i * ηS * (1 + τN))/(μ * (1 + τS));


R1F[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_,
   wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_, R1_] := R1 == (λ * wS)/(1 + τS) - wL;
R2F[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_, R2_] :=
  R2 == wL - (1 + τN) * wS;

R3F[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_, R3_] :=
  R3 == (λ * wS (i * (1 + τN) + μ))/(i + μ (1 + τS)) - wNM;

R4F[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_, R4_] :=
  R4 == (wL (i + μ (1 + τS)))/(μ + i (1 + τN)) - wSM;

R5F[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_, Aμ_,
   wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_, R5_] := R5 == 1 - (α * β * λ) ;

R6F[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_,
   Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_, R6_] := R6 == wL - wNM;

R7F[i_, μ_, τN_, τS_, dr_, α_, β_, λ_, Ai_,
   Aμ_, wNM_, wSM_, ηS_, ϵ_, sN_, wL_, wS_, wH_, cN_, R7_] := R7 == wS - wSM;
```

In[38]:= $\mathtt{WELNF}[\mathtt{i\_}, \mu\_, \tau\mathtt{N\_}, \tau\mathtt{S\_}, \mathtt{dr\_}, \alpha\_, \beta\_, \lambda\_, \mathtt{Ai\_}, \mathtt{A}\mu\_, \mathtt{wNM\_}, \mathtt{wSM\_}, \eta\mathtt{S\_}, \epsilon\_, \mathtt{sN\_}] :=$

$$\frac{1}{0.01\grave{} + \mathrm{dr}} \left(\frac{\mathrm{i}\,\mathrm{Log}[\lambda]}{0.01\grave{} + \mathrm{dr}} + \frac{\mu\,\mathrm{Log}\left[\frac{\mathrm{i}\,\eta\mathrm{S}\,(1-\alpha\,\beta\,\lambda)\,(1+\tau\mathrm{N})}{\mu\,(1+\tau\mathrm{S})\,\left(\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}\right)}\right]}{\mathrm{i}+\mu} + \frac{\mathrm{i}\,\mathrm{Log}\left[\frac{\mathrm{i}\,\eta\mathrm{S}\,(1-\alpha\,\beta\,\lambda)}{\lambda\,\mu\,(1+\tau\mathrm{S})\,\left(\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}\right)}\right]}{\mathrm{i}+\mu} \right);$$

$\mathtt{WELSF}[\mathtt{i\_}, \mu\_, \tau\mathtt{N\_}, \tau\mathtt{S\_}, \mathtt{dr\_}, \alpha\_, \beta\_, \lambda\_, \mathtt{Ai\_}, \mathtt{A}\mu\_, \mathtt{wNM\_}, \mathtt{wSM\_}, \eta\mathtt{S\_}, \epsilon\_, \mathtt{sN\_}] :=$

$$\frac{1}{0.01\grave{} + \mathrm{dr}} \left(\frac{\mathrm{i}\,\mathrm{Log}[\lambda]}{0.01\grave{} + \mathrm{dr}} + \frac{\mu\,\mathrm{Log}\left[\frac{1-\alpha\,\beta\,\lambda}{(1+\tau\mathrm{S})\,\left(\mathrm{wNM}\,(1-\alpha)+\frac{\mathrm{wSM}\,\alpha\,(1-\beta)\,\lambda\,(\mu+\mathrm{i}\,(1+\tau\mathrm{N}))}{\mathrm{i}+\mu\,(1+\tau\mathrm{S})}\right)}\right]}{\mathrm{i}+\mu} + \frac{\mathrm{i}\,\mathrm{Log}\left[\frac{1-\alpha\,\beta\,\lambda}{\lambda\,\left(\mathrm{wSM}\,(1-\beta)+\frac{\mathrm{wNM}\,(1-\alpha)\,\beta\,(\mathrm{i}+\mu\,(1+\tau\mathrm{S}))}{\mu+\mathrm{i}\,(1+\tau\mathrm{N})}\right)}\right]}{\mathrm{i}+\mu} \right)$$

We also note the following restrictions that must hold.

-R8: wL > wLCOMP,

-R9: wS > wSCOMP.

We verify that these restrictions also hold by comparing the values calculated in this file with the values calculated in the Competitive Equilibrium file.

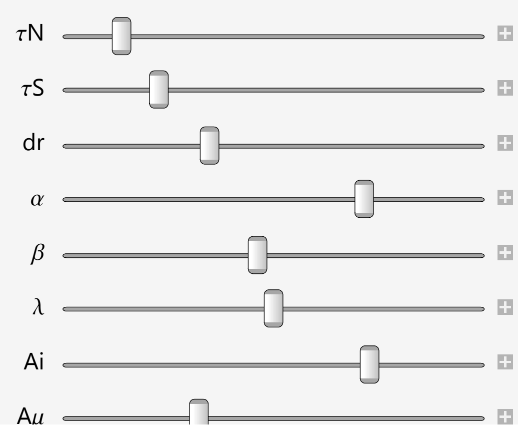
3.1 Numerical Steady-State Equilibrium with wsM>0

We first consider the benchmark case with wSM >0. This corresponds to Table 1 in the paper.

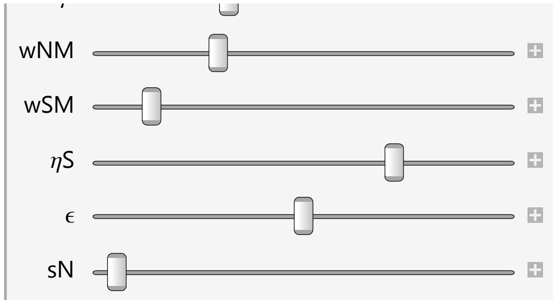
In[40]:=
```
Manipulate[NSolve[
  {FEIN[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ],
   FEIM[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ],
   uNF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, uN],
   uSF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, uS],
   wLF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN],
   wSF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN],
   wHF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN],
   cNF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN],

   R1F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R1],
   R2F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R2],
   R3F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R3],
   R4F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R4],
   R5F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R5],
   R6F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R6],
   R7F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R7] },

  {i, μ, uN, uS, wL, wS, wH, cN, R1, R2, R3, R4, R5, R6, R7}],
 {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14}, {{α, 0.76}, 0.1, 1},
 {{β, 0.51}, 0.1, 1}, {{λ, 2}, 1, 3}, {{Ai, 75}, 1, 100}, {{Aμ, 335}, 50, 1000},
 {{wNM, 0.55}, 0.0, 2}, {{wSM, 0.2}, 0.0, 2}, {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1}, {{sN, 0.01}, 0, 1}
]
```




