Globalization, Rent Protection Institutions, and Going Alone In Freeing Trade

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Abstract: We construct a two-country North-South product cycle model of trade with endogenous growth and trade barriers. We remove the scale effects on growth by incorporating rent protection activities by Northern incumbents. We examine the effects of two forms of globalization – an expansion of the relative size of the South and unilateral trade liberalization by either country. We find that the location of rent protection institutions and the sectoral trade structure determine whether or not globalization raises steady-state economic growth. We demonstrate that for accelerating worldwide economic growth, contrary to conventional wisdom, unilateral Northern trade liberalization is preferable to bilateral trade liberalization.

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Referees' Appendix (not to be published)

Appendix R.1: The PEG Model

In order to verify whether the assumption of RPAs is crucial for the tariff-neutrality result of proposition 2, we replace this route of removing scale effects by the more conventional "permanent-effects-on-growth" (PEG) specification of rising R&D difficulty, which can be found, e.g., in Dinopoulos and Segerstrom (1999a), Dinopoulos and Thompson (1996, 2000), or Şener (2001). In this formulation, R&D difficulty is tied to the exogenous (Northern) population size and is therefore independent of the Northern firm value. Formally, relative to our RPA formulation, the following changes: instead of (7), we have $\pi_N \equiv \pi_N^P$, instead of (10), we have

$$D = kN_N\,, \tag{R.1}$$

(17) is skipped, in (18) $s_N = 0$ (there is no specialized labor since there are no RPAs), (19) is skipped, and (23) is skipped. The rest of the model does not change.

In the PEG model, (25) becomes

$$\frac{c_N\left[1-\frac{w_{LN}}{\lambda(1+\tau_N)}\right]+c_S\eta_S\left(\frac{1}{1+\tau_S}-\frac{w_{LN}}{\lambda}\right)}{\rho+\iota+\mu-n} = w_{LN}a_\iota k \qquad \mathbf{FEIN}\left(c_N, c_S, \iota, \mu, w_{LN}\right), \tag{R.2}$$

(26) becomes

$$\frac{c_N\left(\frac{1}{1+\tau_N}-\frac{1}{w_{LN}}\right)+c_S\eta_S\left[1-\frac{1}{w_{LN}(1+\tau_S)}\right]}{\rho+\iota-n} = a_\mu k \qquad \mathbf{FEIM}\left(c_N, c_S, \iota, w_{LN}\right), \tag{R.3}$$

(30) becomes

$$\left(\frac{c_N}{1+\tau_N}+c_S\eta_S\right)\frac{\iota}{\lambda(\iota+\mu)}+a_\iota \iota k = 1 \qquad \mathbf{LABN}\left(c_N, c_S, \iota, \mu\right), \tag{R.4}$$

(31) becomes

$$\left(\frac{c_S}{1+\tau_S}+\frac{c_N}{\eta_S}\right)\frac{\mu}{w_{LN}(\iota+\mu)}+\frac{a_\mu \mu k}{\eta_S}\frac{\iota}{\iota+\mu} = 1 \qquad \mathbf{LABS}\left(c_N, c_S, w_{LN}, \iota, \mu\right), \tag{R.5}$$

and (22) is unchanged. Using (22) and the definition for Q_N in (R.2) gives

$$\frac{c_N\left[1+\frac{\mu}{\iota(1+\tau_N)}\right]-Q_N w_{LN}}{\rho-n+\iota+\mu} = w_{LN}a_\iota k \qquad \mathbf{FEIN}\left(c_N, w_{LN}, \iota, \mu\right), \tag{R.6}$$

using (22) and the definition for Q_S in (R.3) gives

$$\frac{\frac{c_N}{1+\tau_N}\left[1+\frac{\mu(1+\tau_S)}{\iota}\right]-Q_S}{\rho-n+\iota}=a_\mu k \qquad \mathbf{FEIM}\left(c_N, w_{LN}, \iota, \mu\right). \tag{R.7}$$

Solving (R.6) and (R.7) yields

$$w_{LN}=\frac{\lambda\left[A_{\iota,\mu}\left(\rho-n+\iota+\mu\right)+\rho-n+\iota\right]\left[\iota\left(1+\tau_N\right)+\mu\right]}{\left[\lambda A_{\iota,\mu}\left(\rho-n+\iota+\mu\right)+\rho-n+\iota\right]\left[\iota+\mu\left(1+\tau_S\right)\right]} \qquad \mathbf{w_{LN}}\left(\iota,\mu\right) \tag{R.8}$$

and

$$c_N=\frac{\iota a_\mu\left(1-\sigma_\mu\right)k\left(1+\tau_N\right)\lambda\left[A_{\iota,\mu}\left(\rho-n+\iota+\mu\right)+\rho-n+\iota\right]}{\left(\lambda-1\right)\left[\iota+\mu\left(1+\tau_S\right)\right]} \qquad \mathbf{c_N}\left(\iota,\mu\right), \tag{R.9}$$

where $A_{\iota,\mu}\equiv a_\iota\left(1-\sigma_\iota\right)\big/\left[a_\mu\left(1-\sigma_\mu\right)\right]$. Using (22) and (R.9) in (R.4) gives

$$\frac{\iota k\left\{a_\mu\left(1-\sigma_\mu\right)\left[\left(\rho-n+\iota\right)\left(A_{\iota,\mu}+1\right)+A_{\iota,\mu}\mu\right]+a_\iota\left(\lambda-1\right)\left(\iota+\mu\right)\right\}}{\left(\lambda-1\right)\left(\iota+\mu\right)}=1 \qquad \mathbf{LABN}\left(\iota,\mu\right). \tag{R.10}$$

Using (22), (R.8) and (R.9) in (R.5) gives

$$\frac{\mu a_\mu k\left\{\eta_S\left(1-\sigma_\mu\right)\left[\left(\rho-n+\iota\right)\left(\lambda A_{\iota,\mu}+1\right)+\lambda A_{\iota,\mu}\mu\right]+\left(\lambda-1\right)\iota\right\}}{\eta_S\left(\lambda-1\right)\left(\iota+\mu\right)}=1 \qquad \mathbf{LABS}\left(\iota,\mu\right). \tag{R.11}$$

Again, tariff rates do not show up in these two steady-state equilibrium equations. Therefore, the fact that in the model with RPAs, changes in firm profits trigger proportional changes in RPAs is not responsible for the neutrality of tariff changes with respect to the steady-state industry-wide rates of innovation and imitation.

Comparing (R.6) and (R.7) with (25) and (26), respectively, reveals that the predictions on the effects of unilateral tariff rate changes in the PEG model on w_{LN} and c_N are the same as in the baseline RPA model. Furthermore, it is straightforward to see that increased Southern trade integration has also the same effects in the PEG model as in the baseline RPA model.

Appendix R.2: Asset Ownership Conditions Instead Of The BOT Condition

As an alternative to imposing the BOT condition (22), we can impose an asset market equilibrium condition to solve the system (25) – (31) in order to derive an additional equation for c_N or c_S, respectively. By this we can verify that it is not the chosen particular way to derive this additionally required equation which is driving our results from propositions 1 and 2.

Since π_N/N_N is constant during the incumbency period of any Northern monopolist in the steady state (X/N_N is constant due to (19) in a steady-state equilibrium), the stock-market value per Northern capita is constant over time and equals

$$\frac{v_N}{N_N} = \frac{w_{LN} a_\iota D}{N_N} = \frac{\pi_N/N_N}{\rho + \iota + \mu - n} = \frac{c_N \left(1 - \frac{w_{LN}}{\lambda}\right) + c_S \eta_S \left(\frac{1}{1+\tau_S} - \frac{w_{LN}}{\lambda}\right) - w_{HN} \gamma \frac{X}{N_N}}{\rho + \iota + \mu - n} ,$$

thus the total market value of all Northern firms at time *t*, $V_N = v_N n_N$, is

$$V_N = n_N w_{LN} a_\iota D , \tag{R.12}$$

where *D* is a linearly increasing function of $N_N(t)$ according to (10) and (19). Similarly, the total market value of all Southern firms at time *t*, $V_S = v_S n_S$, is found as

$$V_S = n_S a_\mu D . \tag{R.13}$$

The intertemporal budget constraint of a Northern *consumer* supplying specialized labor is $\dot{B}_{HN} = w_{HN} + \rho B_{HN} - c_{HN} - n B_{HN}$. Since $\dot{B}_{HN}/B_{HN}$ must be constant in a steady-state equilibrium, it follows

$$\frac{\dot{B}_{HN}}{B_{HN}} = \frac{w_{HN} - c_{HN}}{B_{HN}} + \rho - n = 0 \qquad \Leftrightarrow \qquad c_{HN} = w_{HN} + (\rho - n) B_{HN} ,$$

which applies to a fraction s_N of the Northern population. Similarly, we get

$$c_{LN} = w_{LN} + (\rho - n) B_{LN} ,$$

which applies to a fraction $1-s_N$ of the Northern population. Therefore, defining average financial assets of a Northern consumer as $B_N \equiv s_N B_{HN} + (1-s_N) B_{LN}$, it follows:

$$c_N = s_N w_{HN} + (1 - s_N) w_{LN} + (\rho - n) B_N , \tag{R.14}$$

and, analogously,

$$c_S = 1 + (\rho - n) B_S . \tag{R.15}$$

Similar to Dinopoulos and Segerstrom (2007, p. 21), we can assume “balanced asset ownership”, i.e. we impose the assumption that Northern (Southern) consumers only own Northern (Southern) firms, hence

$$B_N = V_N/N_N \qquad \text{and} \qquad B_S = V_S/N_S . \tag{R.16}$$

Plugging (R.12) and (R.16) into (R.14) gives the Northern per-capita consumption expenditures (including payments for the Southern import tariff)

$$c_N = s_N w_{HN} + (1 - s_N) w_{LN} + \left[(\rho - n) n_N w_{LN} a_\iota D / N_N \right]$$

which, after using (23), (10), (19) and (21), can be rewritten as

$$c_N = \left[s_N A_\iota \iota \left(1 + \tfrac{\rho - n}{\iota + \mu}\right) + 1 - s_N \right] w_{LN} . \tag{R.17}$$

Similarly, we derive the Southern per-capita consumption expenditures (including payments for the Northern import tariff)

$$c_S = 1 + \left[(\rho - n) n_S a_\mu D / N_S \right]$$

which, after using (10), (19) and (21), can be rewritten as

$$c_S = 1 + \frac{(\rho - n) A_\mu \mu s_N}{(\iota + \mu) \eta_S} . \tag{R.18}$$

We now have six equations [(25) – (31), and (R.17) – (R.18)] in five unknowns c_N, c_S, ι, μ and w_{LN}. By Walras Law, we can use either (R.17) or (R.18) to derive the steady-state equilibrium of our model. Solving the model yields again (32) and (33) as steady-state solution of our model. Accordingly, performing the same comparative static exercises as in the version of the model with the BOT condition (22) yields exactly the same results in terms of ι, μ and n_N as those stated for the BOT version in Propositions 1 and 2 before.[34]

Alternatively, we can follow Lundberg and Segerstrom (2002, p. 185) by assuming an "unbalanced asset ownership". This would mean to set $B_S = \phi B_W / N_S$ and $B_N = (1-\phi) B_W / N_N$, where $B_W = n_N V_N + n_S V_S$ measures the valuation of worldwide assets, and ϕ is the share of assets owned by Southern consumers (i.e., opposite to the "balanced asset ownership" case, we allow for international cross ownership of firms). Again, solving the model under these assumptions and performing the same comparative static exercises as before yields exactly the same results in terms of ι, μ and n_N as those stated in Proposition 1 and 2 before.[35]

Hence, the bottom line is that allowing for unbalanced asset ownership does not change the steady-state results relative to balanced asset ownership, and more generally, whether we use a BOT

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[34] Contrary to the balanced-trade specification, the sign of $dw_{LN}/d\eta_S$ and $dw_{LN}/d\tau_N$ becomes ambiguous. However, for sufficiently low tariff rates and sufficiently low consumer discount rate $\rho - n$, $dw_{LN}/d\eta_S < 0$ as before. $dw_{LN}/d\tau_S$ remains qualitatively the same as with balanced trade.

[35] As for the effects on w_{LN}, the same reservations as with balanced asset ownership apply.

condition or an asset market equilibrium condition to solve our set of steady-state equilibrium equations does not matter for results in terms of ι, μ and n_N at all.

