R&D Policies, Endogenous Growth and Scale Effects*

by

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Abstract

This paper constructs a scale-free endogenous growth model and studies the determinants of optimal R&D policy. The model combines two of the main approaches to removal of scale effects: the rent protection approach and the diminishing technological opportunities approach. In addition, the model allows for inter-industry knowledge spillovers and inter-industry rent-protection spillovers.

The steady-state rate of innovation is a function of all of the model's parameters including the R&D subsidy/tax rate. Thus, growth is fully endogenous. The effective replacement rate faced by incumbent firms is an increasing function of the innovation-deterring elasticity of rent protection activities, which is also endogenously determined. Numerical simulations imply that it is optimal to tax R&D when innovations are of very small and very large magnitudes, and to subsidize R&D when innovations are of medium size. Under a wide range of empirically relevant calibrations, the subsidy rate turns out to be positive and fluctuates between 5 to 25 percent.

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1. INTRODUCTION

Endogenous growth theory came at a crossroads with the Jones critique in the mid 1990s. First generation endogenous growth models predicted that the long-run growth rate of an economy increases in the level of R&D inputs and thus larger economies should grow at higher rates.[1] In two influential papers Jones (1995a, b) refuted this scale effect prediction by examining the post-war time series data from industrialized countries. In response, a second generation of endogenous growth models has emerged. This literature offers three main approaches to remove scale effects: *i)* Diminishing Technological Opportunities (henceforth **DTOs**) put forward by Jones (1995b), Kortum (1997) and Segerstrom (1998); *ii)* Rent Protection Activities (henceforth **RPAs**) proposed by Dinopoulos and Syropoulos (2006), *iii)* Variety Expansion (henceforth **VE**) proposed by Aghion and Howitt (1998, ch. 12), Dinopoulos and Thompson (1998), Howitt (1999), Peretto (1998) and Young (1998).[2]

In all of the above papers, growth is endogenous in the sense that it is driven by the innovation efforts of profit maximizing entrepreneurs. However the determinants of the steady-state growth rates differ markedly across these approaches. Models based on the DTO approach imply that the steady-state growth rate is exclusively pinned down by the rate of population growth and the rate of exhaustion in technological opportunities, leaving no room for R&D policies to exert an influence. Therefore, these models are often referred to as *semi-endogenous* growth models. In contrast, models using the RPA or VE approach predict that the steady-state growth rate is a function of *all* of the model's parameters including the R&D subsidy/tax rate. Thus, these models are often referred to as *fully-endogenous* growth models.

These stark differences in terms of steady-state outcomes are important not only in their own right but also because of their welfare implications. In a typical endogenous growth model, the search for welfare-maximizing optimal R&D policy involves the comparison of positive and negative externalities associated with a marginal unit of innovation. In the DTO based models, with only a small subset of parameters determining the rate of innovation, the majority of the parameters have no influence on the magnitudes of innovation externalities via their effect on the innovation rate. In contrast, in the RPA or VE based models, the entire set of the parameters do exert an influence through this particular channel. To see the implications for optimal R&D policy, compare for instance

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[1] See Romer (1990), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991a) and Aghion and Howitt (1992).

[2] See Dinopoulos and Sener (2006) for a recent analysis of scale-invariant growth theory. Jones (2006), Dinopoulos and Thompson (1999), and Jones (1999) also provide comprehensive overviews of the scale-invariant endogenous growth literature.

the results from the semi-endogenous growth model of Segerstrom (1998) and fully-endogenous growth model of Dinopoulos and Syropoulos (2006). Segerstrom (1998) finds that for small-sized innovations either R&D taxes or subsidies are optimal, whereas for sufficiently large-sized innovations R&D taxes are welfare maximizing. Quite the contrary, Dinopoulos and Syropoulos (2006) find that R&D taxes are optimal for small and large sized innovations, and R&D subsidies are optimal only for medium-sized innovations. It is easy to find more papers in this literature with major differences in R&D policy recommendations.[3]

Motivated by the above considerations I intend combine the DTO and RPA approaches under a unified setting and explore the implications for steady-state growth and R&D policy. I restrict the focus of the paper to these two approaches in order to facilitate the paper's comparison with the literature. The DTO approach captures the essence of the semi-endogenous growth theory, whereas the RPA approach captures the essence of the fully-endogenous growth theory. Incorporating the VE approach can of course be a fruitful avenue, which for now is left for further research.

Such a unified model can shed light on several important questions central to endogenous growth theory. When we combine the elements that give rise to fully-endogenous growth with those that give rise to semi-endogenous growth, will growth be fully endogenous or semi endogenous? Does the model point to taxes or subsidies as the optimal R&D policy? How do the externalities associated with marginal innovation respond to changes in parameters? When one calibrates the model what is the nature and extent of the optimal R&D policy?

The model is based on a standard quality-ladders growth setting in the tradition of Grossman and Helpman (1991, ch. 4). The economy is characterized by a continuum of structurally-identical industries. In each industry, entrepreneurs participate in R&D races to innovate higher quality products. The winner of an R&D race establishes monopoly power as the quality leader of the industry. Further innovation in the industry implies the emergence of a new quality leader and hence the replacement of the incumbent firm. The replacement rate faced by the incumbent firms is equal to the rate of innovation ι, which is endogenously determined by the profit-maximizing decisions of entrepreneurs.

I allow for positive population growth and remove the scale effects by introducing R&D difficulty at the industry level. I model R&D difficulty as a stock variable whose evolution is governed by four distinct forces. First, as in Dinopoulos and Syropoulos (2006), RPAs undertaken by the incumbent monopolist in a particular industry raise the level of R&D difficulty for outside

[3] See Segerstrom (2006) and Li (2001, 2003) for a comparative analysis of R&D policies implied by different endogenous growth models.

entrepreneurs who target their innovation efforts at this industry. These are costly activities which require the employment of specialized workers such as lawyers and lobbyists. Second, as in Segerstrom (1998, p.1297), within each industry, "the most obvious ideas are discovered first, making it harder to find new ideas subsequently." This sets in motion the DTO mechanism by which current innovation efforts raise the level of R&D difficulty for the subsequent periods. Third, as in Li (2003), inter-industry knowledge spillovers can reduce the level of R&D difficulty in a typical industry by improving the overall quality of technologies available to entrepreneurs. Fourth, inter-industry rent protection spillovers (a brand new consideration) can raise the level of R&D difficulty in a typical industry by hindering the access of entrepreneurs to current technologies.

The model commands a unique steady-state equilibrium in which the rate of innovation remains constant in the presence of positive population growth. Thus, steady-state growth is free of scale effects. The equilibrium rate of innovation responds to all of the model's parameters including the R&D subsidy rate. Hence, the model predicts *fully-endogenous growth*. This is the first central result of the paper.

Even though there is no consensus on whether fully-endogenous or semi-endogenous growth better captures the real world, recent empirical studies by Ha and Howitt (2006) and Zachariadis (2003, 2004) lends more support for the former versus the latter. Ha and Howitt (2006) find that the predictions of the fully-endogenous growth theory (in particular, that the growth rate is a function of the fraction of the resources allocated to R&D and is endogenous) are more consistent with the time-series patterns from advanced countries vis-à-vis the predictions of the semi-endogenous growth theory (in particular, that the growth rate must follow the growth rate in R&D inputs). [4] My paper provides an additional insight by showing that the empirical evidence that favors fully-endogenous growth over semi-endogenous growth does *not* necessarily imply a rejection of the foundations of semi-endogenous growth theory. More specifically, the present model shows that the DTO mechanism that is at the heart of semi-endogenous growth theory can indeed be compatible with fully-endogenous growth when R&D difficulty accumulation has an added component that operates through rent protection by incumbents.

At the steady-state equilibrium of the model, the replacement rate as used in the stock market valuation of the incumbent firm *effectively* increases from ι to $\iota[1 + \eta]$, where η is an elasticity term that measures in percentage terms the effectiveness of RPAs in deterring outside innovation. This

[4] Ha and Howitt (2006) also conduct a comparison of the cointegration relations implied by each theory and conclude that fully-endogenous growth theory outperforms the semi-endogenous growth theory. See this paper and the references therein for an overview of the competing perspectives on this issue.

elasticity is *endogenous* and responds to all of the model's parameters. Hence the model establishes a novel link between the model's parameters and the *effective replacement rate* $\iota[1 + \eta]$ faced by the incumbent firm. This is the second central result of the paper. In Segerstrom (1998), RPAs are not considered and thus $\eta = 0$; whereas in Dinopoulos and Syropoulos (2006), there are RPAs but no consideration of DTOs and inter-industry knowledge/rent protection spillovers; more specifically, their model implies $\eta = 1$.

To understand the implications for R&D policy, I solve the optimization problem of a social planner whose objective is to maximize social welfare. I find that a marginal innovation generates three competing effects on welfare: a positive *consumer-surplus* externality, a negative *business-stealing* externality and a negative *intertemporal R&D spillover* externality. In this setting, the model's parameters influence these externalities directly and also indirectly via their effects on the innovation rate ι and innovation-deterring elasticity η.

The forces at work can be best understood by comparing the model with the two most related models. In Dinopoulos and Syropoulos (2006) there is no consideration of DTOs; therefore, the intertemporal R&D spillover externality is absent. In Segerstrom (1998) all of the three externalities are present; however, the innovation rate is exclusively determined by the two parameters, namely, the population growth rate and the exhaustion rate in technological opportunities. Thus, in Segerstrom there is no room for the rest of the parameters to influence the welfare externalities through the innovation rate. Lastly, in both Segerstrom (1998) and Dinopoulos and Syropoulos (2006) η is fixed; thus, the parameters do not influence the externalities through their effect on η.

Using benchmark values from the U.S., I calibrate the model and calculate the magnitude of optimal R&D policy. The benchmark simulations, taken at face value, imply that it is optimal to subsidize R&D at a rate of 15 percent. I check the robustness of this result by considering high and low values for all parameters. When the size of innovations is set at 1.25 (which implies a 25 percent price-marginal cost mark up) and each parameter is allowed to vary within a 30 percent band, the optimal R&D subsidy rate fluctuates roughly between 10 and 25 percent. When the size of innovations is kept within the interval [1.20, 1.35] (which holds the share of R&D employment below 7 percent) and each parameter is allowed to vary within a band that keeps the innovation rate within the range [0.01, 0.04], the optimal R&D subsidy rate remains between 5 and 25 percent.