```
{{i → 0.0308832, μ → -0.0315771, uN → -35.7946,
  uS → 12.7533, wL → 0.360823, wS → -0.440675, wH → 2.10017,
  cN → -3.52334, R1 → -1.09528, R2 → 0.845565, R3 → -0.248917,
  R4 → -1.25623, R5 → 0.2248, R6 → -0.189177, R7 → -0.640675},
 {i → -0.0835094, μ → 0.0593653, uN → 2.59932,
  uS → -0.517971, wL → 2.34191, wS → 0.54903, wH → 15.356,
  cN → -5.06765, R1 → -1.42686, R2 → 1.73798, R3 → 2.35778,
  R4 → 0.684373, R5 → 0.2248, R6 → 1.79191, R7 → 0.34903},
 {i → -0.0589913 - 0.0389155 ⅈ, μ → 0.0596519 + 0.0345617 ⅈ,
  uN → 12.9528 + 8.46133 ⅈ, uS → -9.57849 - 0.582711 ⅈ,
  wL → 0.237749 - 0.362918 ⅈ, wS → 0.162749 + 0.283732 ⅈ,
```



Out[40]=

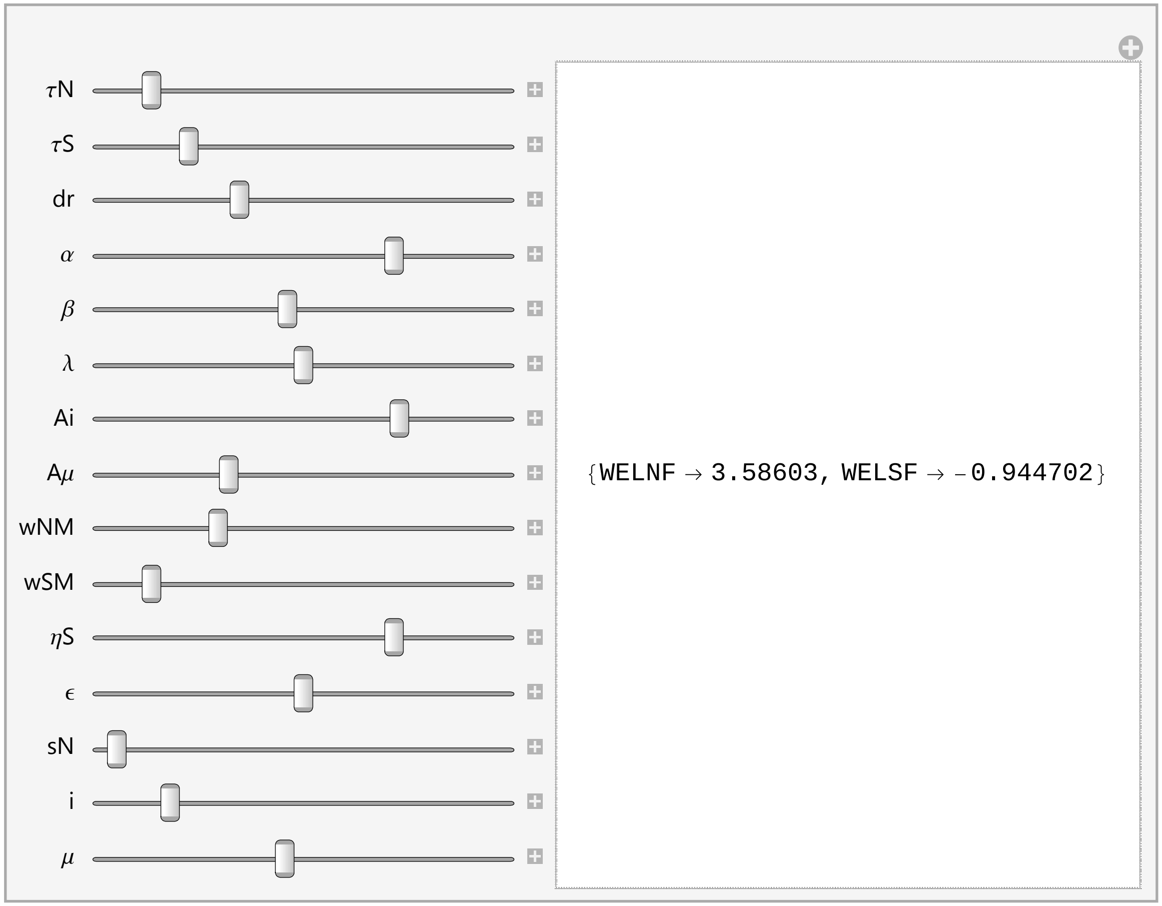
```
wH → 4.32808 + 10.1099 i, cN → -3.68668 - 0.214155 i,
R1 → 0.0334992 + 0.835804 i, R2 → 0.0587251 - 0.675023 i,
R3 → -0.410857 - 0.477524 i, R4 → -0.0730416 + 0.556337 i,
R5 → 0.2248, R6 → -0.312251 - 0.362918 i,
R7 → -0.0372512 + 0.283732 i}, {i → -0.0589913 + 0.0389155 i,
μ → 0.0596519 - 0.0345617 i, uN → 12.9528 - 8.46133 i,
uS → -9.57849 + 0.582711 i, wL → 0.237749 + 0.362918 i,
wS → 0.162749 - 0.283732 i, wH → 4.32808 - 10.1099 i,
cN → -3.68668 + 0.214155 i, R1 → 0.0334992 - 0.835804 i,
R2 → 0.0587251 + 0.675023 i, R3 → -0.410857 + 0.477524 i,
R4 → -0.0730416 - 0.556337 i, R5 → 0.2248,
R6 → -0.312251 + 0.362918 i, R7 → -0.0372512 - 0.283732 i},
{i → -0.0398201 + 0.068831 i, μ → -0.0482097 - 0.0714261 i,
uN → 2.11204 + 2.48616 i, uS → -0.0932766 - 0.46572 i,
wL → 1.18844 - 0.133003 i, wS → 0.750587 + 0.0696027 i,
wH → -6.94071 - 12.0705 i, cN → -1.45374 - 2.98961 i,
R1 → 0.0625409 + 0.249007 i, R2 → 0.362792 - 0.209566 i,
R3 → 0.840049 - 0.175004 i, R4 → 1.07958 + 0.136476 i, R5 → 0.2248,
R6 → 0.638437 - 0.133003 i, R7 → 0.550587 + 0.0696027 i},
{i → -0.0398201 - 0.068831 i, μ → -0.0482097 + 0.0714261 i,
uN → 2.11204 - 2.48616 i, uS → -0.0932766 + 0.46572 i,
wL → 1.18844 + 0.133003 i, wS → 0.750587 - 0.0696027 i,
wH → -6.94071 + 12.0705 i, cN → -1.45374 + 2.98961 i,
R1 → 0.0625409 - 0.249007 i, R2 → 0.362792 + 0.209566 i,
R3 → 0.840049 + 0.175004 i, R4 → 1.07958 - 0.136476 i, R5 → 0.2248,
R6 → 0.638437 + 0.133003 i, R7 → 0.550587 - 0.0696027 i},
{i → 0.0305584, μ → 0.089645, uN → 0.0885762, uS → 0.100746,
wL → 1.17847, wS → 0.771545, wH → 2.05622, cN → 1.22803,
R1 → 0.107433, R2 → 0.329775, R3 → 0.82694,
R4 → 1.12068, R5 → 0.2248, R6 → 0.628474, R7 → 0.571545}}
```

We also conduct a welfare analyis for North and South. We use the benchmark outcomes for i and μ from above. Note that whenever a parameter is changed below, the benchmark values for i and μ also have to be changed using the above manipulate function.

In[41]:=