Appendix R.3: Derivation of the Steady-State Utility Growth Rate

From equation (2) we can derive the common steady-state utility growth rate of both countries as follows: taking into account that only goods with the lowest quality-adjusted price are consumed, and considering the different goods pricing of Northern and Southern firms for both markets, respectively, we get

$$\begin{aligned}\log u_N(t) &= \int_{1-n_N} \log\left[\lambda^{j(\omega,t)} \frac{c_N}{w_{LN}}\right] d\omega + \int_{n_N} \log\left[\lambda^{j(\omega,t)} \frac{c_N}{\lambda(1+\tau_N)}\right] d\omega \\ &= \log\left(\frac{c_N}{w_{LN}}\right) + n_N \log\left[\frac{w_{LN}}{\lambda(1+\tau_N)}\right] + (\log\lambda)\int_0^t \iota(\tau)\,d\tau\end{aligned}$$

as the instantaneous utility of Northern consumers. Similarly, we get

$$\begin{aligned}\log u_S(t) &= \int_{1-n_N} \log\left[\lambda^{j(\omega,t)} \frac{c_S}{w_{LN}(1+\tau_S)}\right] d\omega + \int_{n_N} \log\left[\lambda^{j(\omega,t)} \frac{c_S}{\lambda}\right] d\omega \\ &= \log\left[\frac{c_S}{w_{LN}(1+\tau_S)}\right] + n_N \log\left[\frac{w_{LN}(1+\tau_S)}{\lambda}\right] + (\log\lambda)\int_0^t \iota(\tau)\,d\tau\end{aligned}$$

as the instantaneous utility of Southern consumers. Since w_{LN}, n_N, c_N and c_S are constant in the steady-state equilibrium, differentiation with respect to time gives $\dot{u}_N/u_N = \dot{u}_S/u_S = \iota\log\lambda$. Note in particular that the fraction n_N of Northern industries does not affect this growth rate since R&D is undertaken in all industries $\omega \in [0, 1]$.

Appendix R.4: Consumer Optimization With A Southern Low-Tech Sector (Details)

In order to transform the household's optimization problem of maximizing (1) subject to (37) and (38) into a standard optimal control problem that allows to apply Pontryagin's maximum principle, we define a new state variable Θ with

$$\dot{\Theta}(s) = e^{-[R(s)-R(t)]}\left[c_i(s) + z_i(s)\right], \quad \Theta(0) = 0, \quad \text{and} \quad \lim_{s\to\infty}\Theta(s) = WI_i(t) + FA_i(t).$$

Since the households take the evolution of the innovation index $j(\omega,t)$ and the high-tech goods price $p(\omega,t)$ as given, the term $\int_0^\infty e^{-\rho t}\alpha\{\int_0^1 \log[\lambda^{j(\omega,t)}/p(\omega,t)\,d\omega\}dt$ from (1) and (37) can be neglected, and with $p_Z = 1/b$ the present-value Hamilton function is simply

$$H\left(\Theta, c_i, z_i, \chi, t\right) = e^{-\rho s}\left\{\alpha \log\left[c_i\left(s\right)\right] + \left(1-\alpha\right)\log\left[bz_i\left(s\right)\right]\right\} + \chi\left(s\right)e^{-\left[R(s)-R(t)\right]}\left[c_i\left(s\right) + z_i\left(s\right)\right] \quad \text{(R.19)}$$

with $\chi(s)$ as the new costate variable corresponding to $\Theta(s)$. From the costate equation

$$\partial H/\partial\Theta \;=\; 0 \overset{!}{=} -\dot{\chi} \quad \text{(R.20)}$$

it follows immediately that $\chi(s) = \chi\ \forall s$, i.e. the costate variable is constant over time. Applying Pontryagin's maximum principle yields the other foc:

$$\partial H/\partial c_i\left(s\right) = e^{-\rho s}\left[\alpha/c_i\left(s\right)\right] + \chi e^{-\left[R(s)-R(t)\right]} \overset{!}{=} 0\,, \quad \text{(R.21)}$$

$$\partial H/\partial z_i\left(s\right) = e^{-\rho s}\left[\left(1-\alpha\right)/z_i\left(s\right)\right] + \chi e^{-\left[R(s)-R(t)\right]} \overset{!}{=} 0\,. \quad \text{(R.22)}$$

Differentiating (R.21) with respect to time s gives

$$-\rho e^{-\rho s}\left[\alpha/c_i\left(s\right)\right] - e^{-\rho s}\left[\alpha/c_i\left(s\right)^2\right]\dot{c}_i\left(s\right) - \dot{R}\left(s\right)\chi e^{-\left[R(s)-R(t)\right]} = 0\,,$$

and using the definition $\dot{R}\left(s\right) \equiv r\left(s\right)$ and (R.21) in the third term of the LHS of this equation yields the Keynes-Ramsey rule (4) again. Similarly, differentiating (R.22) with respect to time s, and applying $\dot{R}\left(s\right) \equiv r\left(s\right)$ and (R.22), leads to the optimal low-tech consumption path

$$\frac{\dot{z}_i\left(t\right)}{z_i\left(t\right)} = r\left(t\right) - \rho \qquad \text{for } i = N, S, \quad \text{(R.23)}$$

Finally, dividing (R.21) by (R.22) yields

$$\frac{c_i\left(t\right)}{z_i\left(t\right)} = \frac{\alpha}{1-\alpha}\ , \qquad \text{for } i = N, S, \quad \text{(R.24)}$$

which is (39) in the main text. Finally, the new common steady-state utility growth rate for both countries is $\dot{u}_N/u_N = \dot{u}_S/u_S = \alpha\iota\log\lambda$.

Appendix R.5: The Baseline Model With Only One Type of Northern Labor

This Appendix first shows that the central findings of our baseline model (Propositions 1 and 2 from section 3) are robust to the following change in the labor assignment. Instead of distinguishing between general-purpose and specialized workers in the North, we now assume only one type of (general-purpose) Northern labor that is perfectly mobile between all three activities, which are R&D, manufacturing and (global) RPAs. As before, there is only one type of Southern labor that can be employed in manufacturing of final goods or imitation, which are the only types of activities in the South. We then check for the robustness of our two Main Results as well.

We denote w_{LN} as the Northern wage rate, while the Southern wage rate is still normalized to unity. Equation (7) for the total Northern firm's profits changes to

$$\pi_N = \pi_N^P - w_{LN}\gamma X \, . \qquad \text{(R.25)}$$

Equation (17) for the optimal RPA decision changes to

$$X = \frac{v_N\left(\iota+\mu\right)}{w_{LN}\gamma} \, . \qquad \text{(R.26)}$$

The Northern general-purpose labor market clearing (LABN) condition (18) becomes

$$n_N Q_N\left(c_N, c_S\right) N_N + a_\iota \iota D + n_N \gamma X = N_N \, . \qquad \text{(R.27)}$$

Defining R&D difficulty per Northern capita as $d \equiv D/N_N$, the free-entry in innovation (FEIN) condition (25) changes to

$$\frac{c_N\left[1+\frac{\mu}{\iota(1+\tau_N)}\right]-\tilde{Q}_N\left(c_N\right)w_{LN}}{\rho-n+2\left(\iota+\mu\right)} = a_\iota w_{LN} d \qquad \textbf{FEIN}\left(c_N, w_{LN}, \iota, \mu\right), \qquad \text{(R.28)}$$

and the free-entry in imitation (FEIM) condition (26) changes to

$$\frac{\overbrace{\frac{c_N}{1+\tau_N}\left[1+\frac{\mu(1+\tau_S)}{\iota}\right]}^{\lambda\tilde{Q}_N(c_N)}-\tilde{Q}_S\left(c_N, w_{LN}\right)}{\rho-n+\iota} = a_\mu d \qquad \textbf{FEIM}\left(c_N, w_{LN}, \iota, \mu\right). \qquad \text{(R.29)}$$

Dividing both sides of (R.27) by N_N, using (21) and (22), we derive the LABN condition as:

$$\frac{\tilde{Q}_N\left(c_N\right)\iota}{\iota+\mu} + a_\iota \iota d + \frac{\iota\gamma x}{\iota+\mu} = 1 \qquad \textbf{LABN}\left(c_N, \iota, \mu, x, d\right), \qquad \text{(R.30)}$$

where $x \equiv X_N/N_N$ denotes the per-Northern-worker RPA. Dividing both sides of (20) by N_S, using (21) and (22), we derive the LABS condition as:

$$\frac{\tilde{Q}_S\left(c_N, w_{LN}\right)\mu}{\eta_S\left(\iota+\mu\right)} + \frac{a_\mu \mu d \iota}{\eta_S\left(\iota+\mu\right)} = 1 \qquad \textbf{LABS}\left(c_N, w_{LN}, \iota, \mu, d\right). \qquad \text{(R.31)}$$

Taking the ratio of (R.26) to (13) with D substituted from (10) and using (21) gives the relative profitability condition

$$\frac{v_N(\iota+\mu)}{v_N}=\frac{w_{LN}\gamma X}{w_{LN}a_\iota D} \quad \Leftrightarrow \quad \iota=\frac{n\gamma}{a_\iota\delta}\equiv\frac{1}{A_\iota}, \tag{R.32}$$

which captures the profitability of innovation with respect to RPA. A_ι captures the relative resource requirement in R&D with respect to RPAs. Obviously, as A_ι declines, the rate of innovation increases. This is also highlighted in Dinopoulos and Syropoulos (2007). Note that since the relative profitability is no longer tied to a relative wage, the link between innovation rate and the other parameters of the model vanishes. In particular, the rate of innovation is pinned down by the resource parameters in R&D and RPA.[36] The parameters λ and ρ do not play a role in the determination of ι. The link between ι and the model's entire parameter set (with the exception of the tariff rates, as has been shown in the main text) is reestablished under two conditions that must be fulfilled jointly: first, more than one factor of production is incorporated in either R&D or RPA, and second, there is some labor mobility between either manufacturing and R&D or between manufacturing and RPAs.[37]

Using the definitions for $\tilde{Q}_N$ and $\tilde{Q}_S$, and (10) with $d \equiv D/N_N$, (R.28) and (R.29) can jointly be solved for c_N and w_{LN} as functions in x, ι and μ only, with ι being pinned down by (R.32):

$$c_N=\frac{\iota^2 x\delta\lambda(1+\tau_N)\left\{a_\iota\left[\rho-n+2(\iota+\mu)\right]+a_\mu(\rho-n+\iota)\right\}}{n(\iota+\mu)(\lambda-1)\left[\iota+\mu(1+\tau_S)\right]}, \tag{R.33}$$

$$w_{LN}=\frac{\lambda\left\{a_\iota\left[\rho-n+2(\iota+\mu)\right]+a_\mu(\rho-n+\iota)\right\}\left[\iota(1+\tau_N)+\mu\right]}{\left\{\lambda a_\iota\left[\rho-n+2(\iota+\mu)\right]+a_\mu(\rho-n+\iota)\right\}\left[\iota+\mu(1+\tau_S)\right]}. \tag{R.34}$$

Using the definitions for $\tilde{Q}_N$ and $\tilde{Q}_S$, (10) with $d \equiv D/N_N$, (R.32) to substitute for ι, (R.33) and (R.34) to substitute for c_N and w_{LN}, respectively, the LABN and LABS conditions (R.30) and (R.31) can be written as functions in x and μ only:

[36] In Dinopoulos and Syropoulos (2007) when one type of labor conducts all activities, the rate of population growth exerts no influence on the innovation rate. The reason is that they model R&D difficulty as a flow variable whereas we model it as a stock variable. In our model, higher population growth dilutes R&D difficulty per-capita and boosts steady-state innovation.

[37] In the growth literature, is has become common practice to denote a growth model "(fully) endogenous" under two conditions: first, the growth rate is derived from the optimizing behavior of the economic agents of the model, and second, the growth rate can be affected by public policies. By contrast, a growth model is denoted "semi-endogenous" if its growth rate fulfills the first but not the second condition. In this sense, the growth rate that results from (R.32) is still fully endogenous since e.g. an R&D subsidy rate would reduce the relative resource requirement A_ι in R&D with respect to RPAs.

$$\frac{nx\gamma^2\left\{a_\iota\left[2n\gamma\lambda+a_\iota\delta\left(\rho-n+2\lambda\mu\right)\right]+a_\mu\left[n\gamma+a_\iota\delta\left(\rho-n\right)\right]\right\}}{a_\iota\left(\lambda-1\right)\left(n\gamma+a_\iota\delta\mu\right)^2}=1 \qquad \mathbf{LABN}\left(x,\mu\right), \quad \text{(R.35)}$$

$$\frac{x\gamma\delta\mu\left\{a_\iota\left[a_\mu\delta\left(\rho-n\right)+2n\gamma\lambda\right]+a_\iota^2\delta\lambda\left(\rho-n+2\mu\right)+a_\mu n\gamma\lambda\right\}}{\eta_S\left(\lambda-1\right)\left(n\gamma+a_\iota\delta\mu\right)^2}=1 \qquad \mathbf{LABS}\left(x,\mu\right). \quad \text{(R.36)}$$

Analyzing the Jacobi matrix of the system (R.35) and (R.36) reveals that LABN is unambiguously upward sloping, while LABS is downward sloping for a sufficiently small households' net discount rate $\rho - n$ and $2a_\iota > a_\mu$ (a sufficient but hardly necessary condition).