Allowing for a wider range of innovation size [1.10, 2.75], I find that it is optimal to tax R&D when innovations are of very small and very large magnitudes, and it is optimal to subsidize R&D when innovations are of medium size. This "n-shaped" relationship is a robust feature of the model and is exclusively tied to the fully-endogenous growth nature of the model. This finding implies that combining the RPA approach of Dinopoulos and Syropoulos (2006) with the DTO approach of Segerstrom (1998) resurrects the n-shaped relationship, which was originally proposed by Grossman and Helpman (1991). I also investigate the response of optimal R&D policy to other parameter changes and provide a comparative analysis with respect to the related literature.

The rest of the paper is organized as follows. Section 2 introduces the building blocks of the model and establishes the steady-state equilibrium. Section 3 conducts a comparative steady-state analysis. Section 4 discusses the optimal R&D policies. Section 5 presents the simulation results. Section 6 concludes. Proofs of all propositions are relegated to Appendices, which are available on my website http://minerva.union.edu/senerm/.

2. THE MODEL

2.1. The household's utility maximization

The economy consists of a continuum of identical households with measure of one. The size of each household at time t is $N(t) = e^{nt}$, where the initial level of population is normalized to one and $n > 0$ denotes the population growth rate. Each household takes goods prices, wages, and the interest rate as given and maximizes the following utility function over an infinite horizon

$$U = \int_0^\infty e^{-(\rho - n)t} \log u(t)\, dt\,, \tag{1}$$

where ρ is the subjective discount rate, and $\log u(t)$ is the instantaneous utility of each household member defined as:

$$\log u(t) \equiv \int_0^1 \log \left[\sum_j \lambda^j\, x(j,\omega,t)\right] d\omega\,, \tag{2}$$

where $x(j,\omega,t)$ is the quantity demanded of a product with quality j in industry ω at time t. The size of quality improvements is denoted by $\lambda > 1$. Therefore, the total quality of a good after j innovations is λ^j.

Each household allocates its per capita consumption expenditure $c(t)$ to maximize $u(t)$ given prices at time t. Equation (2) implies that within each industry, products of different quality are perfect substitutes; thus, in each industry households purchase only the product with the lowest quality-adjusted price. Since products enter the utility function symmetrically, households spread their consumption expenditure evenly across the continuum of product lines. Consequently, demand for each product line by a household member is $x(j,\omega,t) = c(t)/P_m$ where P_m is the market price for the product that has the lowest quality-adjusted price.

Given the static demand behavior, the household's dynamic problem is simplified to maximizing

$$\int_0^{\infty} e^{-(\rho - n)t} \log c(t)\, dt, \qquad (3)$$

subject to the budget constraint $\dot{A}(t) = W(t) + r(t)A(t) - c(t)N(t)$, where $A(t)$ denotes the financial assets owned by the household, $W(t)$ is the family's expected wage income and $r(t)$ is the instantaneous rate of return.[5] The solution to this optimization gives the standard differential equation

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho\,. \qquad (4)$$

2.2. Activities and Labor Assignment

Firms conduct three types of activities: innovation, manufacturing of final goods and rent-protection. Labor is the only factor of production and there are two types of labor: general-purpose and specialized labor. General-purpose workers can be employed in either manufacturing or innovation, whereas specialized workers can only be employed in RPAs.[6] The population share of general-purpose workers is $(1 - s)$ and that of specialized workers is s, where $s \in (0,1)$.

2.3. R&D Races

The economy consists of a continuum of structurally-identical industries indexed by $\omega \in (0,1)$. Entrepreneurs participate in industry-specific R&D races to innovate higher quality products. An R&D race in industry ω is aimed at improving the quality of the existing product by a fixed size $\lambda > 1$. The winner of an R&D race gains access to the technology of producing the next-generation product and establishes monopoly power in the product market. Further innovation in the industry results in the replacement of the incumbent firm by a new quality leader.

The arrival of innovations in each industry is governed by a stochastic Poisson process, whose intensity is determined by the profit maximizing decisions of entrepreneurs. Let $R_j(\omega, t)$ represent the innovation intensity of a typical entrepreneur indexed by j targeting industry ω. The instantaneous probability of innovation success by firm j is

[5] I assume that, intra-household transfers ensure that per-capita consumption expenditure is the same for all household members when individually earned wages may differ.

[6] I assume labor mobility between manufacturing and R&D to model the economy's ability to allocate resources in different activities over the long run. This is a standard assumption in the growth literature. On the other hand, I assume a constant share of specialized labor to capture the established institutional set up associated with rent protection activities. With specialized labor, I basically mean lawyers, lobbyists and other individuals who possess rent-protection-activity-specific expertise which is not applicable to manufacturing or R&D. This particular labor assignment scheme follows Dinopoulos and Syropoulos (2006). If one instead assumes complete labor mobility, then the model does not provide a suitable setting to analyze the welfare implications of RPAs. The reason is that in this case the social planner taxes RPAs prohibitively high and thus no labor performs RPAs. The model effectively boils down to that of Segerstrom (1998) as far as welfare implications are concerned.

$$\iota_j(\omega,t) = \frac{R_j(\omega,t)}{D(\omega,t)}, \qquad (5)$$

where $D(\omega,t)$ measures the difficulty of conducting innovation in industry ω. The probability of innovation success is distributed independently across firms and industries. Thus, the instantaneous probability of innovation success at the industry level equals

$$\iota(\omega,t) = \sum_j \iota_j(\omega,t) = \frac{R(\omega,t)}{D(\omega,t)}, \qquad (6)$$

where $R(\omega,t) = \Sigma_j R_j(\omega,t)$.

In each industry, the incumbent monopolist (i.e., the quality leader) hires specialized labor to deter the innovation efforts of outside entrepreneurs. Let $X(\omega,t)$ stand for the level of RPAs undertaken by the incumbent firm in industry ω. Summing up $X(\omega,t)$ and $\iota(\omega,t)$ across structurally-identical industries gives the aggregate level of RPAs as $X_A(t) = \int_0^1 X(\omega,t)d\omega$ and the aggregate rate of innovation as $\iota_A(t) = \int_0^1 \iota(\omega,t)d\omega$.

I model $D(\omega,t)$ as a stock variable with the following equation of motion: [7]

$$\dot{D}(\omega,t) = \delta X(\omega,t) + \mu\iota(\omega,t)D(\omega,t) + \delta_A X_A(t) - \sigma(\dot{Q}/Q)\, D(\omega,t), \qquad (7)$$

where $\delta > 0$, $\mu > 0$, $\delta_A \geq 0$ and $\sigma \geq 0$ are exogenously given. Equation (7) implies that four distinct forces govern the evolution of $D(\omega,t)$ over time.[8]

The first term $\delta X(\omega,t)$ captures the rent protection effect a la Dinopoulos and Syropoulos (2006). The RPAs undertaken by the monopolist firm operating in industry ω raise the stock of R&D difficulty faced by the entrepreneurs who target their R&D efforts at industry ω. Rent protection activities can involve excessive patenting activities, patent enforcement through litigation, practicing

[7] This differs from Dinopoulos and Syropoulos (2006) who model R&D difficulty as a flow variable. I considered the stock formulation for two reasons. First, evidence from the real world suggests that rent protection activities have effects that persist over time since these activities influence the legislative and judicial system. Second, the stock formulation provides a convenient analytical template to combine the DTO and RPA approaches and accommodate inter-industry knowledge/rent protection spillovers.

[8] Equation (7), the equation of motion for $D(\omega,t)$, with its additive form provides a generalized equation of motion for R&D difficulty. When $\delta = \delta_A = \sigma = 0$ and $\mu > 0$, the formulation boils down to that of Segerstrom (1998). When $\mu = \delta_A = \sigma = 0$ and $\delta > 0$, the formulation captures the RPA approach of Dinopoulos and Syropoulos (2006). To generate a positive level of $D(t)$ at the steady-state, the necessary and sufficient conditions are: *i)* the initial level of R&D difficulty at time zero $D_0(t)$ being strictly positive, *ii)* either δ or μ being strictly positive.

trade secrecy, lobbying the government to affect legislation, engaging in corrupt activities to influence the legal/political system, and so on.[9]

The second term $\mu\iota(\omega,t)D(\omega,t)$ captures the DTO effect a la Segerstrom (1998), Jones (1995) and Kortum (1997). In Segerstrom's words (1998, p. 1297) the idea is that "firms start off exploring the *ex ante* most promising projects and when success does not materialize, they gradually switch to *ex ante* less promising projects." Thus, current research efforts raise the stock of R&D difficulty for subsequent periods.

The third term $\delta_A X_A(t)$ introduces spillovers from economy-wide RPAs to industry-level R&D difficulty. Several examples of such spillovers can be observed in the real world. Consider patent fencing activities by quality leaders. A prime example in this regard is du Pont's patenting of over 200 substitute products to protect its major innovation Nylon in the 1940s. Such excessive patenting limits not only the access of du Pont's direct competitors to the leading technology but also of firms operating in related industries. In the same vein consider a patent infringement case vigorously fought by an incumbent firm against a new patent in a chemicals industry on a particular processing method. Conceivably, such litigation efforts can force the entrepreneurs in other industries (such as those in biomedicine, pharmaceuticals and etc.) to seek alternative processing methods and thereby face more research difficulty.