```
Manipulate[
 { "WELNF" -> WELNF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN],
  "WELSF" -> WELSF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN]},
 {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14}, {{α, 0.76}, 0.1, 1},
 {{β, 0.51}, 0.1, 1}, {{λ, 2}, 1, 3}, {{Ai, 75}, 1, 100}, {{Aμ, 335}, 50, 1000},
 {{wNM, 0.55}, 0.0, 2}, {{wSM, 0.2}, 0.0, 2}, {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1},
 {{sN, 0.01}, 0, 1}, {{i, 0.03055839531381263`}, 0, 0.2}, {{μ, 0.08964500229311638`}, 0, 0.2}
]
```

Out[41]=



In[42]:=

3.2. Numerical Steady-State Equilibrium with wsM=0

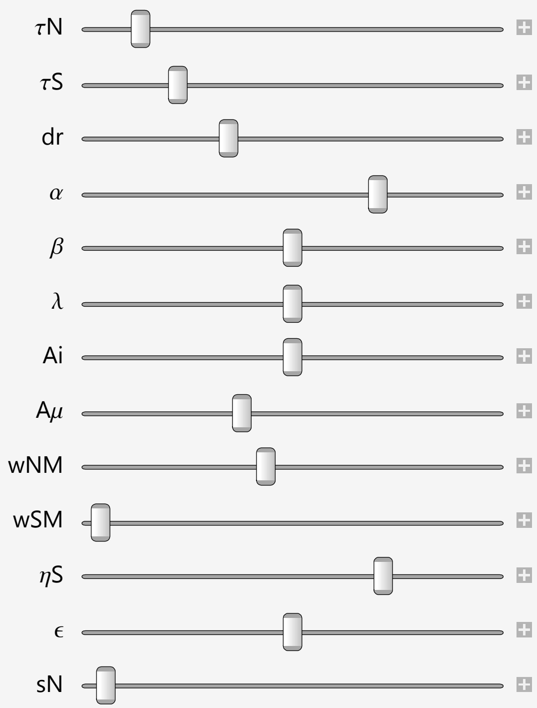
We now consider the benchmark case with wSM =0. This corresponds to Table 1A in Appendix R2 of the paper.

In[43]:=
```
Manipulate[NSolve[
  {FEIN[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ],
   FEIM[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ],
   uNF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, uN],
   uSF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, uS],
   wLF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN],
   wSF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN],
   wHF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN],
   cNF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN],

   R1F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R1],
   R2F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R2],
   R3F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R3],
   R4F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R4],
   R5F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R5],
   R6F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R6],
   R7F[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN, wL, wS, wH, cN, R7] },

  {i, μ, uN, uS, wL, wS, wH, cN, R1, R2, R3, R4, R5, R6, R7}],
 {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14}, {{α, 0.75}, 0.1, 1},
 {{β, 0.55}, 0.1, 1}, {{λ, 2}, 1, 3}, {{Ai, 50}, 1, 100}, {{Aμ, 400}, 50, 1000},
 {{wNM, 0.85}, 0.0, 2}, {{wSM, 0.0}, 0.0, 2}, {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1}, {{sN, 0.01}, 0, 1}
]
```

Out[43]=