We immediately see that tariffs again do not enter (R.35) and (R.36), hence the tariff-neutrality result of Proposition 2 is still valid for this specification of the labor assignment. Appendix R.6 in addition shows that $dx/d\eta_S > 0$, $d\mu/d\eta_S > 0$, $dw_{LN}/d\eta_S < 0$, $dm/d\eta_S > 0$, and $dn_N/d\eta_S < 0$. Hence, all results of Proposition 1 still hold, with the only twist that now, an increase in η_S affects the steady-state level of x instead of ι. Moreover, Appendix R.6 also shows that Southern (Northern) unilateral trade liberalization results in an unambiguous increase (decrease) in the Northern general-purpose wage rate w_{LN}, and hence in the North-South wage gap, as in Proposition 2.

Finally, we check for the robustness of our two Main Results. As is immediately obvious, the innovation effects stated in Proposition 3 and Main Result 2 are effectively muted with only one type of Northern labor. Even when a Southern low-tech sector is added, equations (13) and (R.26) remain valid; hence, the steady-state innovation rate is still pinned down by a subset of exogenous parameters, excluding the tariff rates and η_S, as shown in (R.32). Thus, the η_S effects stated in part ii. of Main Result 1 are muted as well. However, the following shows that part i. of Main Result 1 still remains valid.

When Southern labor is used for imitation-deterring activities like in section 4.1 and Appendix B, and there is only one type of Northern labor that is fully mobile across all three activities, Northern and Southern R&D difficulties are still given by $D_\mu = s_S N_S \delta_\mu/(\gamma_\mu n)$ and $D_\iota = n_N \delta_\iota X_\iota/n$, respectively. Equation (13) becomes

$$v_N = w_{LN} a_\iota D_\iota , \qquad \text{(R.37)}$$

equation (R.25) turns into

$$\pi_N = \pi_N^P - w_{LN}\gamma_\iota X_\iota \ , \qquad \text{(R.38)}$$

equation (R.26) becomes

$$X_\iota = \frac{v_N(\iota+\mu)}{w_{LN}\gamma_\iota} \quad , \tag{R.39}$$

and equation (R.27) now changes into

$$n_N Q_N(c_N, c_S) N_N + a_\iota \iota D_\iota + n_N \gamma_\iota X = N_N \quad , \tag{R.40}$$

for which we define $d_\iota \equiv D_\iota / N_N$. Setting (24) equal to (R.37), using (22), the definitions of D_ι and $\tilde{Q}_N(c_N)$, and simplifying terms gives

$$\frac{c_N\left[1+\frac{\mu}{\iota(1+\tau_N)}\right]-\tilde{Q}_N(c_N)w_{LN}}{\rho-n+2(\iota+\mu)} = a_\iota w_{LN} d_\iota \qquad \textbf{FEIN}(c_N, w_{LN}, \iota, \mu), \tag{R.41}$$

which is almost unchanged relative to (R.28). Equation (14) slightly changes to

$$v_S = a_\mu D_\mu \quad , \tag{R.42}$$

and setting this equal to (16), substituting for D_μ, using (6), $r = \rho$, $\dot{v}_S / v_S = n$, the definitions of $\tilde{Q}_N(c_N)$ and $\tilde{Q}_S(c_N, w_{LN})$, and simplifying yields

$$\frac{\lambda\tilde{Q}_N(c_N)-\tilde{Q}_S(c_N, w_{LN})}{\rho-n+\iota} = \eta_S \hat{A}_\mu s_S \qquad \textbf{FEIM}(c_N, w_{LN}, \iota, \mu), \tag{R.43}$$

where $\hat{A}_\mu \equiv (a_\mu\delta_\mu)/(n\gamma_\mu)$ as in section 4.1. Using (21), the definition $x_\iota \equiv X_\iota/N_N$ and collecting terms, (R.40) can be rewritten as

$$\frac{\iota}{\iota+\mu}\left[\tilde{Q}(c_N) + x_\iota \gamma_\iota\left(\hat{A}_\iota \iota + 1\right)\right] = 1 \qquad \textbf{LABN}(c_N, \iota, \mu, x_\iota), \tag{R.44}$$

where $\hat{A}_\iota \equiv (a_\iota\delta_\iota)/(n\gamma_\iota)$ as in section 4.1. Next, we note that the Southern general-purpose labor market clearing condition (36) remains unchanged, and is here repeated for convenience:

$$\frac{c_N(\iota, \mu, \eta_S)\mu}{w_{LN}(\iota, \mu, \eta_S)\iota} + \frac{\hat{A}_\mu \mu s_S \iota \eta_S}{\iota+\mu} = (1-s_S)\eta_S \qquad \textbf{LABS}(\iota, \mu). \tag{R.45}$$

Note that we have used again the simplifying assumption $\tau_N = \tau_S = 0$ to derive (R.45). From (R.37) and (R.39), we again derive the relative-profitability condition

$$\iota = \frac{n\gamma_\iota}{a_\iota \delta} \equiv \hat{A}_\iota . \tag{R.46}$$

We now solve (R.41) and (R.43) for c_N and w_{LN}, using the definitions of $\tilde{Q}_N(c_N)$ and $\tilde{Q}_S(c_N, w_{LN})$, and our simplifying assumption $\tau_N = \tau_S = 0$:

$$c_N = \frac{\lambda\iota\left\{a_\iota\delta_\iota\left[\rho + 2(\iota+\mu) - n\right] + \eta_S \hat{A}_\mu s_S(\rho+\iota-n)\right\}}{(\lambda-1)(\iota+\mu)}, \tag{R.47}$$

$$w_{LN} = \frac{a_\iota\delta_\iota\left[\rho + 2(\iota+\mu) - n\right] + \eta_S \hat{A}_\mu s_S(\rho+\iota-n)}{a_\iota\delta_\iota\left[\rho + 2(\iota+\mu) - n\right] + \frac{\eta_S \hat{A}_\mu s_S(\rho+\iota-n)}{\lambda}}. \tag{R.48}$$

Using (R.47) and (R.48) in (R.45) gives LABS that determines the Southern industry-wide imitation rate as an increasing function of η_S, given that the Northern innovation rate is pinned down by (R.46):

$$\frac{\frac{\lambda a_\iota\delta_\iota}{\eta_S}\left[\rho + 2(\iota+\mu) - n\right] + \hat{A}_\mu s_S(\rho+\iota-n)}{\lambda-1} + \hat{A}_\mu s_S\iota = \frac{(1-s_S)(\iota+\mu)}{\mu} \qquad \textbf{LABS}(\mu,\eta_S). \tag{R.49}$$

The RHS of (R.49) is decreasing in μ, and the LHS of (R.49) is increasing in μ and decreasing in η_S, hence an increase in η_S requires an increase in μ to restore LABS(μ, η_S) again. Differentiating (R.48) with respect to η_S shows:

$$\frac{\partial w_{LN}}{\partial\eta_S} = \frac{\left[2a_\iota\delta_\iota\frac{\partial\mu}{\partial\eta_S} + \hat{A}_\mu s_S(\rho+\iota-n)\right]den - num\left[2a_\iota\delta_\iota\frac{\partial\mu}{\partial\eta_S} + \frac{\hat{A}_\mu s_S(\rho+\iota-n)}{\lambda}\right]}{den^2} \overset{?}{>} 0$$

$$\Leftrightarrow \quad \frac{\partial\mu}{\partial\eta_S} \overset{?}{<} \frac{\hat{A}_\mu s_S(\rho+\iota-n)\left(1-\frac{w_{LN}}{\lambda}\right)}{2a_\iota\delta_\iota(w_{LN}-1)}, \tag{R.50}$$

where *den* and *num* refer to denominator and numerator of the RHS of (R.48), respectively. In order to check whether condition (R.50) is fulfilled, we apply the implicit function theorem to (R.49):

$$f(\mu,\eta_S) \equiv \frac{\frac{\lambda a_\iota\delta_\iota}{\eta_S}\left[\rho + 2(\iota+\mu) - n\right] + \hat{A}_\mu s_S(\rho+\iota-n)}{\lambda-1} + \hat{A}_\mu s_S\iota - \frac{(1-s_S)(\iota+\mu)}{\mu} = 0$$

$$\Leftrightarrow \frac{\partial\mu}{\partial\eta_S} = -\frac{\partial f/\partial\eta_S}{\partial f/\partial\mu} = \frac{\rho+2(\iota+\mu)-n}{2\eta_S + \frac{(1-s_S)\iota\eta_S^2(\lambda-1)}{\lambda a_\iota\delta_\iota\mu^2}} \overset{?}{<} \frac{\hat{A}_\mu s_S(\rho+\iota-n)\left(1-\frac{w_{LN}}{\lambda}\right)}{2a_\iota\delta_\iota(w_{LN}-1)} \underset{(R.48)}{=} \frac{\rho+2(\iota+\mu)-n}{2\eta_S}.$$

Hence condition (R.50) is fulfilled, and part i. of Main Result 1 ($\partial w_{LN}/\partial\eta_S > 0$) is still valid.

Q.e.d.

In[87]:= "APPENDIX R.6: The Baseline Model With Only One Type of Northern Labor (Mathematica Program) ";

In[88]:= "- This program requires Mathematica Version 5.0. Before evaluating the cells, 'Math Econ' package written by Cliff Huang and Philip Crooke needs to be run. This package accompanies the book 'Mathematics and Mathematica for Economists', 1997, Blackwell Publishers: Oxford, written by the above authors.
- Objective: To conduct comparative steady-state analysis
- Notes: In this program, for convenience we enter the subscripts and superscripts of the main model in regular format. Define dr = ρ-n. The elimination of minus terms help Mathematica to obtain more tidy expressions.";

In[89]:= 1. THE MODEL;

In[90]:= "GENERAL FEATURES:
-- Tariffs are imposed by both the North and the South.
-- Bertrand pricing scheme applies.
-- Trade is balanced.
-- One type of labor in the North that can be employed for either R&D, RPA and manufacturing.
-- wLN represents the relative wage of Northern labor with respect to Southern labor.
-- We use small letters for ai and aμ instead of capital letters, because we will have the D's substituted by Mathematica";

In[91]:= "To simplify, we did the following transformation: dr=ρ-n,";

In[92]:=
```
"We first clear all the variables and functions";
Clear[FEIN, FEIM, LABN, LABS, FLABN, FLABS];
Clear[μ, x, ηS, λ, dr, d, ρ, n, δ, γ, ai, aμ, τN, τS]; Clear[cN, i, μ, wLN];
```

In[95]:=
```
"This is the proportion of Northern industries and aggregate rate of imitation";
nN = i / (i + μ); m = μ * nN;
```

In[97]:= "This is the BOT equation simplified";

$$cS = cN * \frac{1}{\eta S} * \frac{\mu}{i} * \frac{(1 + \tau S)}{(1 + \tau N)}$$

Out[98]= $$\frac{cN\,\mu\,(1 + \tau S)}{i\,\eta S\,(1 + \tau N)}$$

In[99]:= "This is the free entry in innovation condition with both sides divided by NN";

In[100]:=
$$\text{FEIN} = \frac{\left(\text{cN} * \left(1 - \frac{\text{wLN}}{\lambda\,(1+\tau\text{N})}\right)\right) + \left(\text{cS} * \eta\text{S} * \left(\frac{1}{1+\tau\text{S}} - \frac{\text{wLN}}{\lambda}\right)\right)}{\text{dr} + 2\,\text{i} + 2\,\mu} == \text{ai} * \text{wLN} * \text{d} \;//\; \text{FullSimplify}$$

Out[100]=
$$\frac{\text{cN}\,(\text{i}\,(-\text{wLN} + \lambda + \lambda\,\tau\text{N}) - \mu\,(\text{wLN} - \lambda + \text{wLN}\,\tau\text{S}))}{\text{i}\,\lambda\,(\text{dr} + 2\,(\text{i} + \mu))\,(1 + \tau\text{N})} == \text{ai d wLN}$$

In[101]:= "This is the free entry in imitation condition with both sides divided by NN";

In[102]:=
$$\text{FEIM} = \frac{\left(\text{cN}\left(\frac{1}{1+\tau\text{N}} - \frac{1}{\text{wLN}}\right)\right) + \left(\text{cS} * \eta\text{S} * \left(1 - \frac{1}{\text{wLN} * (1+\tau\text{S})}\right)\right)}{\text{dr} + \text{i}} == \text{a}\mu * \text{d} \;//\; \text{FullSimplify}$$

Out[102]=
$$\frac{\text{cN}\,(\text{i}\,(-1 + \text{wLN} - \tau\text{N}) + \mu\,(-1 + \text{wLN} + \text{wLN}\,\tau\text{S}))}{\text{i}\,(\text{dr} + \text{i})\,\text{wLN}\,(1 + \tau\text{N})} == \text{a}\mu\,\text{d}$$

In[103]:= Solve[{FEIN, FEIM}, {cN, wLN}]