Finally, the fourth the term $-\sigma[\dot{Q}(t)/Q(t)]D(t)$ introduces inter-industry knowledge spillovers a la Li (2003) and Kortum (1997).[10] In this expression $Q(t) = \int_0^1 \lambda^{j\omega} d\omega$ stands for the aggregate quality index. Continuous growth in Q can reduce research difficulty in industry ω by raising the aggregate quality of goods/technologies available to researchers. The growth rate of $Q(t)$ equals $(\lambda -$

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[9] For detailed empirical evidence on RPAs, see Dinopoulos and Syropoulos (2006), Sener (2006) and the references therein. See also Levin et al. (1987) and Cohen et al. (2000) for survey-based evidence from the US manufacturing industries. Some empirical evidence from the US can be briefly presented here. According to AIPLA (1997) direct legal costs of patent litigation range between \$1.0 and \$3.0 million (in 1997 dollars) for each side through the trial. Lerner (1995, p. 470) reports that the costs of patent litigation cases started in 1991 will account for 27% of total R&D expenditures of US companies in that year. Time series analysis of Somaya (2002, Figures 3 and 5) suggests that patent litigation has been pervasive in all six broad industries as classified by the USPTO. In a survey of biotech firms Lerner (1995) finds that 55 percent of small firms and 33 percent of large firms cite litigation as a *deterrent* to innovation.

[10] It should be noted that the model does not exhaust all possible dimensions of R&D spillovers. Li (2000) constructs a model in which products can be improved in both *quality* and *variety* dimensions. He considers inter-industry R&D spillovers within and across these dimensions. In my paper, there is no variety improvement, so the spillovers introduced by $\sigma(\dot{Q}/Q)\,D(\omega,t)$ work only through the quality to quality channel across industries. See Li (2002) for a model that considers R&D in a k-dimensional space with $k \geq 2$. The first dimension is variety and the $k-1$ dimensions are quality attributes of each variety.

$1)\iota_A(t)$; hence, $\sigma[\dot{Q}(t)/Q(t)]D(t)$ can be simplified to $\mu_A \iota_A \geq 0$, where $\mu_A = \sigma(\lambda - 1) \geq 0$ is a constant parameter. [11, 12]

2.4. Product Markets

The quality leader in industry ω can produce a product that is λ times better than the current-generation product. Manufacturing one unit of final good requires one unit of general-purpose labor regardless of the quality level of the product. I normalize the wage rate of general-purpose labor $w_G(t)$ to one and hence the unit cost of production. In each industry, a quality leader competes against a follower who can produce the product that is one step down in the quality ladder. Firms compete in prices. The quality leader engages in limit pricing by charging $P = \lambda$ and forces the follower to exit the market. The quality leader's monopoly profits from product sales are:

$$\pi^P(\omega,t) = \frac{\lambda - 1}{\lambda} c(\omega,t)N(t) \tag{8}$$

where $\lambda - 1$ is the profit margin per unit of product and $c(\omega,t)N(t)/\lambda$ is the total demand for product in industry ω. During its tenure, the monopolist firm hires specialized workers to deter the innovation efforts of its rivals. Let γ represent the unit labor requirement of such rent protection activities let $w_S(t)$ represent the wage rate of specialized labor. The total cost of conducting $X(\omega,t)$ units of RPA is $\gamma w_S(t)X(\omega,t)$. With $w_G(t)$ normalized to one, the relative wage between specialized and general purpose labor can be stated as $w(t) \equiv w_S(t)/w_G(t) = w_S(t)$. The monopolist's profit flow net of rent protection costs boils down to: [13]

$$\pi(\omega,t) = \frac{\lambda - 1}{\lambda} c(\omega,t)N(t) - w(t)\gamma X(\omega,t)\,. \tag{9}$$

2.5. Stock Markets

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[11] By the law of large numbers $\dot{Q}(t) = \int_0^1 \iota(\omega,t)(\lambda^{j\omega+1} - \lambda^{j\omega})d\omega = \int_0^1 \iota(\omega,t)[\lambda^{j\omega}(\lambda - 1)]d\omega$. Using $Q(t) = \int_0^1 \lambda^{j\omega} d\omega$ and structural symmetry, it follows that $\dot{Q}(t)/Q(t) = (\lambda - 1)\iota_A(t)$.

[12] Another interpretation is that the term $-\mu_A \iota_A \sigma(\lambda - 1)D(t)$ captures the industry-level *depreciation* in the stock of R&D difficulty as a result of advancements in the aggregate economy. This deprecation is linked to the aggregate rate of innovation and thus endogenously determined. Observe that this is a more flexible setting compared to that of Dinopoulos and Syropoulos (2006) where R&D difficulty is modeled as a flow variable and thus the depreciation rate is 100 percent. One can also introduce to this setting an exogenous rate of depreciation for R&D difficulty, *DEPR*, with $0 \leq DEPR < 1$. The main results are robust to the inclusion of exogenous depreciation, which is omitted to economize on the notation.

[13] As in the standard quality-ladders growth model, it is not profitable for the monopolist to undertake R&D in order to extend its lead over the followers [see for instance Grossman and Helpman (1991c), p. 93].

There exists a stock market that channels the savings of consumers to firms. Consider the stock market valuation of a quality leader $\upsilon(\omega,t)$ operating industry ω at time t. Over a small time interval of dt, the stockholders of the quality leader receive $\pi(\omega,t)dt$ in the form of dividend payments. During the same time period, with probability $\iota(\omega,t)dt$, an outside entrepreneur successfully innovates the next generation product. In this event, the stockholders face a capital loss in the amount of $\upsilon(\omega,t)$. With probability $1 - \iota(\omega,t)dt$, no innovation takes place in the industry, and the stockholders realize an appreciation/depreciation in their holdings by $\dot{\upsilon}(\omega,t)$. In the absence of any arbitrage opportunities, the expected rate of return from holding stocks issued by the quality leader must be equal to the risk-free market rate of return $r(t)$. This implies:

$$\frac{\pi(\omega,t)}{\upsilon(\omega,t)}dt + (1-\iota(\omega,t)dt)\frac{\dot{\upsilon}(\omega,t)}{\upsilon(\omega,t)}dt + \frac{0-\upsilon(\omega,t)}{\upsilon(\omega,t)}\iota(\omega,t)dt = r(t)dt. \qquad (10)$$

Taking limits as $dt \rightarrow 0$ yields:

$$\upsilon(\omega,t) = \frac{\pi(\omega,t)}{r(t)+\iota(\omega,t)-\frac{\dot{\upsilon}(\omega,t)}{\upsilon(\omega,t)}}. \qquad (11)$$

2.6. Free-entry in R&D races

Entrepreneurs hire general-purpose labor to perform innovative activity and participate in R&D races. With w_G normalized to one, the cost of conducting $R_j(\omega,t)$ units of R&D for a typical entrepreneur indexed by j equals $a_\iota R_j(\omega,t)$, where a_ι is the unit labor requirement of R&D. The expected profits of an entrepreneur targeting its innovation efforts at industry ω are:

$$\upsilon(\omega,t)\frac{R_j(\omega,t)}{D(\omega,t)}dt - a_\iota(1-\phi_\iota)R_j(\omega,t)dt \qquad (12)$$

where ϕ_ι is the R&D subsidy rate offered by the government. Over a time interval dt, the entrepreneur realizes a value of $\upsilon(\omega, t)$ with probability $\iota_j(\omega, t)= R_j(\omega, t)/D(\omega, t)dt$ and incurs a cost of $a_\iota(1 - \phi_\iota)R_j(t)dt$. Free entry into R&D races drives the expected profits down to zero. This implies:

$$\frac{\upsilon(\omega,t)}{D(\omega,t)} = a_\iota(1-\phi_\iota). \qquad (13)$$

2.7. Optimal rent protection decisions

The RPAs undertaken by the incumbent firm prolong their monopoly power and thus raise the expected returns on their stocks. The incumbents choose the optimal level of $X(\omega,t)$ by equating the incremental gain in the expected return on their stocks to the incremental cost incurred to hire the additional specialized workers.

To obtain the associated first order condition I first derive the response of R&D intensity that targets industry ω, $\iota(\omega,t)$, to changes in the level of innovation-deterring activities undertaken by the incumbent in industry ω, $X(\omega,t)$. Consider an increase in the level of rent protection activity by dX, beginning at time t and extending over a small time interval dt. This increases the accumulation rate for $D(\omega,t)$ and lowers $\iota(\omega,t)$ via (6). Let $d\iota(\omega,t)$ measure the resulting change in $\iota(\omega,t)$ over the interval dt. To evaluate $d\iota(\omega,t)/ dX(\omega,t)$, the first step is to evaluate the difference in the R&D difficulty levels between time t and $t + dt$, $D(\omega, t + dt) – D(\omega, t)$, due to increased $X(\omega,t)$. Two effects are at work. One is the direct effect that operates through the rent-protection channel. The higher $X(\omega,t)$ increases the rate of accumulation for $D(\omega,t)$ over the interval dt, raising the R&D difficulty level at time $t + dt$. The other is the indirect effect that operates through the DTO channel. The reduction in $\iota(\omega,t)$ by $d\iota$ units over the interval dt decreases the rate of accumulation for $D(\omega,t)$, lowering the R&D difficulty level at time $t + dt$.