```
{{i → -0.0959242, μ → 0.0718756, uN → 5.10738,
  uS → -2.05735, wL → 1.21429, wS → 0.192043, wH → 27.933,
  cN → -4.80785, R1 → -0.894213, R2 → 1.00304, R3 → 0.485714,
  R4 → 0.34917, R5 → 0.175, R6 → 0.364286, R7 → 0.192043},
 {i → -0.0366174 + 0.0648614 ⅈ, μ → -0.049796 - 0.0687786 ⅈ,
  uN → 2.21533 + 2.1994 ⅈ, uS → -0.159456 - 0.394424 ⅈ,
  wL → 1.21429, wS → 0.710234 + 0.151291 ⅈ, wH → -8.70087 - 14.42 ⅈ,
  cN → -1.3179 - 2.87212 ⅈ, R1 → -0.0305616 + 0.252151 ⅈ,
  R2 → 0.433028 - 0.16642 ⅈ, R3 → 0.485714, R4 → 1.29134 + 0.275074 ⅈ,
  R5 → 0.175, R6 → 0.364286, R7 → 0.710234 + 0.151291 ⅈ},
 {i → -0.0366174 - 0.0648614 ⅈ, μ → -0.049796 + 0.0687786 ⅈ,
  uN → 2.21533 - 2.1994 ⅈ, uS → -0.159456 + 0.394424 ⅈ,
  wL → 1.21429, wS → 0.710234 - 0.151291 ⅈ, wH → -8.70087 + 14.42 ⅈ,
  cN → -1.3179 + 2.87212 ⅈ, R1 → -0.0305616 - 0.252151 ⅈ,
  R2 → 0.433028 + 0.16642 ⅈ, R3 → 0.485714, R4 → 1.29134 - 0.275074 ⅈ,
  R5 → 0.175, R6 → 0.364286, R7 → 0.710234 - 0.151291 ⅈ},
 {i → 0.029159, μ → 0.0851755, uN → 0.0867054, uS → 0.107905,
  wL → 1.21429, wS → 0.74828, wH → 2.58111, cN → 1.23328,
  R1 → 0.0328475, R2 → 0.391178, R3 → 0.485714,
  R4 → 1.36051, R5 → 0.175, R6 → 0.364286, R7 → 0.74828}}
```

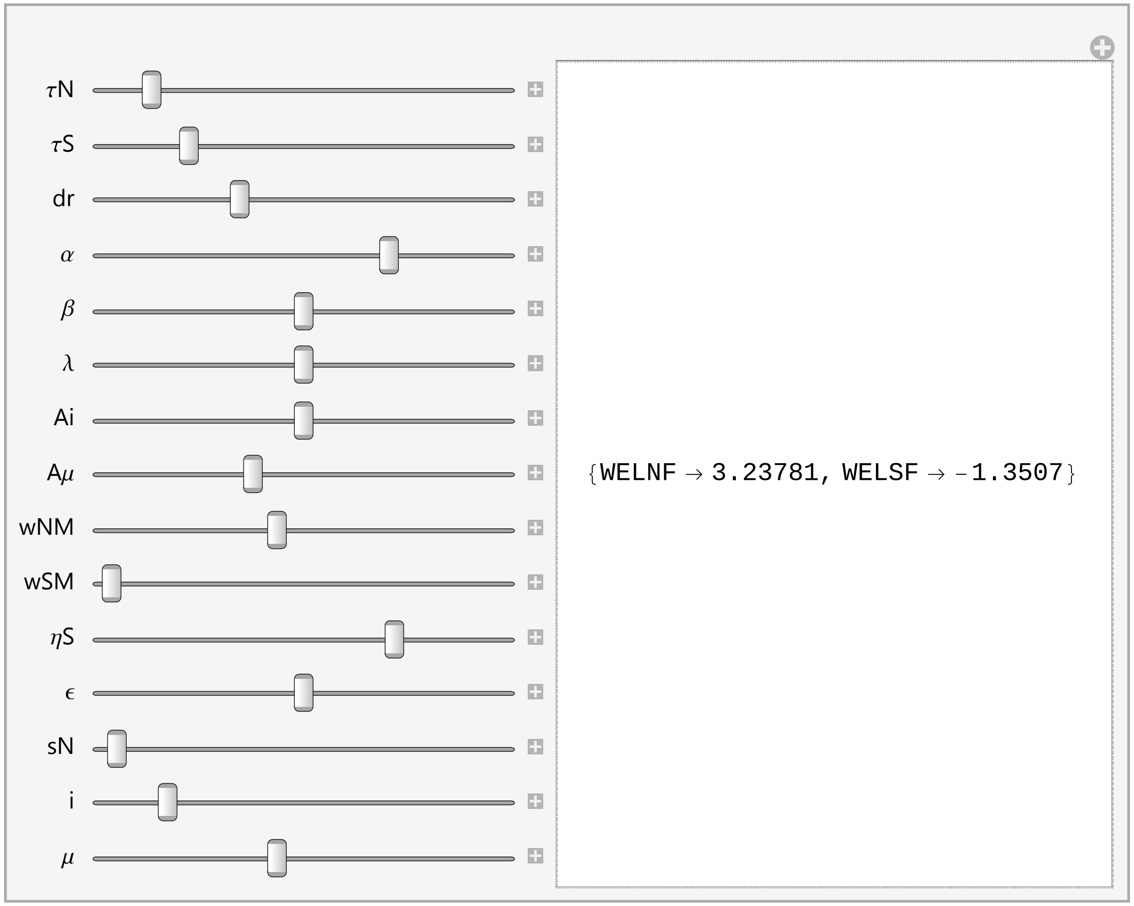
We also conduct a welfare analyis for North and South. We use the benchmark outcomes for i and μ from above. Note that whenever a parameter is changed below, the benchmark values for i and μ also have to be changed using the above manipulate function.

In[44]:=
```
Manipulate[
 { "WELNF" -> WELNF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN],
  "WELSF" -> WELSF[i, μ, τN, τS, dr, α, β, λ, Ai, Aμ, wNM, wSM, ηS, ϵ, sN]},

 {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14}, {{α, 0.75}, 0.1, 1},
 {{β, 0.55}, 0.1, 1}, {{λ, 2}, 1, 3}, {{Ai, 50}, 1, 100}, {{Aμ, 400}, 50, 1000}, {{wNM, 0.85}, 0.0, 2},
 {{wSM, 0.0}, 0.0, 2}, {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1}, {{sN, 0.01}, 0, 1},

 {{i, 0.029159034928924136`}, 0, 0.2}, {{μ, 0.08517549762900549`}, 0, 0.2}
]
```

Out[44]=



North-South Trade, Unemployment and Growth: What's the Role of Labor Unions?

Mathematica Appendix B: The Competitive Labor Market Model

1. The Steady-Steady Equations

We first clear the parameters, variables, and functions.

In[1]:=
```
Clear[wLCF, wSCF, LABS, LABN, cNF, WELN, WELS,
 wLC, wSC, i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN, cN, R1, R2]
```

We enter the wage and labor market equations from the Appendix and cNF

In[2]:=
```
wLCF[i_, μ_, τN_, τS_, dr_, λ_, Ai_, Aμ_, ηS_, ϵ_, wSC_, wLC_] :=
```

$$wLC == \frac{(\lambda - 1)\ \eta S\ (i\ (1 + \tau N) + \mu)}{(1 + \tau S)\ \left(\lambda * \mu\ (dr + (1 + \epsilon)\ (i + \mu))\ Ai * i^{\frac{1-\epsilon}{\epsilon}} + \mu^{\frac{1}{\epsilon}}\ (dr + i)\ A\mu\right)};$$

```
wSCF[i_, μ_, τN_, τS_, dr_, λ_, Ai_, Aμ_, ηS_, ϵ_, wSC_, wLC_] :=
```

$$wSC == \frac{(\lambda - 1)\ \eta S\ (i + \mu\ (1 + \tau S))}{\lambda\ (1 + \tau S)\ \left(\mu\ (dr + (1 + \epsilon)\ (i + \mu)) * Ai * i^{\frac{1-\epsilon}{\epsilon}} + \mu^{\frac{1}{\epsilon}}\ (dr + i)\ A\mu\right)};$$

```
LABN[i_, μ_, τN_, τS_, dr_, λ_, Ai_, Aμ_, ηS_, ϵ_, sN_] :=
```

$$\frac{i\ \left(dr + (1 + \epsilon) * (i + \mu) * Ai * i^{\frac{1-\epsilon}{\epsilon}} + (dr + i) * A\mu * \mu^{\frac{1-\epsilon}{\epsilon}}\right)}{(\lambda - 1)\ (i + \mu)} + Ai * i^{\frac{1}{\epsilon}} - (1 - sN);$$