Out[103]=
$$\Big\{\Big\{\text{cN} \to \frac{\text{d i}\,(\text{ai dr}\,\lambda + \text{a}\mu\,\text{dr}\,\lambda + 2\,\text{ai i}\,\lambda + \text{a}\mu\,\text{i}\,\lambda + 2\,\text{ai}\,\lambda\,\mu)\,(1 + \tau\text{N})}{(-1 + \lambda)\,(\text{i} + \mu + \mu\,\tau\text{S})},$$
$$\text{wLN} \to \big(\text{ai dr i}\,\lambda + \text{a}\mu\,\text{dr i}\,\lambda + 2\,\text{ai i}^2\,\lambda + \text{a}\mu\,\text{i}^2\,\lambda + \text{ai dr}\,\lambda\,\mu + \text{a}\mu\,\text{dr}\,\lambda\,\mu + 4\,\text{ai i}\,\lambda\,\mu + \text{a}\mu\,\text{i}\,\lambda\,\mu + 2\,\text{ai}\,\lambda\,\mu^2 + \text{ai dr i}\,\lambda\,\tau\text{N} +$$
$$\text{a}\mu\,\text{dr i}\,\lambda\,\tau\text{N} + 2\,\text{ai i}^2\,\lambda\,\tau\text{N} + \text{a}\mu\,\text{i}^2\,\lambda\,\tau\text{N} + 2\,\text{ai i}\,\lambda\,\mu\,\tau\text{N}\big) \big/ ((\text{a}\mu\,\text{dr} + \text{a}\mu\,\text{i} + \text{ai dr}\,\lambda + 2\,\text{ai i}\,\lambda + 2\,\text{ai}\,\lambda\,\mu)\,(\text{i} + \mu + \mu\,\tau\text{S}))\Big\}\Big\}$$

In[104]:= "This is the R&D and imitation difficulty per unit of NN, where x = X/NN and d = D/NN";

In[105]:=
$$\text{d} = \frac{\text{nN} * \delta * \text{x}}{\text{n}};$$

In[106]:= "We tell Mathematica what cN and wLN are, with the expression for d already entered";

In[107]:=
$$\text{cN} = \frac{\text{d i}\,(\text{ai dr}\,\lambda + \text{a}\mu\,\text{dr}\,\lambda + 2\,\text{ai i}\,\lambda + \text{a}\mu\,\text{i}\,\lambda + 2\,\text{ai}\,\lambda\,\mu)\,(1 + \tau\text{N})}{(-1 + \lambda)\,(\text{i} + \mu + \mu\,\tau\text{S})};$$
$$\text{wLN} = \big(\text{ai dr i}\,\lambda + \text{a}\mu\,\text{dr i}\,\lambda + 2\,\text{ai i}^2\,\lambda + \text{a}\mu\,\text{i}^2\,\lambda + \text{ai dr}\,\lambda\,\mu + \text{a}\mu\,\text{dr}\,\lambda\,\mu + 4\,\text{ai i}\,\lambda\,\mu + \text{a}\mu\,\text{i}\,\lambda\,\mu + 2\,\text{ai}\,\lambda\,\mu^2 + \text{ai dr i}\,\lambda\,\tau\text{N} +$$
$$\text{a}\mu\,\text{dr i}\,\lambda\,\tau\text{N} + 2\,\text{ai i}^2\,\lambda\,\tau\text{N} + \text{a}\mu\,\text{i}^2\,\lambda\,\tau\text{N} + 2\,\text{ai i}\,\lambda\,\mu\,\tau\text{N}\big) \big/ ((\text{a}\mu\,\text{dr} + \text{a}\mu\,\text{i} + \text{ai dr}\,\lambda + 2\,\text{ai i}\,\lambda + 2\,\text{ai}\,\lambda\,\mu)\,(\text{i} + \mu + \mu\,\tau\text{S}));$$

In[108]:= {cN, wLN} // FullSimplify

Out[108]=
$$\left\{\frac{\text{i}^2\,\text{x}\,\delta\,\lambda\,(\text{a}\mu\,(\text{dr} + \text{i}) + \text{ai}\,(\text{dr} + 2\,(\text{i} + \mu)))\,(1 + \tau\text{N})}{\text{n}\,(-1 + \lambda)\,(\text{i} + \mu)\,(\text{i} + \mu + \mu\,\tau\text{S})},\ \frac{\lambda\,(\text{a}\mu\,(\text{dr} + \text{i}) + \text{ai}\,(\text{dr} + 2\,(\text{i} + \mu)))\,(\text{i} + \mu + \text{i}\,\tau\text{N})}{(\text{a}\mu\,(\text{dr} + \text{i}) + \text{ai}\,\lambda\,(\text{dr} + 2\,(\text{i} + \mu)))\,(\text{i} + \mu + \mu\,\tau\text{S})}\right\}$$

In[109]:= "Taking the ratio of the free-entry in R&D condition
to optimal RPA condition gives the Relative R&D to RPA Profitability Condition";

$$\text{RELP} = \frac{\text{vN}}{\text{vN}\,(\text{i} + \mu)} == \frac{\text{wLN} * \text{ai} * \text{d} * \text{NN}}{\text{wLN} * \gamma * \text{x} * \text{NN}};\ \text{Solve}[\text{RELP}, \text{i}]$$

Out[110]= $\left\{\left\{\text{i} \to \frac{\text{n}\,\gamma}{\text{ai}\,\delta}\right\}\right\}$

In[111]:= "We enter the expression for i"; $\text{i} = \frac{\text{n}\,\gamma}{\text{ai}\,\delta};$

In[112]:= "This is the Northern labor market equilibrium condition with both sides divided by NN";

In[113]:= $\text{LABN} = \frac{(\text{cN} / (1 + \tau\text{N})) + (\text{cS} * \eta\text{S})}{\lambda} \frac{\text{i}}{\text{i} + \mu} + (\text{ai} * \text{i} * \text{d}) + (\gamma * \text{nN} * \text{x}) == 1 \text{ // FullSimplify}$

Out[113]= $\frac{\text{n x}\,\gamma^2\,(\text{a}\mu\,(\text{n}\,\gamma + \text{ai dr}\,\delta) + \text{ai}\,(2\,\text{n}\,\gamma\,\lambda + \text{ai}\,\delta\,(\text{dr} + 2\,\lambda\,\mu)))}{\text{ai}\,(-1 + \lambda)\,(\text{n}\,\gamma + \text{ai}\,\delta\,\mu)^2} == 1$

In[114]:= "This is the Southern labor market equilibrium condition with both sides divided by NS";

In[115]:= LABS = $\frac{\frac{cN}{\eta S} + \frac{cS}{(1+\tau S)}}{wLN}\ \frac{\mu}{i + \mu} + \left(a\mu * \mu * \frac{d}{\eta S} * nN\right) == 1$ // FullSimplify

Out[115]= $\frac{x\,\gamma\,\delta\,\mu\,\left(a\mu\,n\,\gamma\,\lambda + ai\,(a\mu\,dr\,\delta + 2\,n\,\gamma\,\lambda) + ai^2\,\delta\,\lambda\,(dr + 2\,\mu)\right)}{\eta S\,(-1+\lambda)\,(n\,\gamma + ai\,\delta\,\mu)^2} == 1$

In[116]:= "LABN and LABS are now expressed in terms of x and μ only. So, we can
enter these as functions and construct a system of equations for comparative statics.
Observe that LABN and LABS do not contain tariffs, so the tariff neutrality result holds.";

In[117]:= Clear[μ, x, ηS, λ, dr, d, ρ, n, δ, γ, ai, aμ, τN, τS]

In[118]:= FLABN[x_, μ_, dr_, ηS_, λ_, ρ_, n_, δ_, γ_, ai_, aμ_, τN_, τS_] := $\frac{n\,x\,\gamma^2\,(a\mu\,(n\,\gamma + ai\,dr\,\delta) + ai\,(2\,n\,\gamma\,\lambda + ai\,\delta\,(dr + 2\,\lambda\,\mu)))}{ai\,(-1+\lambda)\,(n\,\gamma + ai\,\delta\,\mu)^2} - 1$

In[119]:= FLABS[x_, μ_, dr_, ηS_, λ_, ρ_, n_, δ_, γ_, ai_, aμ_, τN_, τS_] := $\frac{x\,\gamma\,\delta\,\mu\,\left(a\mu\,n\,\gamma\,\lambda + ai\,(a\mu\,dr\,\delta + 2\,n\,\gamma\,\lambda) + ai^2\,\delta\,\lambda\,(dr + 2\,\mu)\right)}{\eta S\,(-1+\lambda)\,(n\,\gamma + ai\,\delta\,\mu)^2} - 1$

In[120]:= "This gives the Jacobian Matrix";

In[121]:= J = {gradf[FLABN[x, μ, dr, ηS, λ, ρ, n, δ, γ, ai, aμ, τN, τS] , {x, μ}],
gradf[FLABS[x, μ, dr, ηS, λ, ρ, n, δ, γ, ai, aμ, τN, τS], {x, μ}]};

In[122]:= FullSimplify[J] // MatrixForm

Out[122]//MatrixForm=

$$\begin{pmatrix} \frac{n\,\gamma^2\,(a\mu\,(n\,\gamma+ai\,dr\,\delta)+ai\,(2\,n\,\gamma\,\lambda+ai\,\delta\,(dr+2\,\lambda\,\mu)))}{ai\,(-1+\lambda)\,(n\,\gamma+ai\,\delta\,\mu)^2} & -\frac{2\,n\,x\,\gamma^2\,\delta\,(a\mu\,(n\,\gamma+ai\,dr\,\delta)+ai\,(n\,\gamma\,\lambda+ai\,\delta\,(dr+\lambda\,\mu)))}{(-1+\lambda)\,(n\,\gamma+ai\,\delta\,\mu)^3} \\ \frac{\gamma\,\delta\,\mu\,\left(a\mu\,n\,\gamma\,\lambda+ai\,(a\mu\,dr\,\delta+2\,n\,\gamma\,\lambda)+ai^2\,\delta\,\lambda\,(dr+2\,\mu)\right)}{\eta S\,(-1+\lambda)\,(n\,\gamma+ai\,\delta\,\mu)^2} & \frac{x\,\gamma\,\delta\,\left(n\,\gamma\,\left(ai\,a\mu\,dr\,\delta+2\,ai\,n\,\gamma\,\lambda+a\mu\,n\,\gamma\,\lambda+ai^2\,dr\,\delta\,\lambda\right)-ai\,\delta\,\left(ai\,a\mu\,dr\,\delta-2\,ai\,n\,\gamma\,\lambda+a\mu\,n\,\gamma\,\lambda+ai^2\,dr\,\delta\,\lambda\right)\mu\right)}{\eta S\,(-1+\lambda)\,(n\,\gamma+ai\,\delta\,\mu)^3} \end{pmatrix}$$

In[123]:= "-- LABN is upward sloping,
-- LABS is upward or downward sloping. However, one can easily show that if dr goes to
zero and 2ai>aμ holds (a sufficient but hardly a necessary condition), LABS is downward sloping.";

In[124]:= **2. COMPARATIVE STATICS FOR ηS;**

In[125]:= "This gives the gradient of the functions FLABN and FLANS with respect to ηS";

```
In[126]:= B = {gradf[FLABN[x, μ, dr, ηS, λ, ρ, n, δ, γ, ai, aμ, τN, τS] , {ηS}],
  gradf[FLABS[x, μ, dr, ηS, λ, ρ, n, δ, γ, ai, aμ, τN, τS] , {ηS}]}
```

Out[126]= $$\left\{\{0\},\left\{-\frac{x\,\gamma\,\delta\,\mu\left(a\mu\,n\,\gamma\,\lambda+ai\,(a\mu\,dr\,\delta+2\,n\,\gamma\,\lambda)+ai^2\,\delta\,\lambda\,(dr+2\,\mu)\right)}{\eta S^2\,(-1+\lambda)\,(n\,\gamma+ai\,\delta\,\mu)^2}\right\}\right\}$$

```
In[127]:= "This follows from Cramer's rule";

In[128]:= impactηS = -Inverse[J].B.{ΔηS};

In[129]:= MatrixForm[impactηS] // FullSimplify
```