To capture the above effects, I need to evaluate the following two terms:

$$D_X \equiv \lim_{\substack{dt\to 0\\ dX\to 0}} \frac{\left[(D(\omega,t+dt)-D(\omega,t))\,\middle|\, X_s = X_t + dX \;\; for\; s\in(t,t+dt)\right]-\left[(D(\omega,t+dt)-D(\omega,t))\,\middle|\, X_s = X_t \;\; for\; s\in(t,t+dt)\right]}{dtdX}$$

which measures the change in D conditional on X being increased by dX units over the interval dt. The second is:

$$D_\iota \equiv \lim_{\substack{dt\to 0\\ d\iota\to 0}} \frac{\left[(D(\omega,t+dt)-D(\omega,t))\,\middle|\, \iota_s = \iota_t + d\iota \;\; for\; s\in(t,t+dt)\right]-\left[(D(\omega,t+dt)-D(\omega,t))\,\middle|\, \iota_s = \iota_t \;\; for\; s\in(t,t+dt)\right]}{dtd\iota}$$

which measures the change in D conditional on ι being increased by $d\iota$ units over the interval dt. It immediately follows from (7) that:

$$D_X \equiv \frac{\partial\left(\frac{dD(\omega,t)}{dt}\right)}{\partial X} = \delta \;\text{ and }\; D_\iota \equiv \frac{\partial\left(\frac{dD(\omega,t)}{dt}\right)}{\partial \iota} = \mu D(\omega,t)\,. \tag{14}$$

Hence $dD(\omega) = D_X dX\, dt + D_\iota d\iota\, dt$, given the changes in X and ι as dX and $d\iota$ over the time interval dt. Totally differentiating $\iota(\omega,t)$ then implies:

$$dtd\iota = -\frac{R(\omega,t)}{D(\omega,t)^2}(D_X dX + D_\iota d\iota)dt\,. \tag{15}$$

Substituting for D_X and D_ι from (14), using (6), and taking limits as $dt \to 0$ gives:

$$\frac{d\iota}{dX} = -\frac{\delta\, R(\omega,t)}{D(\omega,t)^2\,[1+\mu(R(\omega,t)/D(\omega,t))]} = -\frac{\delta\,\iota(\omega,t)}{D(\omega,t)[1+\mu\iota(\omega,t)]}\,, \tag{16}$$

which provides an expression for $d\iota/dX$ and completes the first step of the analysis. Note for future use that according to (16), a diminishing returns relationship exists between $D(\omega,t)$ and $|d\iota/dX|$: as the stock of R&D difficulty increases, the effectiveness of rent protection activity declines. Moreover, (16) implies the following:

Lemma 1: *Ceteris paribus, the presence of DTOs as captured by* $\mu\iota > 0$ *reduces the marginal effectiveness of RPAs in deterring innovation (as measured by* $|d\iota/dX|$*). I call this "the DTO-RPA interaction mechanism."*

Intuitively, whenever an incumbent firm raises its RPAs and thus deters innovation, it indirectly mitigates the impact of DTOs on research difficulty. This is a novel mechanism that arises from the joint modeling of RPAs and DTOs.

The second step is to evaluate the change in the expected return on the incumbent's stocks due to the fall in $\iota(\omega,t)$ induced by an increase in $X(\omega,t)$ over a time interval dt in the amount dX. Differentiating $\pi(\omega,t)dt + [0 - \upsilon(\omega,t)]\,\iota(\omega,t)dt + \dot{\upsilon}\,[1 - (\iota(\omega,t)dt]dt$ with respect to $\iota(\omega,t)$ yields the incremental gain in the expected return as:

$$-\upsilon(\omega,t)\frac{d\iota}{dX}dXdt - \dot{\upsilon}\frac{d\iota}{dX}dXdtdt\,. \tag{17}$$

At the optimal level of $X(\omega,t)$, this must equal the incremental expenditure on specialized labor $w(t)\gamma dXdt$ over a time interval dt. Imposing this condition and taking limits as $dt \to 0$ gives:

$$-\upsilon(\omega,t)\frac{d\iota}{dX} = w(t)\gamma\,. \tag{18}$$

Substituting for $d\iota/dX$ from (16) into (18) yields the first order condition for optimal $X(\omega,t)$:

$$\frac{\delta\iota(\omega,t)\upsilon(\omega,t)}{D(\omega,t)[1+\mu\iota(\omega,t)]} = w(t)\gamma\,. \tag{19}$$

2.8. Labor Markets

Demand for general-purpose labor comes from manufacturing and R&D. In each industry, entrepreneurs hire $R(\omega,t)a_\iota$ units of labor to conduct innovative activity, and the incumbent firm hires $c(t)N(t)/\lambda$ units of labor for manufacturing purposes. The economy-wide demand for general purpose labor is $\int_0^1 [R(\omega,t)a_\iota + c(t)N(t)/\lambda]\,d\omega = R_A(t)a_\iota + c(t)N(t)/\lambda$. The equilibrium condition for the general-purpose labor market can then be stated as:

$$(1-s)N(t) = R_A(t)a_\iota + \frac{c(t)N(t)}{\lambda}\,. \tag{20}$$

Demand for specialized labor comes from RPAs. In each industry, incumbent firm hires $\gamma X(\omega,t)$ units of specialized labor to conduct such activities. The economy-wide demand for specialized labor is $\int_0^1 \gamma X(\omega,t)\,d\omega = \gamma X_A(t)$. The equilibrium condition for the specialized labor market then becomes:

$$sN(t) = \gamma X_A(t). \tag{21}$$

2.9. Steady-State Equilibrium

I now solve the model for a steady-state equilibrium in which all endogenous variables attain strictly positive values and the rate of innovation $\iota(t)$ remains constant over time. The stability of this equilibrium is shown in Appendix A. At the steady-state $c(t)$, $w(t)$ and $r(t)$ remain constant over time, and $X(t)$, $\upsilon(t)$, $D(t)$, and $\pi(t)$ grow at the rate of n. From this point on, I drop the time index for the variables that remain constant at the steady-state.

Given the structurally-symmetry and measure one of industries, it follows that $R_A(t)= R(\omega,t)= R(t)$, $\iota_A = \iota(\omega,t)= \iota$ and $X_A(\omega,t) = X(\omega,t) = X(t)$. To simplify notation, I henceforth drop the industry index ω as well. Imposing $\dot{D}/D = n$ on equation (7) and solving for $D(t)$ gives:

$$D(t) = \frac{(\delta + \delta_A)X(t)}{n - \iota(\mu - \mu_A)}, \qquad \Rightarrow \qquad \frac{D(t)}{X(t)} = \frac{(\delta + \delta_A)}{n - \iota(\mu - \mu_A)} \tag{22}$$

which implies that $D(t) > 0$ requires $\iota < n/(\mu - \mu_A)$. I make the reasonable assumption that $\mu - \mu_A > 0$, that is, the DTO effects within an industry prevail over inter-industry knowledge spillovers. Let $\eta(\iota)$ be defined as:

$$\eta(\iota) \equiv -\frac{d\iota}{dX}\frac{X}{\iota}$$

where $\eta(\iota)$ represents *the innovation-deterring elasticity of RPAs*. Substituting for $d\iota/dX$ from (16) and $D(t)$ from (22) into the $\eta(\iota)$ above gives:

$$\eta(\iota) \equiv \frac{1}{(1 + \mu\iota)}\frac{\delta X(t)}{D(t)} = \frac{\delta[n - \iota(\mu - \mu_A)]}{(\delta + \delta_A)(1 + \mu\iota)}, \tag{23}$$

Lemma 2: *At the steady-state, the partial derivatives of the innovation-deterring elasticity $\eta(\iota)$ are as follows:*

- *$\partial\eta(\iota)/\partial\iota < 0$ because of two effects. First, a higher ι increases $D(t)/X(t)$ through the DTO channel via (22). Second, a higher ι triggers the "DTO-RPA interaction mechanism" as identified in Lemma 1. Both effects reduce the effectiveness of rent protection and hence $\eta(\iota)$.*
- *$\partial\eta(\iota)/\partial\mu < 0$ because of the same two effects identified above.*
- *$\partial\eta(\iota)/\partial\mu_A > 0$ because a higher μ_A reduces $D(t)/X(t)$ and increases the effectiveness of rent protection and thus $\eta(\iota)$.*
- *$\partial\eta(\iota)/\partial\delta_A < 0$ because a higher δ_A increases $D(t)/X(t)$ and reduces the effectiveness of rent protection and thus $\eta(\iota)$.*

Lemma 2 demonstrates the endogeneity of $\eta(\iota)$. Observe that the joint modeling of DTOs and RPAs play a crucial role in generating this outcome. In Dinopoulos and Syropoulos (2006), $\eta(\iota) = 1$, and in Segerstrom (1998), $\eta(\iota) = 0$. [14] Hence, endogeneity of $\eta(\iota)$ constitutes a major departure from the literature where $\eta(\iota)$ is modeled a rigid parameter.

Substituting for $D(t)$ from (22) into (19) simplifies the first order condition for optimal $X(t)$ as:

$$w\gamma X(t) = \iota\eta(\iota)\upsilon(t). \qquad (24)$$

Equation (24) has a straightforward interpretation. At the steady-state, the level of rent protection expenditures $w\gamma X(t)$ increases when the incumbent faces a larger threat of replacement (higher ι), when RPAs become more effective (higher $\eta(\iota)$), or when the incumbent has more capital loss at stake due to replacement (higher $\upsilon(t)$). It follows from Lemma 2 and (24) that one can further decompose the effects that operate through the $\eta(\iota)$ channel. On the one hand, outside innovation efforts ($\iota > 0$), DTOs ($\mu > 0$) and inter-industry rent protection spillovers ($\delta_A > 0$) have a restraining impact on RPA expenditures since they reduce $\eta(\iota)$. On the other hand, inter-industry knowledge spillovers ($\mu_A > 0$) have an encouraging impact on RPA expenditures since they increase $\eta(\iota)$. Intuitively, when either ι, μ or δ_A is larger or when μ_A is smaller, the stock of R&D difficulty in a given industry attains a higher level on its own. This renders the incumbent firm's rent protection efforts less effective and thereby reduces $\eta(\iota)$. In addition, when ι or μ is larger, this triggers the RPA-DTO interaction mechanism identified in Lemma 1, which also works to reduce $\eta(\iota)$.

Substituting $\pi(t)$ from (9) into (11) using $w\gamma X(t)$ from (24) gives the stock market valuation of the firms as:

$$\upsilon(t) = \frac{[(\lambda - 1)/\lambda]cN(t)}{\rho - n + \iota[1 + \eta(\iota)]}. \qquad (25)$$

In equation (25), the numerator is the incumbent's profit flow from product sales and the denominator is the adjusted discount rate which takes into account the replacement rate faced by the incumbent firm.

Lemma 3: *The effective replacement rate faced by a monopolist equals* $\iota[1 + \eta(\iota)]$.

[14] I should note that imposing $\mu = \mu_A = \delta_A = 0$ gives $\eta(\iota) = n$. This differs from Dinopoulos and Syropoulos (2006), where $\eta(\iota) = 1$. In Dinopoulos and Syropoulos (2006), R&D difficulty is modeled as a flow variable and thus the effectiveness of innovation deterring $-d\iota/dX$ does not get discounted by getting multiplied by n when the steady-state level of $D(t)$ is substituted. In the present paper, I model R&D difficulty as a stock variable and assume that firms choose their optimal RPA levels by looking dt periods ahead. Thus, when $D(t)$ is substituted from (22) into (16), the n term pops up as a coefficient of discount for $-d\iota/dX$. Under the stock formulation, if one assumes instead that incumbents have a perfect foresight of the steady-state equilibrium and choose their rent protection efforts $X(t)$ based on their steady-state impact, one obtains $\eta(\iota) = 1$ here as well. The main results are robust to this alternative behavioral assumption.