```
LABS[i_, μ_, τN_, τS_, dr_, λ_, Ai_, Aμ_, ηS_, ϵ_, sN_] :=
```

$$\frac{\left(\lambda * \mu * (dr + (1 + \epsilon) * (i + \mu)) * Ai * i^{\frac{1-\epsilon}{\epsilon}}\right) + \left((dr + i) * A\mu * \mu^{\frac{1}{\epsilon}}\right)}{(\lambda - 1)\ (i + \mu)} + \frac{i * A\mu * \mu^{\frac{1}{\epsilon}}}{i + \mu} - \eta S;$$

```
cNF[i_, μ_, τN_, τS_, dr_, λ_, Ai_, Aμ_, ηS_, ϵ_, sN_, wLC_, wSC_, cN_] :=
```

$$cN == \frac{i * \eta S * (1 + \tau N)}{\mu * (1 + \tau S)};$$

We enter the expressions that enter the welfare functions with an added “W” to differentiate them from above. Hence, cN becomes cNW, wL becomes wLW, and wS becomes wSW.

In[7]:=
$$cS = 1;\ cNW = \frac{i * \eta S * (1 + \tau N)}{\mu * (1 + \tau S)};$$
$$wLW = ((\lambda - 1)\ \eta S\ (i\ (1 + \tau N) + \mu)) \Big/ \left((1 + \tau S)\ \left(\lambda * \mu\ (dr + (1 + \epsilon)\ (i + \mu))\ Ai * i^{\frac{1-\epsilon}{\epsilon}} + \mu^{\frac{1}{\epsilon}}\ (dr + i)\ A\mu\right)\right);$$
$$wSW = ((\lambda - 1)\ \eta S\ (i + \mu\ (1 + \tau S))) \Big/ \left(\lambda\ (1 + \tau S)\ \left(\mu\ (dr + (1 + \epsilon)\ (i + \mu))\ Ai * i^{\frac{1-\epsilon}{\epsilon}} + \mu^{\frac{1}{\epsilon}}\ (dr + i)\ A\mu\right)\right);$$

In[10]:=
$$WELN = \frac{1}{dr + 0.01}\left(\frac{i * Log[\lambda]}{dr + 0.01} + \frac{\mu}{i + \mu}\ Log\left[\frac{cNW}{wLW}\right] + \frac{i}{i + \mu} * Log\left[\frac{cNW}{\lambda * wS * (1 + \tau N)}\right]\right)$$

Out[10]=
$$\frac{\frac{i\ Log[\lambda]}{0.01+dr} + \frac{\mu\ Log\left[\frac{i\ \left(A\mu\ (dr+i)\ \mu^{\frac{1}{\epsilon}} + Ai\ i^{\frac{1-\epsilon}{\epsilon}}\ \lambda\ \mu\ (dr+(1+\epsilon)\ (i+\mu))\right)\ (1+\tau N)}{(-1+\lambda)\ \mu\ (\mu+i\ (1+\tau N))}\right]}{i+\mu} + \frac{i\ Log\left[\frac{i\ \eta S}{wS\ \lambda\ \mu\ (1+\tau S)}\right]}{i+\mu}}{0.01 + dr}$$

In[11]:=
$$WELS = \frac{1}{dr + 0.01}\left(\frac{i * Log[\lambda]}{dr + 0.01} + \frac{\mu}{i + \mu}\ Log\left[\frac{cSW}{wLW\ (1 + \tau S)}\right] + \frac{i}{i + \mu} * Log\left[\frac{cSW}{\lambda * wSW}\right]\right)$$

Out[11]=
$$\frac{\frac{i\ Log[\lambda]}{0.01+dr} + \frac{\mu\ Log\left[\frac{cSW\ \left(A\mu\ (dr+i)\ \mu^{\frac{1}{\epsilon}} + Ai\ i^{\frac{1-\epsilon}{\epsilon}}\ \lambda\ \mu\ (dr+(1+\epsilon)\ (i+\mu))\right)}{\eta S\ (-1+\lambda)\ (\mu+i\ (1+\tau N))}\right]}{i+\mu} + \frac{i\ Log\left[\frac{cSW\ \left(A\mu\ (dr+i)\ \mu^{\frac{1}{\epsilon}} + Ai\ i^{\frac{1-\epsilon}{\epsilon}}\ \mu\ (dr+(1+\epsilon)\ (i+\mu))\right)\ (1+\tau S)}{\eta S\ (-1+\lambda)\ (i+\mu\ (1+\tau S))}\right]}{i+\mu}}{0.01 + dr}$$

In[12]:=
$$WELNF[i\_,\ \mu\_,\ \tau N\_,\ \tau S\_,\ dr\_,\ \lambda\_,\ Ai\_,\ A\mu\_,\ \eta S\_,\ \epsilon\_,\ sN\_] :=$$
$$\frac{\frac{i\ Log[\lambda]}{0.01`+dr} + \frac{\mu\ Log\left[\frac{i\ \left(A\mu\ (dr+i)\ \mu^{\frac{1}{\epsilon}} + Ai\ i^{\frac{1-\epsilon}{\epsilon}}\ \lambda\ \mu\ (dr+(1+\epsilon)\ (i+\mu))\right)\ (1+\tau N)}{(-1+\lambda)\ \mu\ (\mu+i\ (1+\tau N))}\right]}{i+\mu} + \frac{i\ Log\left[\frac{i\ \left(A\mu\ (dr+i)\ \mu^{\frac{1}{\epsilon}} + Ai\ i^{\frac{1-\epsilon}{\epsilon}}\ \mu\ (dr+(1+\epsilon)\ (i+\mu))\right)}{(-1+\lambda)\ \mu\ (i+\mu\ (1+\tau S))}\right]}{i+\mu}}{0.01` + dr};$$
$$WELSF[i\_,\ \mu\_,\ \tau N\_,\ \tau S\_,\ dr\_,\ \lambda\_,\ Ai\_,\ A\mu\_,\ \eta S\_,\ \epsilon\_,\ sN\_] :=$$