Out[129]//MatrixForm=

$$\begin{pmatrix} \frac{2\,ai\,x\,\delta\,\Delta\eta S\,\mu\left(a\mu\,n\,\gamma\,\lambda+ai\,(a\mu\,dr\,\delta+2\,n\,\gamma\,\lambda)+ai^2\,\delta\,\lambda\,(dr+2\,\mu)\right)(a\mu\,(n\,\gamma+ai\,dr\,\delta)+ai\,(n\,\gamma\,\lambda+ai\,\delta\,(dr+\lambda\,\mu)))}{\eta S\,(n\,\gamma+ai\,\delta\,\mu)\left(a\mu^2\,n^2\,\gamma^2\,\lambda+ai\,a\mu\,n\,\gamma\,(1+\lambda)\,(a\mu\,dr\,\delta+2\,n\,\gamma\,\lambda)+ai^3\,\delta\,(a\mu\,dr\,\delta+2\,n\,\gamma\,\lambda)\,(dr+dr\,\lambda+4\,\lambda\,\mu)+ai^4\,\delta^2\,\lambda\left(dr^2+4\,dr\,\mu+4\,\lambda\,\mu^2\right)+ai^2\left(a\mu^2\,dr^2\,\delta^2+4\,n^2\,\gamma^2\,\lambda^2+2\,a\mu\,n\,\gamma\,\delta\,\lambda\,(3\,dr+2\,\mu)\right)\right)} \\ \frac{\Delta\eta S\,\mu\left(a\mu\,n\,\gamma\,\lambda+ai\,(a\mu\,dr\,\delta+2\,n\,\gamma\,\lambda)+ai^2\,\delta\,\lambda\,(dr+2\,\mu)\right)(a\mu\,(n\,\gamma+ai\,dr\,\delta)+ai\,(2\,n\,\gamma\,\lambda+ai\,\delta\,(dr+2\,\lambda\,\mu)))}{\eta S\left(a\mu^2\,n^2\,\gamma^2\,\lambda+ai\,a\mu\,n\,\gamma\,(1+\lambda)\,(a\mu\,dr\,\delta+2\,n\,\gamma\,\lambda)+ai^3\,\delta\,(a\mu\,dr\,\delta+2\,n\,\gamma\,\lambda)\,(dr+dr\,\lambda+4\,\lambda\,\mu)+ai^4\,\delta^2\,\lambda\left(dr^2+4\,dr\,\mu+4\,\lambda\,\mu^2\right)+ai^2\left(a\mu^2\,dr^2\,\delta^2+4\,n^2\,\gamma^2\,\lambda^2+2\,a\mu\,n\,\gamma\,\delta\,\lambda\,(3\,dr+2\,\mu)\right)\right)} \end{pmatrix}$$

```
In[130]:= {dx, dμ} = %;
```

In[131]:= **"Clearly $\frac{dx}{d\eta S}>0$ and also $\frac{d\mu}{d\eta S}>0$, without requiring further restrictions.";**

```
In[132]:= wLN // FullSimplify
```

Out[132]= $$\frac{\lambda\left(a\mu\,n\,\gamma+ai\,(2\,n\,\gamma+a\mu\,dr\,\delta)+ai^2\,\delta\,(dr+2\,\mu)\right)(ai\,\delta\,\mu+n\,\gamma\,(1+\tau N))}{(a\mu\,(n\,\gamma+ai\,dr\,\delta)+ai\,\lambda\,(2\,n\,\gamma+ai\,\delta\,(dr+2\,\mu)))\,(n\,\gamma+ai\,\delta\,\mu\,(1+\tau S))}$$

In[133]:=
```
dwLN = D[wLN, x] dx + D[wLN, μ] dμ + D[wLN, ηS] ΔηS // FullSimplify
```

Out[133]=
```
-(ai δ ΔηS λ μ (aμ n γ λ + ai (aμ dr δ + 2 n γ λ) + ai^2 δ λ (dr + 2 μ)) (aμ (n γ + ai dr δ) + ai (2 n γ λ + ai δ (dr + 2 λ μ)))
   (aμ^2 n^3 γ^3 (τN + τS + τN τS) + ai^4 δ^2 (2 aμ dr δ (-1 + λ) μ^2 (1 + τS) + n γ λ (dr + 2 μ)^2 (τN + τS + τN τS)) +
    2 ai aμ n^2 γ^2 (aμ dr δ (τN + τS + τN τS) + n γ (-1 + λ + 2 λ τN + (1 + λ) (1 + τN) τS)) + ai^2 n γ (aμ^2 dr^2 δ^2 (τN + τS + τN τS) +
      4 n^2 γ^2 λ (τN + τS + τN τS) + aμ n γ δ (dr (-2 + 2 λ + τN + 5 λ τN + 3 (1 + λ) (1 + τN) τS) + 4 μ (-1 + λ (1 + τN) (1 + τS)))) + ai^3 n γ δ
     (4 n γ λ (dr + 2 μ) (τN + τS + τN τS) + aμ δ (2 (-1 + λ) μ^2 (1 + τS) + dr^2 (1 + λ) (τN + τS + τN τS) + 4 dr μ (-1 + λ (1 + τN) (1 + τS)))))) /
  (ηS (aμ (n γ + ai dr δ) + ai λ (2 n γ + ai δ (dr + 2 μ)))^2 (aμ^2 n^2 γ^2 λ + ai aμ n γ (1 + λ) (aμ dr δ + 2 n γ λ) + ai^3 δ (aμ dr δ + 2 n γ λ)
     (dr + dr λ + 4 λ μ) + ai^4 δ^2 λ (dr^2 + 4 dr μ + 4 λ μ^2) + ai^2 (aμ^2 dr^2 δ^2 + 4 n^2 γ^2 λ^2 + 2 aμ n γ δ λ (3 dr + 2 μ))) (n γ + ai δ μ (1 + τS))^2)
```

In[134]:=
```
" Clearly dwLN/dηS<0 without requiring further restrictions.";
```

In[135]:=
```
dm = D[m, μ] dμ + D[m, ηS] ΔηS;
```

In[136]:=
```
dnN = D[nN, μ] dμ + D[nN, ηS] ΔηS;
```

In[137]:=
```
FullSimplify[{dm, dnN}] // MatrixForm
```

Out[137]//MatrixForm=
```
( n^2 γ^2 ΔηS μ (aμ n γ λ+ai (aμ dr δ+2 n γ λ)+ai^2 δ λ (dr+2 μ)) (aμ (n γ+ai dr δ)+ai (2 n γ λ+ai δ (dr+2 λ μ)))
  / (ηS (n γ+ai δ μ)^2 (aμ^2 n^2 γ^2 λ+ai aμ n γ (1+λ) (aμ dr δ+2 n γ λ)+ai^3 δ (aμ dr δ+2 n γ λ) (dr+dr λ+4 λ μ)+ai^4 δ^2 λ (dr^2+4 dr μ+4 λ μ^2)+ai^2 (aμ^2 dr^2 δ^2+4 n^2 γ^2 λ^2+2 aμ n γ δ λ (3 dr+2 μ))))

  -ai n γ δ ΔηS μ (aμ n γ λ+ai (aμ dr δ+2 n γ λ)+ai^2 δ λ (dr+2 μ)) (aμ (n γ+ai dr δ)+ai (2 n γ λ+ai δ (dr+2 λ μ)))
  / (ηS (n γ+ai δ μ)^2 (aμ^2 n^2 γ^2 λ+ai aμ n γ (1+λ) (aμ dr δ+2 n γ λ)+ai^3 δ (aμ dr δ+2 n γ λ) (dr+dr λ+4 λ μ)+ai^4 δ^2 λ (dr^2+4 dr μ+4 λ μ^2)+ai^2 (aμ^2 dr^2 δ^2+4 n^2 γ^2 λ^2+2 aμ n γ δ λ (3 dr+2 μ)))) )
```

In[138]:=
```
"Clearly dm/dηS>0 , and also dnN/dηS<0 ";
```

In[139]:=
```
3. COMPARATIVE STATICS FOR τS;
```

In[140]:=
```
BτS = {gradf[FLABN[x, μ, dr, ηS, λ, ρ, n, δ, γ, ai, aμ, τN, τS] , {τS}],
  gradf[FLABS[x, μ, dr, ηS, λ, ρ, n, δ, γ, ai, aμ, τN, τS] , {τS}]}
```

Out[140]=
```
{{0}, {0}}
```

In[141]:=
```
impactτS = -Inverse[J].BτS.{ΔτS};
```

In[142]:= `MatrixForm[impactτS] // FullSimplify`

Out[142]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[143]:= `{dx, dμ} = %;`

In[144]:= "Clearly $\frac{dx}{d\tau S}$=0 and also $\frac{d\mu}{d\tau S}$=0
This is because there is no tariff expression in the FLABN and FLABS equations.";

In[145]:= `dwLN = D[wLN, μ] dμ + D[wLN, τS] ΔτS // FullSimplify`

Out[145]=

$$-\left(ai\,\delta\,\Delta\tau S\,\lambda\,\mu\,\left(a\mu\,n\,\gamma + ai\,(2\,n\,\gamma + a\mu\,dr\,\delta) + ai^2\,\delta\,(dr + 2\,\mu)\right)\,(ai\,\delta\,\mu + n\,\gamma\,(1 + \tau N))\right) \Big/ \left((a\mu\,(n\,\gamma + ai\,dr\,\delta) + ai\,\lambda\,(2\,n\,\gamma + ai\,\delta\,(dr + 2\,\mu)))\,(n\,\gamma + ai\,\delta\,\mu\,(1 + \tau S))^2\right)$$

In[146]:= "Clearly $\frac{dwLN}{d\tau S}$<0.
Tariff reduction by the South increases the North-South wage gap ";

In[147]:= `dm = D[m, μ] dμ + D[m, τS] ΔτS;`

In[148]:= `dnN = D[nN, μ] dμ + D[nN, τS] ΔτS;`

In[149]:= `FullSimplify[{dm, dnN}] // MatrixForm`

Out[149]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[150]:= "Clearly $\frac{dm}{d\tau S}$=0 and also $\frac{dnN}{d\tau S}$=0
Again, this is because tariffs do not affect i and μ.";

In[151]:= **4. COMPARATIVE STATICS FOR τN;**

In[152]:=
```
BτN = {gradf[FLABN[x, μ, dr, ηS, λ, ρ, n, δ, γ, ai, aμ, τN, τS] , {τN}],
  gradf[FLABS[x, μ, dr, ηS, λ, ρ, n, δ, γ, ai, aμ, τN, τS] , {τN}]}
```

Out[152]= {{0}, {0}}

In[153]:=
```
impactτN = -Inverse[J].BτN.{ΔτN};
```

In[154]:=
```
MatrixForm[impactτS] // FullSimplify
```

Out[154]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[155]:=
```
{dx, dμ} = %;
```

In[156]:= "Clearly $\frac{dx}{d\tau N}$=0 and also $\frac{d\mu}{d\tau N}$=0
This is because there is no tariff expression in the FLABN and FLABS equations.";

In[157]:=
```
dwLN = D[wLN, μ] dμ + D[wLN, τN] ΔτN // FullSimplify
```

Out[157]=

$$\frac{n\,\gamma\,\Delta\tau N\,\lambda\,\left(a\mu\,n\,\gamma + ai\,(2\,n\,\gamma + a\mu\,dr\,\delta) + ai^2\,\delta\,(dr + 2\,\mu)\right)}{(a\mu\,(n\,\gamma + ai\,dr\,\delta) + ai\,\lambda\,(2\,n\,\gamma + ai\,\delta\,(dr + 2\,\mu)))\,(n\,\gamma + ai\,\delta\,\mu\,(1 + \tau S))}$$

In[158]:= "Clearly $\frac{dwLN}{d\tau N}$>0.
Tariff reduction by the North reduces the North-South wage gap ";

In[159]:=
```
dm = D[m, μ] dμ + D[m, τN] ΔτN;
```

In[160]:=
```
dnN = D[nN, μ] dμ + D[nN, τN] ΔτN;
```

In[161]:=
```
FullSimplify[{dm, dnN}] // MatrixForm
```

Out[161]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[162]:= "Clearly $\frac{dm}{d\tau N}$=0 and also $\frac{dnN}{d\tau N}$=0
Again, this is because tariffs do not affect i and μ.";

Appendix R.7: The Baseline Model With Two Types of Northern Labor and a Cobb-Douglas Production Function For RPAs

This Appendix first shows that the central findings of our baseline model (Propositions 1 and 2 from section 3) are robust to the following change in the labor assignment: instead of using general-purpose workers in the North only for manufacturing and R&D, there is now perfectly mobile between all three activities (R&D, manufacturing and global RPAs), while Northern specialized workers still exclusively conduct RPAs. For the unit-cost function of RPAs, a Cobb-Douglas technology is used. As before, there is only one type of Southern labor that can be employed in manufacturing of final goods or imitation, which are the only types of activities in the South. We then check for the robustness of our Main Result 1 as well.[38]

To conduct RPAs, each Northern incumbent hires Northern specialized labor at a wage rate of w_{HN} and also Northern general-purpose workers at a wage rate of w_{LN}. The unit cost function of RPAs is derived from a Cobb-Douglas technology:

$$B\left(w_{HN}, w_{LN}\right) = \gamma w_{HN}^{\beta} w_{LN}^{1-\beta}\ , \tag{R.51}$$

where $0 \leq \beta \leq 1$ and $\gamma > 0$. Equation (7) for the total Northern firm's profits changes to

$$\pi_N = \pi_N^P - B\left(w_{HN}, w_{LN}\right) X\ . \tag{R.52}$$

Equation (17) for the optimal RPA decision changes to

$$X = \frac{v_N\left(\iota + \mu\right)}{B\left(w_{HN}, w_{LN}\right)}\ . \tag{R.53}$$

The Northern general-purpose labor market clearing (LABN) condition (18) becomes

$$n_N Q_N\left(c_N, c_S\right) N_N + a_\iota \iota D + n_N b_L X = \left(1 - s_N\right) N_N\ , \tag{R.54}$$

where by Shephard's lemma, $b_L \equiv \partial B(.)/\partial w_{LN}$ is the RPA unit labor requirement for general-purpose workers. The Northern specialized labor market clearing condition (19) becomes

$$n_N b_H X = s_N N_N\ , \tag{R.55}$$

where by Shephard's lemma, $b_H \equiv \partial B(.)/\partial w_{HN}$ is the RPA unit labor requirement for specialized workers. Taking the ratio of (R.53) to (13) with D substituted from (10) and using (21) gives the

[38] A version of our model with two types of Northern labor and a Cobb-Douglas technology for RPAs in the North, and an additional Southern low-tech sector is no longer analytically tractable. This is why our Proposition 3 and Main Result 2 cannot be checked analytically for this labor assignment.

relative profitability condition

$$\iota = \frac{1}{A_\iota}\left(\frac{w_{HN}}{w_{LN}}\right)^{\beta}, \tag{R.56}$$

where $A_\iota \equiv (a_\iota\delta)/(n\gamma)$ again captures the relative resource requirement in R&D with respect to RPAs. Obviously, the rate of innovation increases in the relative cost of RPAs with respect to R&D, i.e., the RHS of (R.56). Observe that we consider in the main text the special case $\beta = 1$, i.e. RPAs use only specialized labor.