Lemma 3 establishes a novel negative link between innovation-deterring elasticity $\eta(\iota)$ and the stock market valuation of quality leaders. Intuitively, any increase in η, say by $d\eta$ units, *holding all else constant*, raises the effectiveness of RPAs and induces the monopolist to increase its expenditure on rent protection by $\iota\upsilon(t)d\eta$ units *at each point in time* [via (24)]. This incremental expenditure flow leads to a fall in the firm's stock market valuation, which amounts to an increase in the effective replacement rate by $\iota d\eta$ units [via (25)].[15] Thus, shocks to μ, δ_A, and μ_A as well as changes in the endogenous innovation rate ι have an additional impact on firm value through the $\eta(\iota)$ term.

I establish the steady-state equilibrium in (c, ι) space by obtaining two steady-state relationships: the competitive equilibrium free-entry in R&D condition, $\mathbf{RD^{CE}}$, and the general purpose labor market equilibrium condition **LM**. Substituting $\upsilon(t)$ from (25), $D(t)$ from (22), $\eta(\iota)$ from (23) and $X(t)$ from (21) into (13) gives RD^{CE}:

$$\frac{(\delta+\delta_A)s}{\gamma[n-\iota(\mu-\mu_A)]}a_\iota(1-\phi_\iota)=\frac{[(\lambda-1)/\lambda]c}{\rho-n+\iota\left[1+\dfrac{\delta(n-\iota(\mu-\mu_A))}{(\delta+\delta_A)(1+\mu\iota)}\right]} \quad \mathbf{RD^{CE}} \qquad (26)$$

Clearly, $(dc/d\iota)\Big/_{RD^{CE}} > 0$. For a given c, an increase in ι exerts two main effects on R&D profitability. First, it increases R&D difficulty $D(t)$ through the DTO channel and renders R&D more costly. Second, an increase in ι, increases the effective replacement rate $\iota[1+\eta(\iota)]$ despite the mitigating factor stemming from the fall in $\eta(\iota)$ (see Lemma 2). This reduces the rewards from R&D. Both effects work to decrease R&D profitability. To restore equilibrium, the rewards from R&D must increase through an increase in c.

I now derive the LM condition. Note that in (20) $R(t) = \iota D(t)$ follows from (6) and structural symmetry. Substituting for $D(t)$ from (22) into (20) using $X(t) = sN(t)/\gamma$ from (21) gives:

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[15] Here one may wonder why firms undertake RPAs if this effectively increases their replacement rate from ι to $\iota(1+\eta(\iota))$. This is not a proper assessment though because ι is endogenous and its equilibrium levels may differ across models. In the scale-dependent endogenous growth models, ι increases with the population size N. Thus, the replacement rate can increase without bound if there is population growth! In the present model, ι does not depend on N, but on the rate of growth in $N(t)$ (see Proposition 1). Hence, the replacement rate remains constant in the presence of population growth. On the other hand, it is possible to make a comparison with Segerstrom's (1998) scale-free growth model. To simplify, I set the spillover parameters to zero $\mu_A = \delta_A = 0$. Segerstrom's replacement rate is $\iota = n/\mu$. Using the expression for $\eta(\iota)$ from (23), it can be easily shown that $\iota(1+\eta(\iota)) < n/\mu$ holds when the condition for an interior equilibrium with positive RPAs $\iota < n/\mu$ is satisfied. Note that for $D > 0$, it follows from (22) that $n - \iota\mu < 0$ must hold. Intuitively, the presence of RPAs generates an additional factor that contributes to the accumulation of R&D difficulty and pushes the innovation rate that sustains balanced growth downward. To see this, differentiate (6) with respect to time and use (7) along with the steady-state conditions.

$$(1-s)=\frac{\iota(\delta+\delta_A)s}{\gamma(n-\iota(\mu-\mu_A))}a_\iota+\frac{c}{\lambda} \qquad \textbf{LM} \qquad (27)$$

Clearly, $(dc/d\iota)\,/_{LM} > 0$. For a given c, an increase in ι directly raises the demand for R&D labor. At the same time, a higher ι raises $D(t)$ through the DTO channel, increasing the resource requirement in R&D. Both effects work to increase the demand for general purpose labor. To restore labor market equilibrium, labor demand must fall through a decline in c.

I illustrate the steady-state equilibrium in Figure 1 with the intersection of the RD^{CE} and LM curves in (ι, c) space. Denote with "*" the steady-state equilibrium levels. It is straightforward to show that there exists a unique equilibrium for (ι^*, c^*) under the parametric condition: $(\lambda-1)(1-s)\gamma n > sa_\iota(\rho-n)(\delta+\delta_A)(1-\phi_\iota)$.[16] One can then determine the equilibrium values for the rest of the endogenous variables in a recursive fashion. Substituting c^* into (8) gives $\pi_P{}^*(t)$. Substituting $X^*(t)$ from (21) and ι^* into (22) gives $D(t)^*$. Substituting ι^* and c^* into (25) gives $\upsilon^*(t)$. To find w^*, substitute $\upsilon/D(t)$ from (13) into (19) and solve for w. This gives: $w^* = a_\iota\delta(1-\phi_\iota)\iota^*/\gamma(1+\mu\iota^*)$.

3. COMPARATIVE STATICS

Using the LM and RD^{CE} conditions and Figure 1, it is straightforward to establish the following:

Proposition 1: *The steady-state innovation rate* ι^*

- *increases with the innovation size* λ*, the R&D subsidy rate* ϕ_ι*, the inter-industry knowledge spillover rate* μ_A*, the population growth rate n, and the unit labor requirement in rent protection* γ*,*
- *decreases with the population share of specialized labor s, the rent-protection effectiveness parameter* δ*, the inter-industry rent protection spillover rate* δ_A*, the subjective discount rate* ρ*, and the unit labor requirement in R&D* a_ι*,*
- *changes in an ambiguous direction with the rate at which DTOs accumulate* μ*.*

Proposition 1 implies that the steady-state growth rate is fully endogenous. In other words, the entire set of parameters including the R&D subsidy rate exerts an influence on the rate of innovation. Qualitatively, these results mirror those of Dinopoulos and Syropoulos (2006); however, Dinopoulos and Syropoulos (2006) do not have results for μ, μ_A, and δ_A. On the other hand, these results differ in a

[16] Note that on the LM curve, as $\iota\to 0$, $c\to\lambda(1-s)$ and as $\iota\to\iota^{max}=n/(\mu-\mu_A)$, $c\to-\infty$. On the RD^{CE} curve, as $\iota\to 0$, $c\to c^0=\lambda sa_\iota(1-\phi_\iota)(\rho-n)(\delta+\delta_A)/(\lambda-1)n\gamma$ and as $\iota\to\iota^{max}=n/(\mu-\mu_A)$, $c\to\infty$. Hence, for a unique equilibrium, we need to have the intercept of the LM curve be strictly higher than that of the RD^{CE} curve: $\lambda(1-s)>c_0 \Rightarrow \lambda-1>[sa_\iota(1-\phi_\iota)(\rho-n)(\delta+\delta_A)]/[(1-s)n\gamma]$

major way from Segerstrom (1998) where $\iota^* = n/\mu$ and thus variations in γ, s, δ, μ_A, and δ_A exert no influence on ι^*.[17, 18]

To highlight the new features of my model, I discuss only the shocks to μ, μ_A and δ_A. An increase in μ exerts two competing effects on R&D profitability. First, it increases the marginal cost of R&D by raising the level of R&D difficulty $D(t)$. Second, it increases the rewards from R&D by reducing the innovation deterring elasticity $\eta(\iota)$ and thereby the effective replacement rate. The net impact on innovation profitability and thus on the RD^{CE} curve is ambiguous. On the other hand, in the general purpose labor market a larger μ raises $D(t)$, and for a given c, this leaves fewer resources for innovation. Thus, the LM curve shifts to the left. The ambiguous effect of an increase in μ differs from Segerstrom (1998) where $d\iota^*/d\mu < 0$. In my model, a larger μ introduces a new effect by reducing the innovation-deterring elasticity and thereby raising the value of a successful firm. If this effect is sufficiently large then the FE curve shifts right and it becomes theoretically possible to have $d\iota^*/d\mu > 0$. Numerical simulations imply that for a wide range of parameters, the elasticity effect turns out to be quite modest and thus $d\iota^*/d\mu < 0$ holds.

The steady-state impact of changes in δ_A and μ_A are more straightforward. A larger δ_A raises the marginal cost of R&D by increasing $D(t)$. It also increases the rewards from R&D by reducing $\eta(\iota)$ and thus the effective replacement rate. The net impact is a reduction in R&D profitability. For a given c, this requires a fall in ι, shifting the RD^{CE} curve to the left. In the general purpose labor market, a larger δ_A raises $D(t)$ and thus the R&D labor requirement. For a given c, this requires a fall in ι, shifting the LM curve to the left. At the new equilibrium, the rate of innovation attains a lower level. The effects of an increase in μ_A work through the same channels but simply in the opposite direction.

4. WELFARE ANALYSIS

I now consider the problem of a social planner who allocates the economy's resources to maximize consumer's welfare over an infinite horizon as measured by (1). Recall from consumer's static optimization that in each industry consumers buy only the highest-quality good and per capita demand for each good is given by $x(j, \omega, t) = c(t)/\lambda$. Substituting this into (2) gives:

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[17] In an extension of Segerstrom's (1998) model, Li (2003) introduces inter-industry knowledge spillovers and finds that $d\iota^*/d\mu^A > 0$.