$$\frac{\frac{i\ Log[\lambda]}{0.01`+dr} + \frac{\mu\ Log\left[\frac{A\mu\ (dr+i)\ \mu^{\frac{1}{\epsilon}} + Ai\ i^{\frac{1-\epsilon}{\epsilon}}\ \lambda\ \mu\ (dr+(1+\epsilon)\ (i+\mu))}{\eta S\ (-1+\lambda)\ (\mu+i\ (1+\tau N))}\right]}{i+\mu} + \frac{i\ Log\left[\frac{\left(A\mu\ (dr+i)\ \mu^{\frac{1}{\epsilon}} + Ai\ i^{\frac{1-\epsilon}{\epsilon}}\ \mu\ (dr+(1+\epsilon)\ (i+\mu))\right)\ (1+\tau S)}{\eta S\ (-1+\lambda)\ (i+\mu\ (1+\tau S))}\right]}{i+\mu}}{0.01` + dr};$$

We also note the restrictons that should apply in the competitive case

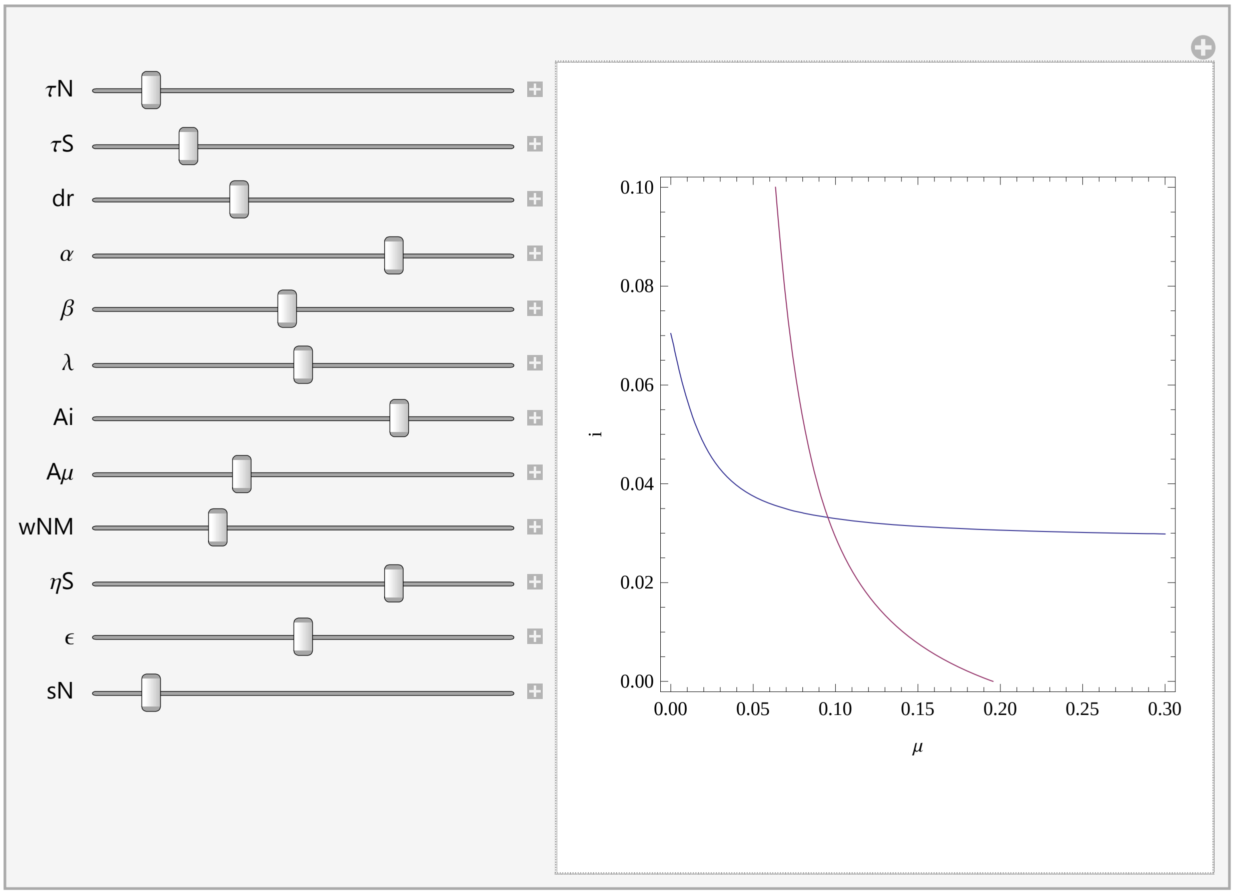
In[14]:=
$$R1CF[i\_,\ \mu\_,\ \tau N\_,\ \tau S\_,\ dr\_,\ \lambda\_,\ Ai\_,\ A\mu\_,\ \eta S\_,\ \epsilon\_,\ sN\_,\ wLC\_,\ wSC\_,\ cN\_,\ R1\_] := R1 == \frac{\lambda * wSC}{(1 + \tau S)} - wLC;$$
$$R2CF[i\_,\ \mu\_,\ \tau N\_,\ \tau S\_,\ dr\_,\ \lambda\_,\ Ai\_,\ A\mu\_,\ \eta S\_,\ \epsilon\_,\ sN\_,\ wLC\_,\ wSC\_,\ cN\_,\ R2\_] := R2 == wLC - (1 + \tau N) * wSC;$$

2. Graphical Representation of the Model in (μ,ι) Space

2.1. We borrow the parameters from the case with wSM>0.

In[16]:=
```
Manipulate[ContourPlot[{LABN[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN] == 0,
    LABS[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN] == 0},
   {μ, 0.0, 0.3}, {i, 0.00, 0.1}, FrameLabel → {"μ", "i"}],
  {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14}, {{α, 0.76}, 0.1, 1},
  {{β, 0.51}, 0.1, 1}, {{λ, 2}, 1, 3}, {{Ai, 75}, 0, 100}, {{Aμ, 335}, 0, 1000},
  {{wNM, 0.55}, 0.0, 2}, {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1}, {{sN, 0.01}, 0, 0.1}]
```

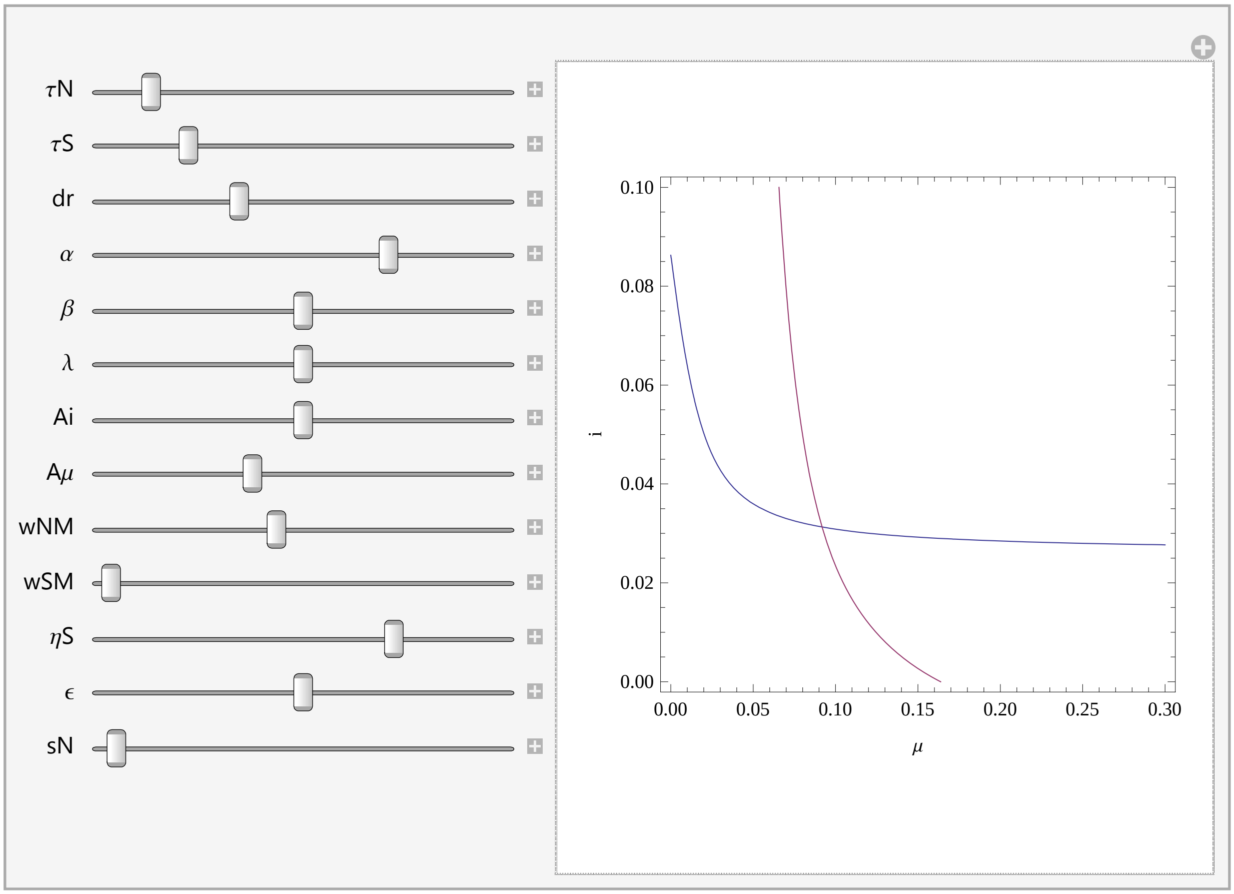
Out[16]=



2.2. We borrow the parameters from the case with wSM=0.

In[17]:=
```
Manipulate[ContourPlot[{LABN[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN] == 0,
    LABS[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN] == 0},
   {μ, 0.0, 0.3}, {i, 0.00, 0.1}, FrameLabel → {"μ", "i"}],
  {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14},
  {{α, 0.75}, 0.1, 1}, {{β, 0.55}, 0.1, 1}, {{λ, 2}, 1, 3}, {{Ai, 50}, 1, 100},
  {{Aμ, 400}, 50, 1000}, {{wNM, 0.85}, 0.0, 2}, {{wSM, 0.0}, 0.0, 2},
  {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1}, {{sN, 0.01}, 0, 1}
]
```

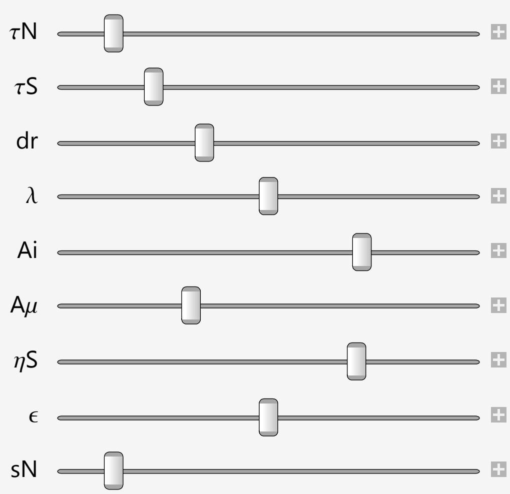
Out[17]=



3. Numerical Representation of the Model

3.1. We borrow the parameters from the case with wSM>0.

In[18]:=
```
Manipulate[NSolve[
  {LABN[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN],
   LABS[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN],
   wLCF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, wSC, wLC],
   wSCF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, wSC, wLC],
   cNF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN, wLC, wSC, cN],
   R1CF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN, wLC, wSC, cN, R1],
   R2CF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN, wLC, wSC, cN, R2]},
  {i, μ, wSC, wLC, cN, R1, R2}],
 {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14},
 {{λ, 2}, 1, 3}, {{Ai, 75}, 1, 100}, {{Aμ, 335}, 50, 1000},
 {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1}, {{sN, 0.01}, 0, 0.1}]
```

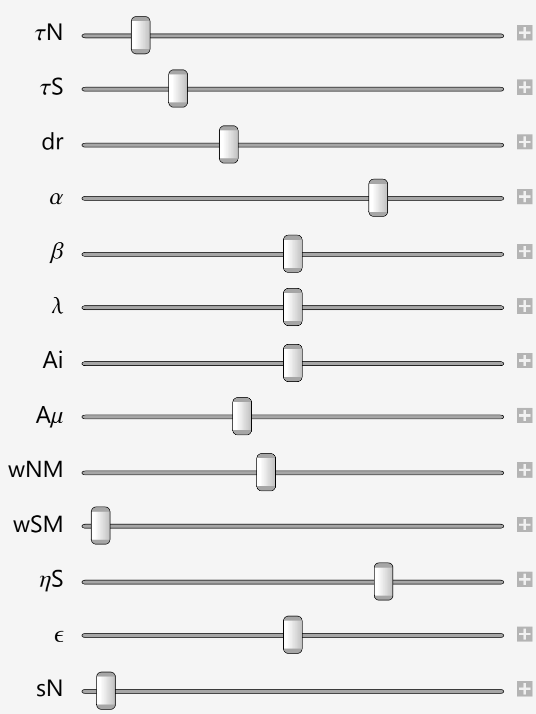