The FEIN and FEIM conditions are identical to (R.28) and (R.29), thus c_N and w_{LN} as functions of ι and μ only are still given by (R.33) and (R.34). Plugging (R.34) into (R.56) allows to solve for w_{HN} as a function solely in ι and μ:

$$w_{HN} = \left(\frac{a_\iota\iota\delta}{n\gamma}\right)^{\frac{1}{\beta}} \frac{\lambda\left\{a_\iota\left[\rho-n+2(\iota+\mu)\right]+a_\mu(\rho-n+\iota)\right\}\left[\iota(1+\tau_N)+\mu\right]}{\left\{\lambda a_\iota\left[\rho-n+2(\iota+\mu)\right]+a_\mu(\rho-n+\iota)\right\}\left[\iota+\mu(1+\tau_S)\right]}. \tag{R.57}$$

Using (R.56) to substitute for w_{LN}/w_{HN} in equation (R.55), and using (21) to substitute for n_N, allows to derive the per-Northern-worker RPA as

$$x = \frac{s_N (A_\iota\iota)^{\frac{1-\beta}{\beta}}(\iota+\mu)}{\beta\gamma\iota}\ . \tag{R.58}$$

By using n_N from (21), the definition for Q_N from (18), the solution for c_N from (R.33), the BOT condition (22) to substitute for c_S, (10) to substitute for D, (R.58) to substitute for $x \equiv X/N_N$, and using $b_L \equiv \partial B(.)/\partial w_{LN}$ from Shephard's lemma, LABN (R.54) can be written as a function solely in ι and μ:

$$\frac{s_N\left\{a_\iota\iota\delta\left[\rho-n+(1+\lambda)(\iota+\mu)\right]+a_\mu\iota\delta(\rho-n+\iota)+n(1-\beta)\gamma A_\iota\iota(\lambda-1)(\iota+\mu)\right\}}{(A_\iota\iota)^{\frac{\beta-1}{\beta}} n\beta\gamma(\lambda-1)(\iota+\mu)} = 1-s_N \qquad \mathbf{LABN}(\iota,\mu). \tag{R.59}$$

The Southern labor market clearing condition (20) still applies. By using n_S from (21), the definition for Q_S from (20), the solution for c_N from (R.33), the BOT condition (22) to substitute for c_S, (10) to substitute for D, (R.58) to substitute for $x \equiv X/N_N$, LABS can also be written as a function solely in ι and μ:

$$\frac{s_N \delta \left(A_\iota \iota\right)^{\frac{1-\beta}{\beta}} \mu \left\{ a_\iota \lambda \left[\rho - n + 2\left(\iota + \mu\right) \right] + a_\mu \left(\rho - n + \iota \lambda \right) \right\}}{n \beta \gamma \eta_S \left(\lambda - 1\right)\left(\iota + \mu\right)} = 1 \qquad \mathbf{LABS}\left(\iota, \mu\right). \tag{R.60}$$

Analyzing the Jacobi matrix of the system (R.59) and (R.60) reveals that LABN is unambiguously upward sloping, while LABS is downward sloping for a sufficiently small households' net discount rate $\rho - n$, which corresponds to the condition (34) in our baseline model in the main text.

We immediately see that tariffs again do not enter (R.59) and (R.60), hence the tariff-neutrality result of Proposition 2 is still valid for this specification of the labor assignment. Appendix R.8 in addition shows that for sufficiently low $\rho - n$, $d\iota/d\eta_S > 0$, $d\mu/d\eta_S > 0$, $dw_{LN}/d\eta_S < 0$, $dm/d\eta_S > 0$, and $dn_N/d\eta_S < 0$. Hence, all results of Proposition 1 still hold for sufficiently low $\rho - n$. Finally, Appendix R.8 also shows that Southern (Northern) unilateral trade liberalization results in an unambiguous increase (decrease) in the Northern general-purpose wage rate w_{LN}, and hence in the North-South wage gap, as in Proposition 2.

Finally, we check for the robustness of our Main Result 1. When Southern labor is used for imitation-deterring activities like in section 4.1 and Appendix B, and there are two types of Northern workers with Cobb-Douglas technology in RPAs as defined in (R.51), the following equations changes: equation (R.51) becomes

$$B\left(w_{HN}, w_{LN}\right) = \gamma_\iota w_{HN}^{\beta} w_{LN}^{1-\beta}, \tag{R.61}$$

equation (R.52) becomes

$$\pi_N = \pi_N^P - B\left(w_{HN}, w_{LN}\right) X_\iota, \tag{R.62}$$

equation (R.53) becomes

$$X_\iota = \frac{v_N\left(\iota + \mu\right)}{B\left(w_{HN}, w_{LN}\right)}, \tag{R.63}$$

equation (R.54) becomes

$$n_N Q_N\left(c_N, c_S\right) N_N + a_\iota \iota D_\iota + n_N b_L X_\iota = \left(1 - s_N\right) N_N \tag{R.64}$$

with $D_\iota = n_N \delta_\iota X_\iota / n$, and equation (R.55) becomes

$$n_N b_H X_\iota = s_N N_N. \tag{R.65}$$

Equation (R.37) applies again as the Northern free-entry in R&D condition. Solving (R.63) for v_N and dividing by (R.37) yields the relative profitability condition

$$\iota = \left(\frac{w_{HN}}{w_{LN}}\right)^{\beta} \frac{1}{\hat{A}_\iota} \ . \tag{R.66}$$

The FEIN condition (R.41) and the FEIM condition (R.43) still apply, and for $\tau_N = \tau_S = 0$ these are still solved for c_N given in (R.47) and w_{LN} given in (R.48). Using (R.48) in (R.66) allows to solve for $w_{HN}(\iota, \mu)$. Equations (R.65) and (R.66) can be solved for

$$x_\iota \equiv \frac{X_\iota}{N_N} = \frac{s_N (\iota + \mu)\left(\iota \hat{A}_\iota\right)^{\frac{1-\beta}{\beta}}}{\iota \beta \gamma_\iota} \ . \tag{R.67}$$

Using (21), the definition for $Q_N(c_N, c_S)$ from (18), the BOT condition (22), (R.47) to substitute for c_N, $D_\iota = n_N \delta_\iota X_\iota / n$, $b_L \equiv \partial B(.)/\partial w_{LN}$ and (R.67) to substitute for x_ι, we can rewrite (R.64) as

$$\frac{\iota\left\{a_\iota \delta_\iota \left[\rho + 2(\iota + \mu) - n\right] + \eta_S \hat{A}_\mu s_S (\rho + \iota - n)\right\}}{(\lambda - 1)(\iota + \mu)} + \frac{\left(\iota \hat{A}_\iota\right)^{\frac{1}{\beta}} s_N (2 - \beta)}{\beta} = 1 - s_N \qquad \mathbf{LABN}(\iota, \mu) . \tag{R.68}$$

The LHS of (R.68) is rising in ι and declining in μ, hence LABN(ι, μ) is positively sloped in (ι, μ)-space. Since an increase in η_S raises the LHS, a reduction of ι is required for any given μ to restore the Northern labor market equilibrium, hence the LABN curve shifts downward. Finally, the Southern labor market equilibrium condition is still given by (R.49), which we rewrite here for convenience:

$$\frac{\mu}{\iota + \mu}\left\{\frac{\frac{\lambda a_\iota \delta_\iota}{\eta_S}\left[\rho + 2(\iota + \mu) - n\right] + \hat{A}_\mu s_S (\rho + \iota - n)}{\lambda - 1} + \hat{A}_\mu s_S \iota\right\} = 1 - s_S \qquad \mathbf{LABS}(\mu, \eta_S) . \tag{R.69}$$

The LHS of (R.69) is unambiguously increasing in μ, and the LHS is also increasing in ι provided that condition

$$\rho - n \ < \ \frac{\lambda \mu \hat{A}_\mu s_S}{\hat{A}_\mu s_S + \frac{\lambda a_\iota \delta_\iota}{\eta_S}} \tag{R.70}$$

is fulfilled. Under this condition, the LABS(ι, μ) is negatively sloped in (ι, μ)-space. Since an increase in η_S reduces the LHS of (R.69), an increase in ι is required for any given μ to restore the Southern labor market equilibrium, hence the LABS curve shifts upward. The Figure R.1 below illustrates the steady-state labor market effects of increased Southern trade integration in this model.

It follows that Southern imitation is unambiguously rising, and Northern innovation can go either up or down. Totally differentiating (R.68) and (R.69) for $d\iota = 0$ gives the sizes of the horizontal shifts:

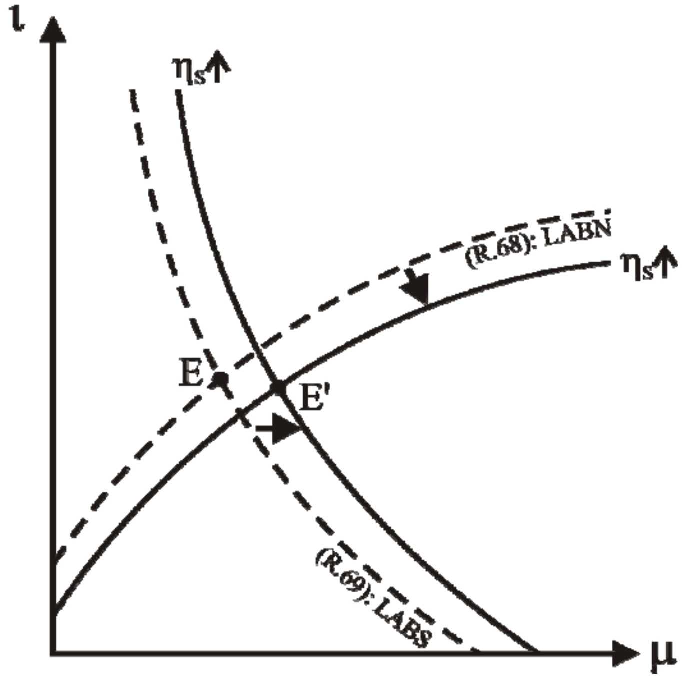

Figure R1: Steady-state effects of increased Southern trade integration

$$\left.\frac{d\mu}{d\eta_S}\right|_{(\text{R.68})} = \frac{\iota(\iota+\mu)}{\frac{a_\iota \delta_\iota (\rho-n)}{\hat{A}_\mu s_S (\rho+\iota-n)} + \eta_S} , \tag{R.71}$$

$$\left.\frac{d\mu}{d\eta_S}\right|_{(\text{R.69})} = \frac{\mu(\iota+\mu)\lambda a_\iota \delta_\iota \left[\rho+2(\iota+\mu)-n\right]}{\eta_S\left\{\lambda a_\iota \delta_\iota \left[2\mu(\iota+\mu)+\iota\left[\rho+2(\iota+\mu)-n\right]\right]+\eta_S \hat{A}_\mu s_S (\rho+\iota-n)\right\}} . \tag{R.72}$$

Comparing these two shifts show that depending on parameter values, the net effect on ι can indeed be positive or negative. Hence, part ii. of Main Result 1 is still valid. Given that contrary to Appendix R.5 an increase in η_S not only increases μ but also changes ι (in ambiguous direction), the net effect on w_{LN} (where this wage rate is still given in (R.48)) cannot be signed analytically, hence part i. of Main Result 1 cannot be checked analytically for this version of the model.