[18] Even though Segerstrom (1998) predicts that steady-state growth responds only to n and μ, it is worth pointing out that in his model's transition path, all of the parameters play an active role in affecting the endogenous variables. In particular, during the transition phase $d\iota/d\lambda > 0$, $d\iota/d\phi_\iota > 0$, $d\iota/da_\iota < 0$, and $d\iota/d\rho < 0$, which are in line with Proposition 1.

$$log\ u(t) = \int_0^1 \ log\lambda^{j(\omega,t)}d\omega + log[c(t)/\lambda]. \qquad (28)$$

Consider now the social planner's allocation decision of a *given* amount of aggregate R&D resources across industries at time t. The planner's goal is to maximize the growth rate of the first term in (28) $\frac{d\int_0^1 log\ \lambda^{j(\omega,t)}d\omega}{dt} = log\ \lambda \int_0^1 \iota(\omega,t)d\omega = log\ \lambda \int_0^1 \frac{R(\omega,t)}{D(\omega,t)}d\omega$, where I have used (6) for $\iota(\omega,t)$. Hence, for a given level of $X(\omega,t)$, the planner devotes all R&D resources to industries with the lowest $D(\omega,t)$; similarly, for a given level of $\iota(\omega,t)$, the planner devotes all specialized labor to industries with the lowest $D(\omega,t)$. Over time, this will imply $D(\omega,t) = D(t)$, $\iota(\omega,t) = \iota(t)$, $X(\omega,t) = X(t)$ for all ω and t. With $\frac{d\int_0^1 log\ \lambda^{j(\omega,t)}d\omega}{dt} = \iota(t) log\ \lambda$, it follows that $\int_0^1 \ log\lambda^{j(\omega,t)}d\omega = log\lambda\Phi(t)$, where $\Phi(t) = \int_0^t \iota(\tau)d\tau$ stands for the expected number of innovations before time t. Thus, the instantaneous utility at time t captured by (28) boils down to:

$$log\ u(t) = \Phi(t) log\lambda + log[c(t)/\lambda]. \qquad (29)$$

Using the general purpose labor market condition along with $D(\omega,t) = D(t)$ and $\iota(\omega,t) = \iota(t)$ and measure one of industries, it follows that $c(t)/\lambda = (1 - s) - a_\iota\, d(t)\, \iota(t)$, where $d(t)=D(t)/N(t)$ stands for per capita R&D difficulty. Substituting (29) into (1) using the expression for $c(t)/\lambda$, I can now state the social planner's problem as:

$$\max_{\{\iota\}} \int_0^\infty e^{-(\rho-n)t}\{\Phi(t) log\ \lambda + log[1 - s - a_\iota d(t)\iota(t)]\}dt \qquad (30)$$

subject to the state equations $\dot{\Phi} = \iota(t)$ and $\dot{d} = [(\delta + \delta_A)s/\gamma] + (\mu - \mu_A)\iota(t)d(t) - nd(t)$; the initial conditions $\Phi(0) = 0$, $d(0) = d_0 > 0$; and the control constraint, $(1 - s)/a_\iota d(t) \geq \iota(t) \geq 0$ for all t. To derive the $\dot{d}$ equation I have used (7), (21), $\dot{N}/N = n$, along with $\iota(\omega,t) = \iota(t)$, $X(\omega,t) = X(t)$ and measure one of industries.

I solve this optimization problem in Appendix B. I find that there exists a unique balanced growth solution for c and ι characterized by (27) and the *socially optimum R&D condition* given by:

$$\frac{(\delta+\delta_A)s}{\gamma[n-\iota(\mu-\mu_A)]}\frac{a_\iota\rho}{[\rho-\iota(\mu-\mu_A)]} = \frac{[log\ \lambda/\lambda]c}{\rho-n}. \qquad \mathbf{RD^{SO}} \qquad (31)$$

Observe that the RD^{SO} equation is the analog of RD^{FE}, this time though, the marginal cost of and marginal returns from R&D are measured from the perspective of the social planner. In particular, the RD^{SO} and RD^{FE} conditions differ with respect to three terms:

- $log\lambda$ as the consumer's valuation of a higher quality good in RD^{SO} versus $\lambda - 1$ as the profit margin enjoyed by a successful innovator in RD^{CE},
- $\rho - n$ as the discount factor of a representative household in RD^{SO} vs. $\rho - n + \iota[1 + \eta(\iota)]$ as the replacement-rate-adjusted discount factor of a quality leader in RD^{CE},
- $\rho /[\rho - \iota(\mu - \mu_A)] > 1$ as a coefficient that magnifies the social planner's perceived R&D cost in RD^{SO} versus a coefficient of unity in RD^{CE}.

With $\lambda > 1$ and $\iota > 0$, it is clear that the socially optimum and competitive equilibrium outcomes differ and thus market intervention becomes desirable. Let ~ represent the socially optimal levels of endogenous variables. RD^{SO} and LM conditions determine $\tilde{\iota}$ and $\tilde{c}$, and RD^{FE} and LM conditions determine the competitive equilibrium levels ι^* and c^*. To replicate the socially optimum outcome, the optimal R&D policy must imply $\iota^* = \tilde{\iota}$ and $c^* = \tilde{c}$. Given that the LM condition is the same, this simply requires that R&D^{FE} imply $\iota^* = \tilde{\iota}$, in which case $c^* = \tilde{c}$ would hold automatically. Substituting for c from RD^{SO} into RD^{FE} and using the expression for $\eta(\iota)$ gives the equation that characterizes the optimal subsidy rate ϕ_ι^{SO} as:

$$(1-\phi_\iota^{SO})\frac{log\,\lambda}{\rho - n} = \frac{(\lambda - 1)}{\left[\rho - n + \iota\left(1 + \dfrac{\delta[n - \iota(\mu - \mu_A)]}{(\delta + \delta_A)(1 + \mu\iota)}\right)\right]}\frac{\rho}{[\rho - \iota(\mu - \mu_A)]} \tag{32}$$

It is easy to see from (32) that depending on the parameters of the model, the optimal policy can be a tax $\phi_\iota^{SO} < 0$, or a subsidy $0 < \phi_\iota^{SO} < 1$. The parameters λ, ρ, n, μ, μ_A, δ and δ_A exert a direct impact on the subsidy rate. Moreover, all of the parameters influence ϕ_ι^{SO} indirectly through their effect on ι. For a generic parameter α, it follows that $d\phi_\iota^{SO}/d\alpha = \partial\phi_\iota^{SO}/\partial\alpha + (\partial\phi_\iota^{SO}/\partial\iota)(d\iota/d\alpha)$, where the partials come from (32) and $d\iota/d\alpha$ comes from Proposition 1. It is easy to show that $\partial\phi_\iota^{SO}/\partial\iota >< 0$ and further substitution of the partial derivatives do not resolve the ambiguity. In the end, the analytical relationship between ϕ_ι^{SO} and the model's parameters remains indeterminate.

In Segerstrom (1998), $\iota = n/\mu$; thus, no parameter other than n and μ exerts an indirect influence on ϕ_ι^{SO} through altering ι. Moreover, in his setting, $\delta = \delta_A = \mu_A = 0$ and $\eta(\iota) = 0$; therefore, rent protection and inter-industry knowledge/rent protection spillovers do not enter into the social planner's decision. In Dinopoulos and Syropoulos (2006), all of their model's parameters play a role in determining ι and thus exert an indirect effect on ϕ_ι^{SO} through this channel. However, in their model $\eta(\iota) = 1$; hence, RPAs exerts a rigid impact on ϕ_ι^{SO}. Moreover, in Dinopoulos and Syropoulos (2006), $\mu = \mu_A = \delta_A = 0$; thus, neither DTOs nor inter-industry knowledge/rent protection spillovers enter into the social planner's decision.

4.1. Marginal Welfare Analysis

To understand the forces at work, I follow Grossman and Helpman (2001) and Segerstrom (1998) and consider the effects of a marginal innovation by an external entrepreneur on welfare as measured by (1). This equals (see Appendix C for the derivation):

$$MU_{\Phi} = \frac{dU}{d\Phi} = \underbrace{\frac{\log\lambda}{\rho - n}}_{CS} - \underbrace{\frac{\lambda - 1}{\rho - n + \iota(1+\eta(\iota))}}_{BS} - \underbrace{\overbrace{\frac{\lambda - 1}{[\rho - n + \iota(1+\eta(\iota))]}}^{BS} \times \overbrace{\frac{\iota(\mu - \mu_A)}{[\rho - \iota(\mu - \mu_A)]}}^{Spillover\ \ Comp.}}_{IS} \qquad (33)$$

The first term in (33) measures the *consumer surplus externality* (henceforth **CS**). With each additional innovation, consumers enjoy a higher level of utility because product quality increases and yet prices remain constant. Furthermore, these utility gains accumulate over time because each successful innovation adds to the knowledge base and paves the way for the subsequent R&D race that is aimed at innovating the next-generation product. Entrepreneurs do not take into account in their R&D decisions these utility gains that accrue to consumers over an infinite horizon. Hence, the CS effect captures *a positive externality* associated with additional innovation, calling for an R&D subsidy. This externality increases with λ and decreases with $(\rho - n)$.

The second term in (33) measures the *business stealing externality* (henceforth **BS**). In each industry, successful innovation implies the replacement of the incumbent producer with a new quality leader. As a result, the stockholders of the incumbent firm suffer a loss in their asset valuations, which equals the expected discounted value of the forfeited stream of monopoly profits. Consequently, incomes and consumer expenditures decline for all industries. This creates a multiplier effect, further lowering incomes and expenditures and so on. Entrepreneurs do not take into account in their R&D decisions the losses incurred by the incumbent firms and its reverberations throughout the economy. Thus, the BS term measures *a negative externality* associated with additional innovation, calling for an R&D tax.

How does the BS externality respond to variations in parameters? On the one hand, for a given ι, the BS externality increases with λ and n, and decreases with ρ.[19] On the other hand, ceteris paribus, an increase in ι, increases the effective replacement rate $\iota[1+\eta(\iota)]$ despite the mitigating factor coming from the lower $\eta(\iota)$. Since ι and $\eta(\iota)$ are tied to all of the parameters (Lemma 2 and Proposition 1), a shock to any of the model's parameter can affect BS through these channels. In Segerstrom, $\iota = n/\mu$, and $\eta = 0$; thus, both channels are effectively muted, except for shocks to n and

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[19] The result with regards to λ is obtained by considering the impact of λ on inter-industry spillovers via the spillover coefficient $\mu_A = \sigma(\lambda - 1)$.