```
{{i → 0.391312, μ → -0.125122,
  wSC → 0.580405, wLC → -0.993809,
  cN → -11.2666, R1 → 1.96115,
  R2 → -1.63225}, {i → -0.192266,
  μ → 0.0797922, wSC → 1.00649,
  wLC → 13.503, cN → -8.68052,
  R1 → -11.8255, R2 → 12.3958},
 {i → 0.00250849, μ → -0.00248847,
  wSC → -7.70118, wLC → 12.0751,
  cN → -3.63148, R1 → -24.9104,
  R2 → 20.5464}, {i → -0.067561,
  μ → -0.135515, wSC → 1.75733,
  wLC → 1.79683, cN → 1.79603,
  R1 → 1.13205, R2 → -0.136231},
 {i → 0.0331701, μ → 0.0954102,
  wSC → 0.702599, wLC → 1.06877,
  cN → 1.25244, R1 → 0.102224,
  R2 → 0.295916}, {i → -0.0585817,
  μ → 0.0448373, wSC → 1.14976,
  wLC → 4.40869, cN → -4.7068,
  R1 → -2.49242, R2 → 3.14395}}
```

3.2. We borrow the parameters from the case with wSM=0.

In[19]:=
```
Manipulate[NSolve[
  {LABN[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN],
   LABS[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN],
   wLCF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, wSC, wLC],
   wSCF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, wSC, wLC],
   cNF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN, wLC, wSC, cN],
   R1CF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN, wLC, wSC, cN, R1],
   R2CF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN, wLC, wSC, cN, R2]},
  {i, μ, wSC, wLC, cN, R1, R2}],
 {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14},
 {{α, 0.75}, 0.1, 1}, {{β, 0.55}, 0.1, 1}, {{λ, 2}, 1, 3}, {{Ai, 50}, 1, 100},
 {{Aμ, 400}, 50, 1000}, {{wNM, 0.85}, 0.0, 2}, {{wSM, 0.0}, 0.0, 2},
 {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1}, {{sN, 0.01}, 0, 1}
]
```

Out[19]=