In[163]:= "APPENDIX R.8: The Baseline Model With Two Types of Northern Labor and a Cobb-Douglas Production Function For RPAs (Mathematica Program)";

In[164]:= 1. THE MODEL;

In[165]:= "GENERAL FEATURES:
-- Tariffs are imposed by both the North and the South.
-- Bertrand pricing scheme applies.
-- Trade is balanced.
-- Northern general purpose workers: Mobile across all activities
-- Northern Specialized workers : Performing only RPA
-- Northern RPA uses a Cobb Douglass production function
-- β is the share of specialized workers in RPA production.
-- There is no RPA in the South.";

In[166]:= "To simplify, we did the following transformations dr=ρ-n";

```
In[167]:= "We first clear all the variables and functions";
Clear[FEIN, FEIM, LABN, LABS, FLABN, FLABS];
Clear[ηS, λ, dr, ρ, n, δ, ai, aμ, sN, τN, τS]; Clear[cN, i, μ, wLN, wHN, x, d];

In[170]:= τS =.; τN =.; dr =.;

In[171]:= "This is the proportion of Northern industries and the aggregate rate of imitation";
nN = i / (i + μ); m = μ * nN;

In[173]:= "This is the BOT equation simplified";
```

$$cS = cN * \frac{1}{\eta S} * \frac{\mu}{i} * \frac{(1 + \tau S)}{(1 + \tau N)}$$

Out[174]=

$$\frac{cN\, \mu\, (1 + \tau S)}{i\, \eta S\, (1 + \tau N)}$$

In[175]:= "This is the free entry in innovation condition with both sides divided by NN. Note that d = D/NN";

In[176]:= $\text{FEIN} = \frac{\left(\text{cN}\left(1 - \frac{\text{wLN}}{\lambda(1+\tau\text{N})}\right)\right) + \left(\text{cS} * \eta\text{S} * \left(\frac{1}{1+\tau\text{S}} - \frac{\text{wLN}}{\lambda}\right)\right)}{\text{dr} + 2\,\text{i} + 2\,\mu} == \text{ai} * \text{wLN} * \text{d}$ // FullSimplify

Out[176]= $\frac{\text{cN}\,(\text{i}\,(-\text{wLN} + \lambda + \lambda\,\tau\text{N}) - \mu\,(\text{wLN} - \lambda + \text{wLN}\,\tau\text{S}))}{\text{i}\,\lambda\,(\text{dr} + 2\,(\text{i} + \mu))\,(1 + \tau\text{N})} == \text{ai d wLN}$

In[177]:= "This is the free entry in imitation condition with both sides divided by NN";

In[178]:= $\text{FEIM} = \frac{\left(\text{cN}\left(\frac{1}{1+\tau\text{N}} - \frac{1}{\text{wLN}}\right)\right) + \left(\text{cS} * \eta\text{S} * \left(1 - \frac{1}{\text{wLN} * (1+\tau\text{S})}\right)\right)}{\text{dr} + \text{i}} == \text{a}\mu * \text{d}$ // FullSimplify

Out[178]= $\frac{\text{cN}\,(\text{i}\,(-1 + \text{wLN} - \tau\text{N}) + \mu\,(-1 + \text{wLN} + \text{wLN}\,\tau\text{S}))}{\text{i}\,(\text{dr} + \text{i})\,\text{wLN}\,(1 + \tau\text{N})} == \text{a}\mu\ \text{d}$

In[179]:= Solve[{FEIN, FEIM}, {cN, wLN}] // FullSimplify

Out[179]= $\left\{\left\{\text{cN} \to \frac{\text{d i}\,\lambda\,(\text{a}\mu\,(\text{dr} + \text{i}) + \text{ai}\,(\text{dr} + 2\,(\text{i} + \mu)))\,(1 + \tau\text{N})}{(-1 + \lambda)\,(\text{i} + \mu + \mu\,\tau\text{S})},\ \text{wLN} \to \frac{\lambda\,(\text{a}\mu\,(\text{dr} + \text{i}) + \text{ai}\,(\text{dr} + 2\,(\text{i} + \mu)))\,(\text{i} + \mu + \text{i}\,\tau\text{N})}{(\text{a}\mu\,(\text{dr} + \text{i}) + \text{ai}\,\lambda\,(\text{dr} + 2\,(\text{i} + \mu)))\,(\text{i} + \mu + \mu\,\tau\text{S})}\right\}\right\}$

In[180]:= "-- By solving the FEIN and FEIM equations simultaneously for
cN and wLN, we obtain the above expressions. We enter these solutions in the program.";

In[181]:= $\text{cN} = \frac{\text{d i}\,\lambda\,(\text{a}\mu\,(\text{dr} + \text{i}) + \text{ai}\,(\text{dr} + 2\,(\text{i} + \mu)))\,(1 + \tau\text{N})}{(-1 + \lambda)\,(\text{i} + \mu + \mu\,\tau\text{S})}$; $\text{wLN} = \frac{\lambda\,(\text{a}\mu\,(\text{dr} + \text{i}) + \text{ai}\,(\text{dr} + 2\,(\text{i} + \mu)))\,(\text{i} + \mu + \text{i}\,\tau\text{N})}{(\text{a}\mu\,(\text{dr} + \text{i}) + \text{ai}\,\lambda\,(\text{dr} + 2\,(\text{i} + \mu)))\,(\text{i} + \mu + \mu\,\tau\text{S})}$;

In[182]:= "The unit cost of production is"; $\text{B} = \gamma * \text{wHN}^{\beta} * \text{wLN}^{(1-\beta)}$;
"Using Shephard's Lemma, the unit labor requirement in RPA for specialized and general purpose workers can be found as";

In[184]:= $\text{bH} = \gamma * \beta * \left(\frac{\text{wLN}}{\text{wHN}}\right)^{1-\beta}$; $\text{bL} = \gamma * (1 - \beta) * \left(\frac{\text{wHN}}{\text{wLN}}\right)^{\beta}$;

In[185]:= "This is the R&D and imitation difficulty per unit of NN, where x = X/NN and d = D/NN";

In[186]:= $\text{d} = \frac{\text{nN} * \delta * \text{x}}{\text{n}}$;

In[187]:= "Taking the ratio of the foc for RPA to free-entry in R&D gives the Relative Profitability Condition";

$$\text{RELP} = \frac{\text{vN}\,(\text{i}+\mu)}{\text{vN}} == \frac{\text{B} * \text{x} * \text{NN}}{\text{wLN} * \text{ai} * \text{d} * \text{NN}} \text{ // FullSimplify}$$

Out[187]=
$$\text{i} + \mu == \frac{\text{n}\,\text{wHN}^{\beta}\,\gamma\,(\text{i}+\mu)\left(\frac{\lambda\,(\text{a}\mu\,(\text{dr}+\text{i})+\text{ai}\,(\text{dr}+2\,(\text{i}+\mu)))\,(\text{i}+\mu+\text{i}\,\tau\text{N})}{(\text{a}\mu\,(\text{dr}+\text{i})+\text{ai}\,\lambda\,(\text{dr}+2\,(\text{i}+\mu)))\,(\text{i}+\mu+\mu\,\tau\text{S})}\right)^{-\beta}}{\text{ai}\,\text{i}\,\delta}$$

In[188]:= "Solve[RELP,wHN]//FullSimplify"; "This gives";

In[189]:=
$$\text{wHN} = \left(\frac{\text{ai}\,\text{i}\,\delta}{\text{n}\,\gamma}\right)^{\frac{1}{\beta}}\left(\frac{\lambda\,(\text{a}\mu\,(\text{dr}+\text{i})+\text{ai}\,(\text{dr}+2\,(\text{i}+\mu)))\,(\text{i}+\mu+\text{i}\,\tau\text{N})}{(\text{a}\mu\,(\text{dr}+\text{i})+\text{ai}\,\lambda\,(\text{dr}+2\,(\text{i}+\mu)))\,(\text{i}+\mu+\mu\,\tau\text{S})}\right);$$

In[190]:= "This is the Northern general purpose labor market equilibrium condition with both sides divided by NN.";

In[191]:=
$$\text{LABN} = \frac{(\text{cN}/(1+\tau\text{N}))+(\text{cS}*\eta\text{S})}{\lambda}\,\frac{\text{i}}{\text{i}+\mu} + (\text{ai}*\text{i}*\text{d}) + (\text{bL}*\text{nN}*\text{x}) == 1 - \text{sN} \text{ // FullSimplify}$$

Out[191]=
$$\text{sN} + \left(\text{i}\,\text{x}\left(\text{a}\mu\,\text{i}\,(\text{dr}+\text{i})\,\delta - \text{n}\,(-1+\beta)\,\gamma\left(\left(\frac{\text{ai}\,\text{i}\,\delta}{\text{n}\,\gamma}\right)^{\frac{1}{\beta}}\right)^{\beta}(-1+\lambda)\,(\text{i}+\mu) + \text{ai}\,\text{i}\,\delta\,(\text{dr}+(1+\lambda)\,(\text{i}+\mu))\right)\right) \Big/ \left(\text{n}\,(-1+\lambda)\,(\text{i}+\mu)^2\right) == 1$$

In[192]:= "This is the Northern specialized labor market equilibrium condition with both sides divided by NN";

$$\text{LABNSP} = \text{sN} == \text{bH}*\text{x}*\text{nN} \text{ // FullSimplify}$$

Out[193]=
$$\text{sN} == \frac{\text{i}\,\text{x}\,\beta\,\gamma\left(\left(\frac{\text{ai}\,\text{i}\,\delta}{\text{n}\,\gamma}\right)^{-1/\beta}\right)^{1-\beta}}{\text{i}+\mu}$$

In[194]:= Solve[LABNSP, x] // FullSimplify

Out[194]=
$$\left\{\left\{\text{x} \rightarrow \frac{\text{sN}\left(\left(\frac{\text{ai}\,\text{i}\,\delta}{\text{n}\,\gamma}\right)^{-1/\beta}\right)^{-1+\beta}(\text{i}+\mu)}{\text{i}\,\beta\,\gamma}\right\}\right\}$$

In[195]:= "We enter the expression for x ";

In[196]:=
$$\text{x} = \frac{\text{sN}\left(\left(\frac{\text{ai}\,\text{i}\,\delta}{\text{n}\,\gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta}(\text{i}+\mu)}{\text{i}\,\beta\,\gamma};$$

In[197]:= "Now, we write LABN with x substituted from above"; LABN // FullSimplify

Out[197]= $sN + \left(sN \left(\left(\frac{ai\, i\, \delta}{n\, \gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta} \left(a\mu\, i\, (dr + i)\, \delta - n\, (-1+\beta)\, \gamma \left(\left(\frac{ai\, i\, \delta}{n\, \gamma}\right)^{\frac{1}{\beta}}\right)^{\beta} (-1+\lambda)\, (i+\mu) + ai\, i\, \delta\, (dr + (1+\lambda)\, (i+\mu))\right)\right) \Big/ (n\, \beta\, \gamma\, (-1+\lambda)\, (i+\mu)) == 1$

In[198]:= "This is the Southern general purpose labor market equilibrium condition with both sides divided by NS.";

In[199]:= $LABS = \frac{\frac{cN}{\eta S} + \frac{cS}{(1+\tau S)}}{wLN} \frac{\mu}{i+\mu} + \left(a\mu * \mu * \frac{d}{\eta S} * nN\right) == 1$ // FullSimplify

Out[199]= $\frac{sN\, \delta \left(\left(\frac{ai\, i\, \delta}{n\, \gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta} \mu\, (a\mu\, (dr + i\, \lambda) + ai\, \lambda\, (dr + 2\, (i+\mu)))}{n\, \beta\, \gamma\, \eta S\, (-1+\lambda)\, (i+\mu)} == 1$

In[200]:= "LABN and LABS are now expressed in terms of i and μ only. Note that they are independent of tariffs and hence the tariff neutrality result holds. So, we can enter these as functions and construct a system that is ready for comparative statics. Note that for LABN we did a simplification(1/β)^β=1 to help Mathematica ";

In[201]:= FLABN[i_, μ_, dr_, ηS_, λ_, ai_, aμ_, δ_, n_, β_, sN_, τN_, τS_] :=

$sN + \frac{1}{n\, \beta\, \gamma\, (-1+\lambda)\, (i+\mu)} sN \left(\left(\frac{ai\, i\, \delta}{n\, \gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta} \left(a\mu\, i\, (dr + i)\, \delta - n\, (-1+\beta)\, \gamma \left(\frac{ai\, i\, \delta}{n\, \gamma}\right) (-1+\lambda)\, (i+\mu) + ai\, i\, \delta\, (dr + (1+\lambda)\, (i+\mu))\right) - 1$