μ which only works through the ι channel. In Dinopoulos and Segerstrom (2005), ι depends on all of the parameters but η turns out t o equal to one. Thus, the channel through ι is open; but, the indirect channel that works through η is completely muted.

The third term in (33) measures the *intertemporal R&D spillover externality* (henceforth **IS**) associated with DTOs. R&D investment by entrepreneurs in the current period leaves ex-ante less promising projects for future entrepreneurs and raises the difficulty of research in the subsequent periods. This implies that more resources are devoted to R&D and fewer resources remain for the production of final goods. In equilibrium, lower production translates into lower consumption expenditure and thus lower profits. This triggers a multiplier effect, further decreasing incomes and expenditures and so on. Entrepreneurs do not take into account in their R&D decisions the negative implications of their current research activities for future innovation efforts and their reverberations throughout the economy. Thus, the IS term captures a *negative externality* associated with marginal innovation, calling for R&D taxes. The first component of the IS externality $(\lambda - 1)/[\rho - n + \iota(1+\eta(\iota)]$ is essentially the BS externality and thus exhibits the same responses to the parameters as identified above. The second component $\iota(\mu - \mu_A) / [\rho - \iota(\mu - \mu_A)]$, which I will refer as the *spillover component*, decreases with ρ and increases with $\iota(\mu - \mu_A)$. Recall that $\iota(\mu - \mu_A)$ measures the rate at which DTOs accumulate within each industry adjusted for inter-industry knowledge spillovers. The spillover factor decreases with λ and σ [note that $\mu_A = \sigma(\lambda - 1)$].

4.2. The relationship between marginal welfare and the parameters

Would it be possible to sign $dMU_\Phi/d\alpha$ for a generic parameter α? Observe that each parameter influences the welfare innovation externalities directly and also indirectly via their impact on ι. Often times the direct and indirect effects work against each other *within* each externality. Moreover, the sign of $\partial MU_\Phi/\partial\iota$ is indeterminate to begin with and thus the indirect effect as a whole is ambiguous. Consequently, it does not appear to be feasible to sign $dMU_\Phi/d\alpha$ analytically. However, it proves useful to contrast the underlying mechanisms with those from the literature. To do this, I focus on the impact of four parameters λ, μ, μ_A and δ_A.

4.3. Innovation size λ and marginal welfare

First, I consider the direct effects. For a given ι, a higher λ increases the positive CS effect by raising the utility gains of consumers from each innovation. It also increases the negative BS externality by magnifying the profit margins. This impact is mitigated (but not overturned) by the increased magnitude of inter-industry knowledge spillovers induced by a higher λ.[20] At the same time, a higher λ generates an ambiguous impact on the negative IS externality. More specifically, a higher λ increases the BS component of this externality but at the same time reduces its spillover component by raising μ_A.[21] Second, I consider the indirect effects of a higher λ that operates through increasing ι. A larger ι increases the effective replacement rate and thereby reduces the negative BS externality (note that $d[\iota(1 + \eta(\iota)]/d\iota > 0$). On the other hand, a larger ι generates an ambiguous impact on the IS externality. In particular, a larger ι, induced by a higher λ reduces its BS component and at the same time increases its spillover component. Observe that within the BS and IS externalities, all indirect effects of a higher λ work against the direct effects. With multiple competing effects present one cannot obtain an analytical result on relationship between the optimal subsidy rate ϕ_ι^{SO} and the innovation size λ.

In Segerstrom (1998), ι is independent of λ and hence the indirect effects are muted. Moreover, with $\mu_A = 0$, no inter-industry knowledge spillovers are considered, and with $\delta = \delta_A = 0$, no interaction between inter-industry knowledge spillovers and DTOs are observed. Hence, Segerstrom's model considers only the direct effects identified above (excluding the direct effect of λ that operates through the inter-industry knowledge spillover channel). In Segerstrom's model, given $d[(\lambda - 1)log\lambda]/d\lambda > 0$ *for* $\lambda > 1$, it follows that ϕ_ι^{SO} is a decreasing function of λ, where (32) is used for the derivation.

In Dinopoulos and Syropoulos (2006), a change in λ exerts both a direct impact and an indirect impact through ι on marginal welfare. However, in their setting $\mu = 0$, and thus the intertemporal R&D spillover effects are absent in their entirety. Hence, λ's direct and indirect effects through the IS externality disappear. In addition, in Dinopoulos and Syropoulos, with no inter-industry knowledge spillovers and no DTOs, $\eta(\iota) = 1$; thus changes in λ exert no influence through this channel. Their model implies a n-shaped relationship between the subsidy rate ϕ_ι^{SO} and innovation size λ. This means that at low levels of λ, ϕ_ι^{SO} increases with λ and at high levels of λ, ϕ_ι^{SO} decreases with λ.

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[20] Note that a higher λ raises μ_A and thus $\eta(\iota)$. This in turn raises the effective replacement rate $\iota(1+\eta(\iota))$, *mitigating* the rise in the business stealing effect. To see this, totally differentiate the business stealing effect with respect to λ.

[21] When we assume that $\sigma = 0$, that is, the case no inter-industry knowledge spillovers, a higher λ increases the CS effect, which is a positive externality and thus calls for a subsidy. Second, a higher λ increases the BS and IS effect which is a negative externality and thus calls for tax. One can show that around the optimal R&D policy the combined impact of BS and IS effects dominates the CS effect.

4.4. DTO parameter μ and marginal welfare

I now consider the effects on other parameters. I will be brief as the mechanisms are similar to the ones observed for λ. Consider an increase in μ. First, I analyze the direct effects holding ι constant. It follows from Lemma 2 that a larger μ reduces $\eta(\iota)$ and thereby the effective replacement rate. This increases the negative BS and IS externalities. In addition, a larger μ directly increases the spillover component in the negative IS externality. With all negative externalities increasing, a larger μ's direct impact calls for a decline in the R&D subsidy rate. Second, I consider the indirect effects that operate through ι. For illustrative purposes I assume that $d\iota^*/d\mu < 0$. A decline in ι induced by the higher μ increases the negative BS externality. At the same time, a lower ι increases the BS component and reduces the spillover component of the negative IS externality, exerting a net ambiguous effect. To sum up, within the BS externality, the indirect effect works against the direct effect, whereas within the IS externality, the indirect effect may work for or against the direct effect. Again, with multiple competing effects present one cannot obtain an analytical relationship between ϕ_ι^{SO} and μ.

In Segerstrom (1998), with $\iota = n/\mu$, the effects of changes in μ on marginal welfare are more straightforward. A higher μ reduces ι, leading to a fall in the replacement rate. This increases both the negative BS and IS externalities. On the other hand, with $\iota = n/\mu$ and $\mu_A = 0$, the spillover component collapses to $n/(\rho - n)$, which implies that the effects of a higher μ on the spillover component exactly cancel out. Consequently, μ exerts no further impact on the IS externality. Thus the subsidy rate ϕ_ι is unambiguously a decreasing function of μ. In Dinopoulos and Syropoulos (2006) $\mu = 0$; thus, their model does not allow for studying the relationship between ϕ_ι^{SO} and μ.

4.5. Inter-industry knowledge spillover parameter μ_A and marginal welfare

Consider an increase in μ_A triggered by a rise in σ for a given λ. First, I analyze the direct effects holding ι constant. It follows from Lemma 2 that a larger μ_A increases $\eta(\iota)$ and thereby the effective replacement rate. This reduces the negative BS and IS externalities. Moreover, a larger μ_A reduces the spillover component in the negative IS externality. With all negative externalities falling, the direct impact calls for a rise in the R&D subsidy rate. Second, I consider the indirect effects that operate through ι. It follows from Proposition 1 that $d\iota/\mu_A > 0$. Thus, a higher ι induced by a higher μ_A reduces both the negative BS and IS externalities. At the same time a higher ι increases the spillover component, further increasing the IS externality. The net impact on marginal welfare remains indeterminate and hence the relationship between ϕ_ι^{SO} and μ_A.

In Segerstrom (1998) and Dinopoulos and Syropoulos (2006), $\mu_A = 0$ and thus there are no inter-industry knowledge spillovers. Li (2003) allows for such spillovers and finds that the subsidy rate increases with μ_A.

4.6. Inter-industry RPA spillover parameter δ_A and marginal welfare

Finally, consider an increase in δ_A. First, I analyze the direct effects holding ι constant. It follows from Lemma 2 that a larger δ_A reduces $\eta(\iota)$ and thereby the effective replacement rate. This increases both the negative BS and IS externalities. With all negative externalities rising, a larger δ_A's direct impact calls for a reduction in R&D subsidy rate. Second, I consider the indirect effects that operate through ι. It follows from Proposition 1 that $d\iota/ d\delta_A < 0$. A lower ι induced by a higher δ_A increases the negative BS and IS externalities. At the same time the lower ι decreases the spillover component and thereby the negative IS externality. Again, the relationship between ϕ_ι and δ_A remains indeterminate. The effects for δ are the same, simply working in the opposite direction.

In Segerstrom (1998) $\delta_A = \delta = 0$, and thus RPAs are not considered. In Dinopoulos and Syropoulos (2006), $\delta > 0$ and $\delta_A = 0$, that is, there are RPAs but no associated spillovers.

5. SIMULATION RESULTS

The analytical exposition above gives two results: *i)* the optimal R&D policy ϕ_ι^{SO} can be a tax or a subsidy depending on the parameters of the model, *ii)* the parameters exert an ambiguous impact on ϕ_ι^{SO} . Thus, a numerical simulation exercise becomes necessary to gain further insights about the model's welfare implications. For this purpose, I choose the following benchmark parameters:

$\lambda = 1.25$, $\rho = 0.07$, $n = 0.01$, $s = 0.00023$,

$a_\iota = 70$, $\mu = 0.20$, $\sigma = 0.01$, $\delta = 1$, $\delta_A = 0.01$, $\gamma = 1$.