```
{{i → -0.334692, μ → 0.0878781,
  wSC → 0.956819, wLC → -14.3664,
  cN → -13.7205, R1 → 15.9611,
  R2 → -15.4189}, {i → 0.525422,
  μ → -0.112694, wSC → 0.663202,
  wLC → -1.45521, cN → -16.7962,
  R1 → 2.56054, R2 → -2.18473},
 {i → -0.0557465, μ → 0.0433509,
  wSC → 3.37816, wLC → 8.64321,
  cN → -4.63259, R1 → -3.01294,
  R2 → 4.92723}, {i → -0.0745685,
  μ → -0.1255, wSC → 1.80626,
  wLC → 2.14752, cN → 2.1405,
  R1 → 0.86291, R2 → 0.160635},
 {i → 0.00204092, μ → -0.00202059,
  wSC → -7.06589, wLC → 9.59959,
  cN → -3.63875, R1 → -21.3761,
  R2 → 17.3721}, {i → 0.0312937,
  μ → 0.0918398, wSC → 0.675197,
  wLC → 1.09297, cN → 1.22752,
  R1 → 0.032354, R2 → 0.350258}}
```

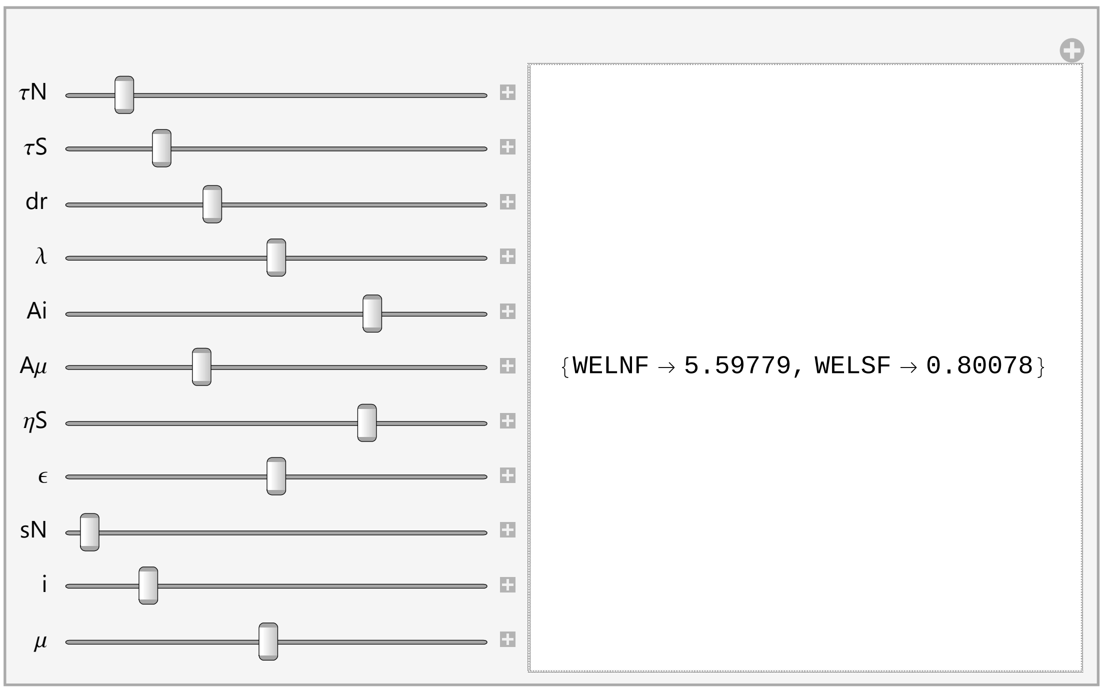
4. Welfare Analysis

4.1. We borrow the parameters from the case with wSM>0.

In[20]:=
```
Manipulate[
 { "WELNF" -> WELNF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN],
  "WELSF" -> WELSF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN]},

 {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14},
 {{λ, 2}, 1, 3}, {{Ai, 75}, 1, 100}, {{Aμ, 335}, 50, 1000},
 {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1}, {{sN, 0.01}, 0, 1},
 {{i, 0.03317006510588066`}, 0, 0.2}, {{μ, 0.09541017891849637`}, 0, 0.2}
]
```

Out[20]=



4.1. We borrow the parameters from the case with wSM=0.

In[21]:=
```
Manipulate[
 { "WELNF" -> WELNF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN],
  "WELSF" -> WELSF[i, μ, τN, τS, dr, λ, Ai, Aμ, ηS, ϵ, sN]},

 {{τN, 0.1}, 0, 1}, {{τS, 0.2}, 0, 1}, {{dr, 0.06}, 0.02, 0.14},
 {{α, 0.75}, 0.1, 1}, {{β, 0.55}, 0.1, 1}, {{λ, 2}, 1, 3}, {{Ai, 50}, 1, 100},
 {{Aμ, 400}, 50, 1000}, {{wNM, 0.85}, 0.0, 2}, {{wSM, 0.0}, 0.0, 2},
 {{ηS, 3.93}, 1, 5}, {{ϵ, 0.5}, 0, 1}, {{sN, 0.01}, 0, 1},
 {{i, 0.031293708531937636`}, 0, 0.2}, {{μ, 0.09183977053502725`}, 0, 0.2}
]
```

Out[21]=