In[202]:= FLABS[i_, μ_, dr_, ηS_, λ_, ai_, aμ_, δ_, n_, β_, sN_, τN_, τS_] := $\frac{sN\, \delta \left(\left(\frac{ai\, i\, \delta}{n\, \gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta} \mu\, (a\mu\, (dr + i\, \lambda) + ai\, \lambda\, (dr + 2\, (i+\mu)))}{n\, \beta\, \gamma\, \eta S\, (-1+\lambda)\, (i+\mu)} - 1$

In[203]:= "This gives the Jacobian Matrix";

In[204]:= J = {gradf[FLABN[i, μ, dr, ηS, λ, ai, aμ, δ, n, β, sN, τN, τS], {i, μ}],
gradf[FLABS[i, μ, dr, ηS, λ, ai, aμ, δ, n, β, sN, τN, τS], {i, μ}]};

In[205]:= FullSimplify[J] // MatrixForm;

In[206]:= "The resulting expressions, which are supressed due to their excessive length, suggest that the slopes of LABN and LABS are ambiguous. We check whether setting dr=0 can resolve the ambiguity";

In[207]:= **dr = 0; FullSimplify[J] // MatrixForm**

Out[207]//MatrixForm=

$$\begin{pmatrix} \frac{sN\,\delta \left(\left(\frac{ai\, i\, \delta}{n\,\gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta} \left(-ai\,(\beta\,(-1+\lambda)-2\,\lambda)\,(i+\mu)^2+a\mu\, i\,(i+\mu+\beta\,\mu)\right)}{n\,\beta^2\,\gamma\,(-1+\lambda)\,(i+\mu)^2} & -\frac{a\mu\, i^2\, sN\,\delta \left(\left(\frac{ai\, i\, \delta}{n\,\gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta}}{n\,\beta\,\gamma\,(-1+\lambda)\,(i+\mu)^2} \\ -\frac{sN\,\delta \left(\left(\frac{ai\, i\, \delta}{n\,\gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta} \lambda\,\mu \left(a\mu\, i\,(i\,(-1+\beta)-\mu)+2\, ai\,(-1+\beta)\,(i+\mu)^2\right)}{i\, n\,\beta^2\,\gamma\,\eta S\,(-1+\lambda)\,(i+\mu)^2} & \frac{sN\,\delta \left(\left(\frac{ai\, i\, \delta}{n\,\gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta} \lambda \left(a\mu\, i^2+2\, ai\,(i+\mu)^2\right)}{n\,\beta\,\gamma\,\eta S\,(-1+\lambda)\,(i+\mu)^2} \end{pmatrix}$$

In[208]:= **"-- LABN is upward sloping. Note that in the above expressions $\beta(-1+\lambda)-2\,\lambda < 0$,**
-- LABS is downward sloping. ";

In[209]:= **2. COMPARATIVE STATICS FOR ηS;**

In[210]:= **"This gives the gradient of the functions FLABN and FLABS with respect to ηS"; "We first set dr free"; dr =.;**

In[211]:= **B = {gradf[FLABN[i, μ, dr, ηS, λ, ai, aμ, δ, n, β, sN, τN, τS] , {ηS}],**
gradf[FLABS[i, μ, dr, ηS, λ, ai, aμ, δ, n, β, sN, τN, τS] , {ηS}]}

Out[211]= $\left\{\{0\}, \left\{-\frac{sN\,\delta \left(\left(\frac{ai\, i\, \delta}{n\,\gamma}\right)^{\frac{1}{\beta}}\right)^{1-\beta} \mu\,(a\mu\,(dr+i\,\lambda)+ai\,\lambda\,(dr+2\,(i+\mu)))}{n\,\beta\,\gamma\,\eta S^2\,(-1+\lambda)\,(i+\mu)}\right\}\right\}$

In[212]:= **"This follows from the Cramer's rule";**

In[213]:= **impactηS = -Inverse[J].B.{ΔηS};**

In[214]:= **MatrixForm[impactηS] // FullSimplify;**

In[215]:= **"The resulting expressions, which are suppressed due to their length,**
suggest that the terms have ambiguous signs. We therefore simplify by setting dr=0"; dr = 0;

In[216]:= **MatrixForm[impactηS] // FullSimplify**

Out[216]//MatrixForm=

$$\begin{pmatrix} -\dfrac{a\mu\, i^2\, \beta\, \Delta\eta S\, \mu\, (a\mu\, i + 2\, ai\, (i+\mu))}{\eta S\, (i+\mu)\, \left(-a\mu^2\, i^2 + ai\, a\mu\, i\, (i\, (\beta\, (-1+\lambda) - 2\, (1+\lambda)) - 4\, \mu) + 2\, ai^2\, (\beta\, (-1+\lambda) - 2\, \lambda)\, (i+\mu)^2\right)} \\ \dfrac{\Delta\eta S\, \mu\, (a\mu\, i + 2\, ai\, (i+\mu))\, \left(ai\, (\beta\, (-1+\lambda) - 2\, \lambda)\, (i+\mu)^2 - a\mu\, i\, (i+\mu+\beta\, \mu)\right)}{\eta S\, (i+\mu)\, \left(-a\mu^2\, i^2 + ai\, a\mu\, i\, (i\, (\beta\, (-1+\lambda) - 2\, (1+\lambda)) - 4\, \mu) + 2\, ai^2\, (\beta\, (-1+\lambda) - 2\, \lambda)\, (i+\mu)^2\right)} \end{pmatrix}$$

In[217]:= **{di, dμ} = %;**

In[218]:= **"Clearly $\frac{di}{d\eta S}$>0 and also $\frac{d\mu}{d\eta S}$>0 . Note that β (-1+λ)-2 λ < 0";**

In[219]:= **dwLN = D[wLN, i] di + D[wLN, μ] dμ + D[wLN, ηS] ΔηS // FullSimplify**

Out[219]=

$$\Big(i\, \Delta\eta S\, \lambda\, \mu\, (a\mu\, i + 2\, ai\, (i+\mu))\, (-a\mu\, i + ai\, (\beta\, (-1+\lambda) - 2\, \lambda)\, (i+\mu))\, \big(a\mu^2\, i^2\, (\tau N + \tau S + \tau N\, \tau S) + 4\, ai^2\, \lambda\, (i+\mu)^2\, (\tau N + \tau S + \tau N\, \tau S) + 2\, ai\, a\mu\, \big((-1+\lambda)\, \mu^2\, (1+\tau S) + i^2\, (-1+\lambda+2\, \lambda\, \tau N + (1+\lambda)\, (1+\tau N)\, \tau S) + 2\, i\, \mu\, (-1+\lambda\, (1+\tau N)\, (1+\tau S))\big)\big)\Big) \Big/ \Big(\eta S\, (a\mu\, i + 2\, ai\, \lambda\, (i+\mu))^2\, \big(a\mu^2\, i^2 - 2\, ai^2\, (\beta\, (-1+\lambda) - 2\, \lambda)\, (i+\mu)^2 + ai\, a\mu\, i\, (i\, (2+\beta+2\, \lambda - \beta\, \lambda) + 4\, \mu)\big)\, (i+\mu+\mu\, \tau S)^2\Big)$$

In[220]:= **"Thus, with dr→0, it follows that $\frac{dwLN}{d\eta S}$<0. ";**

In[221]:= **dm = D[m, i] di + D[m, μ] dμ + D[m, ηS] ΔηS;**

In[222]:= **dnN = D[nN, i] di + D[nN, μ] dμ + D[nN, ηS] ΔηS;**

In[223]:= **FullSimplify[{dm, dnN}] // MatrixForm**

Out[223]//MatrixForm=

$$\begin{pmatrix} \dfrac{i^2\, \Delta\eta S\, \mu\, (a\mu\, i + 2\, ai\, (i+\mu))\, (ai\, (\beta\, (-1+\lambda) - 2\, \lambda)\, (i+\mu) - a\mu\, (i+\beta\, \mu))}{\eta S\, (i+\mu)^2 \left(-a\mu^2\, i^2 + ai\, a\mu\, i\, (i\, (\beta\, (-1+\lambda) - 2\, (1+\lambda)) - 4\, \mu) + 2\, ai^2\, (\beta\, (-1+\lambda) - 2\, \lambda)\, (i+\mu)^2\right)} \\ \dfrac{i\, \Delta\eta S\, \mu\, (a\mu\, i + 2\, ai\, (i+\mu))\, (a\mu\, i - ai\, (\beta\, (-1+\lambda) - 2\, \lambda)\, (i+\mu))}{\eta S\, (i+\mu)^2 \left(-a\mu^2\, i^2 + ai\, a\mu\, i\, (i\, (\beta\, (-1+\lambda) - 2\, (1+\lambda)) - 4\, \mu) + 2\, ai^2\, (\beta\, (-1+\lambda) - 2\, \lambda)\, (i+\mu)^2\right)} \end{pmatrix}$$

In[224]:= **"Clearly $\frac{dm}{d\eta S}>0$, and also $\frac{dnN}{d\eta S}<0$ ";**

In[225]:= **"We again set dr free"; dr =.;**

In[226]:= **3. COMPARATIVE STATICS FOR τS;**

In[227]:=
```
BτS = {gradf[FLABN[i, μ, dr, ηS, λ, ai, aμ, δ, n, β, sN, τN, τS], {τS}],
  gradf[FLABS[i, μ, dr, ηS, λ, ai, aμ, δ, n, β, sN, τN, τS] , {τS}]}
```

Out[227]= {{0}, {0}}

In[228]:= **impactτS = -Inverse[J].BτS.{ΔτS};**

In[229]:= **MatrixForm[impactτS] // FullSimplify**

Out[229]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[230]:= **{di, dμ} = %;**

In[231]:= **"Clearly $\frac{di}{d\tau S}=0$ and also $\frac{d\mu}{d\tau S}=0$**
This is because there is no tariff expression in the FLABN and FLABS equations.";

In[232]:=
```
dwLN = D[wLN, i] di + D[wLN, μ] dμ + D[wLN, τS] ΔτS // FullSimplify
```

Out[232]=

$$-\frac{\Delta\tau S\,\lambda\,\mu\,(a\mu\,(dr+i)+ai\,(dr+2\,(i+\mu)))\,(i+\mu+i\,\tau N)}{(a\mu\,(dr+i)+ai\,\lambda\,(dr+2\,(i+\mu)))\,(i+\mu+\mu\,\tau S)^2}$$

In[233]:=
"Clearly $\frac{dwLN}{d\tau S}<0$.
Tariff reduction by the South increases the North-South wage gap ";

In[234]:=
```
dm = D[m, i] di + D[m, μ] dμ + D[m, τS] ΔτS;
```

In[235]:=
```
dnN = D[nN, i] di + D[nN, μ] dμ + D[nN, τS] ΔτS;
```

In[236]:=
```
FullSimplify[{dm, dnN}] // MatrixForm
```

Out[236]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[237]:=
"Clearly $\frac{dm}{d\tau S}=0$ and also $\frac{dnN}{d\tau S}=0$
Again, this is because tariffs do not affect i and μ.";

In[238]:=
4. COMPARATIVE STATICS FOR τN;

In[239]:=
```
BτN = {gradf[FLABN[i, μ, dr, ηS, λ, ai, aμ, δ, n, β, sN, τN, τS] , {τN}],
  gradf[FLABS[i, μ, dr, ηS, λ, ai, aμ, δ, n, β, sN, τN, τS], {τN}]}
```

Out[239]= {{0}, {0}}

In[240]:=
```
impactτN = -Inverse[J].BτN.{ΔτN};
```

In[241]:= `MatrixForm[impactτS] // FullSimplify`

Out[241]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[242]:= `{di, dμ} = %;`

In[243]:= "Clearly $\frac{di}{d\tau N}$=0 and also $\frac{d\mu}{d\tau N}$=0
This is because there is no tariff expression in the FLABN and FLABS equations.";

In[244]:= `dwLN = D[wLN, i] di + D[wLN, μ] dμ + D[wLN, τN] ΔτN // FullSimplify`

Out[244]=

$$\frac{i \, \Delta\tau N \, \lambda \, (a\mu \, (dr + i) + ai \, (dr + 2 \, (i + \mu)))}{(a\mu \, (dr + i) + ai \, \lambda \, (dr + 2 \, (i + \mu))) \, (i + \mu + \mu \, \tau S)}$$

In[245]:= "Clearly $\frac{dwLN}{d\tau N}$>0.
Tariff reduction by the North reduces the North-South wage gap ";

In[246]:= `dm = D[m, i] di + D[m, μ] dμ + D[m, τN] ΔτN;`

In[247]:= `dnN = D[nN, i] di + D[nN, μ] dμ + D[nN, τN] ΔτN;`

In[248]:= `FullSimplify[{dm, dnN}] // MatrixForm`

Out[248]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[249]:= "Clearly $\frac{dm}{d\tau N}$=0 and also $\frac{dnN}{d\tau N}$=0
Again, this is because tariffs do not affect i and μ.";

Additional References For The Appendices

Dinopoulos, E. and Thompson, P., 1996. A Contribution to the Empirics of Endogenous Growth. Eastern Economic Journal 22(4), 389-400.

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