The size of innovations, λ, measures the gross mark up (the ratio of the price to the marginal cost) enjoyed by innovators and is estimated as ranging between 1.05 and 1.4 [see Basu, 1996, and Norrbin, 1993]. The population growth rate, n, is calculated as the annual rate of population growth in the US between 1975 and 1995 according to the World Development Indicators (World Bank, 2003). The subjective discount rate, ρ, is set at 0.07 to reflect a real interest rate of 7 percent, consistent with the average real return on the US stock market over the past century as calculated by Mehra and Prescott (1985). The percentage of specialized labor s is set at 0.23 percent to generate a fifty percent wage differential between specialized and general-purpose labor. The goal is to capture the relatively higher earnings of lobbyists/lawyers with respect to other workers. The unit labor requirement parameter for innovation, a_ι, is set at 70 to generate a growth rate $g = \iota log\lambda$ in the neighborhood of 0.5 percent. This is due to Denison (1985) who calculates the rate of growth driven by knowledge advancements to be in the neighborhood of 0.5 percent. The choice of μ = 0.20 follows from Dinopoulos and Segerstrom (1999) and Steger (2003). The parameters δ and μ are normalized to one. Proportional changes in these parameters change leave results unchanged. I set σ = 0.01 and δ_A = 0.01 to allow for reasonable levels of inter-industry knowledge and rent protection spillovers. Estimates for such spillover rates as utilized by the model are not readily available in the empirical literature.[22]

The benchmark simulation taken at face value implies that the competitive equilibrium innovation rate is ι^{CE} = 0.0218364 and the employment share of R&D workers is s_R^{CE} = 0.0624339. These values are below the socially optimal levels ι^{SO} = 0.0247836 and s_R^{SO} = 0.0789395. Obviously, the optimal R&D policy is a subsidy and turns out to equal ϕ_ι^{SO} = 0.148785. How does ϕ_ι^{SO} change with λ and other parameters? To answer this question, I map ϕ_ι^{SO} against λ and considered high and low values for *each* parameter within a 30 percent range as a rule of thumb.[23] The simulations reveal a robust n-shaped relationship between ϕ_ι^{SO} and λ as shown in Figure 2, meaning that for very small and very large values of λ, it is optimal to tax R&D; and for medium-size values of λ it optimal to subsidize R&D. 3D numerical simulations which are presented in Appendix D further confirm the robustness of this n-shaped relationship. When λ is kept at it benchmark level 1.25 and each parameter is allowed to vary within a 30 percent band, the optimal R&D subsidy rate fluctuates roughly between 10 and 25 percent.

Table 1 provides a summary of the main findings with respect to the rest of the parameters and compares them to Segerstrom (1998) and Dinopoulos and Syropoulos (2006). The "+" signs indicate that the change in positive welfare externalities outweigh the negative ones for an increase in that particular parameter. The opposite is true for the "–" sign. To economize on space, instead of giving a full account of the competing effects, I refer the reader to the previous section and to Appendix E for further details.

5.1 Optimal R&D policy in the real world

Figure 2 implies that the direction and magnitude of optimal R&D policy is highly sensitive to the choice of parameters. The question then is: would it be possible to identify a plausible range for the parameters that help the policy maker in designing R&D policy? One solution is to obtain a range for each parameter from the empirical studies, run simulations for upper and lower bounds, and

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[22] In general, the benchmark parameters and outcomes are in line with the recent theoretical growth papers that use numerical simulations [see Jones 2002, Lundborg and Segerstrom 1999; Sayek and Sener 2006 Dinopoulos and Segerstrom 1999, Steger 2003, Segerstrom 2006].

[23] The only exception is the case of δ_A which is shocked by 20 times since there was no visible change in the ϕ_ι^{SO} curve for 30 percent shocks.

identify an optimal R&D policy interval. However, this approach is likely to present multiple problems. Some parameters will yield off-target predictions for the endogenous variables, and some parameters will be incompatible with the existence and uniqueness conditions. This indeed turns out to be the case and should not come as a surprise, since by construction the parameters come from empirical studies that have different theoretical underpinnings.[24]

So, what is a researcher to do? I argue that a better strategy is to choose a parameter range with the goal of keeping the calibrated values of the model's central endogenous variables within an empirically relevant band. In the context of the present endogenous growth model, I consider these variables to be: *i)* the steady-state growth rate g, and *ii)* the steady-state employment share of R&D workers s_R.[25] After all, the model at hand is an endogenous growth model that links economic growth to the amount of resources allocated to R&D.

I propose in particular the following 3-step methodology. First, run a benchmark simulation using average values/estimates from the literature and adjust the free parameter a_ι to set $g = 0.05$ (as already done in the previous section). Second, identify a range for λ that keeps s_R below 0.07 . As illustrated λ plays a pivotal role in determining the direction R&D policy; hence, the analysis should allow for a certain amount of variation in λ. For this purpose, I choose the interval $\lambda \in [1.20, 1.35]$ which keeps s_R within the interval [0.03, 0.07].[26] Third, given the range for λ, perturb each parameter such that the rate of innovation ι remains within the interval [0.01, 0.04].[27, 28]

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[24] Consider for instance the range for innovation size λ, which is estimated to be between 1.05 and 1.4 by Norrbin (1993) and Basu (1996). In the present model, (given the other parameters) $\lambda = 1.05$ does not satisfy the necessary condition to generate a positive level of ι, and $\lambda = 1.4$ implies that the employment share of R&D workers s_R exceeds 11.7%. This is unrealistically high compared to the observed levels from the US and other advanced countries—which are in the neighborhood of 1%. Similarly, consider the range for the interest rate ρ, which is taken to be between 0.04 and 0.14 by Jones and Williams (2000). In my model (given the other parameters) $\rho = 0.04$ implies an s_R value of 15.5%, and $\rho = 0.14$ does not satisfy the necessary condition to generate a positive level of ι.

[25] Segerstrom (2006) strongly emphasizes the importance of generating empirically relevant levels for s_R.

[26] This range for s_R is above the observed level for the US and other advanced countries, which is in the neighborhood of 0.01. However, it is well-known that the R&D employment intensity definition is too narrow to account for all employment involved in creating, refining and disseminating new ideas. In Jones' [(2002), p. 226] words, "the research behind the creation of new consumer products like Odwalla or Jamba juice fruit drinks is not included" in this definition. Moreover, In the US, the employment intensity measure focuses on science and engineering and emphasizes research that requires the equivalent of a 4-year degree. Again in Jones's words "the research undertaken by the young Steve Jobs, Bill Gates and Marc Andreessen was probably excluded" from this statistics.

[27] This allows for a considerably wide range for the growth rate $g \in [0.00182322, 0.0120042]$. To see this, note that $g = \iota log\lambda$; thus $g^{LOW} = 0.01*\log[1.2] = 0.00182322$ and $g^{HIGH} = 0.04*\log[1.35] = 0.0120042$. Recall that the estimate for long-run growth attributable to technology advancements is 0.5% [Denison, 1995], and the average US per-capita income growth rate over the last 125 year is a steady 1.8% [Jones, 2002].

The simulation results for the restricted range $\lambda \in [1.20, 1.35]$ are shown in Figure 3. In most cases the optimal R&D policy is a subsidy and lies within a 5-25 percent band. The only exceptions are for the upper bound values of ρ and μ. When $\rho = 0.0095$, the subsidy rate is positive but clearly below 0.05. When $\mu = 0.4$, the subsidy rate is only slightly above zero for $\lambda < 1.3$ and becomes negative (hence a tax) for $\lambda < 1.3$. These findings are in line with the existing literature which tends to recommend subsidizing R&D while recognizing the competing welfare effects of a marginal innovation.[29]

How do these finding compare with the real world R&D subsidy rates? For the OECD countries, the average percentage of business enterprise R&D expenditure funded by the government is in the neighborhood of 10%. [30] Hence, the developed countries may not be far off from their optimal levels but yet there may be room for pushing the R&D subsidy rates upward to maximize welfare. The simulations of the present paper cannot resolve this magnitude issue once and for all; however, my findings strengthen the case for R&D subsidies and may pave the way for future research aimed at fine tuning the magnitude of optimal R&D policy.

It should also be acknowledged that the model in this paper does not account for all of the conceivable externalities associated with a marginal innovation or the mechanisms that influence these externalities. Li (2003) constructs a quality-ladders growth model with a CES utility function and removes the scale effects by assuming that R&D difficulty increases as the products become more complex with each innovation and also allows for DTO effects. Li's model introduces two additional welfare externalities. One is the "*across-industry business stealing externality*" by which new innovations reduce the profit flow of leaders in other industries through lowering consumer demand. The other is an effect that reinforces "*the intertemporal R&D spillover externality*". Each innovation success adds to the product complexity and raises R&D difficulty in the subsequent periods. In addition, with CES preferences Li's model allows for unconstrained monopoly pricing for large-sized innovations (instead of limit pricing). This links product prices to the elasticity of substitution in the

[28] The idea here is to allow for some of the variation to emanate from λ and some to emanate from the parameter in question. Simulations show that s_R remains below 13% for the upper bound of growth rate 0.04.

[29] See among others Segerstrom (2006), Jones and Williams (2000), Stokey (1995), Li (2001), Romer (1990), the variety-expansion based model of Grossman and Helpman (1991). See Jones and Williams (1998) for an empirical paper that reports that actual R&D investment in the US is 25 to 50 percent of the optimal R&D investment. It should be noted that Dinopoulos and Syropoulos (2006) do not provide a quantitative evaluation of their model; hence, the present paper is the first attempt to quantify optimal R&D policy in an endogenous growth setting with RPAs.

[30] OECD (2000, p.31, Science and Technology Outlook).

utility function, limiting the role of innovation size in affecting the BS externality through profit margins.

Segerstrom (2006) uses Li's (2001) setting to construct a model in which both incumbent firms and outside entrepreneurs undertake R&D. In addition, Segerstrom's (2006) model allows for high quality products to be copied at an exogenous rate. He finds that as the rate of copying and hence the rate of replacement increases, the magnitudes of the negative externalities (the intertemporal R&D spillover effect, and the across and within business stealing effects) decline and optimal R&D policy moves toward a subsidy. Jones and Williams (2000) construct a model of variety-expansion based endogenous growth that incorporates DTOs. They introduce creative destruction by assuming a link between old and new varieties via "*innovation clusters*". With this mechanism, the successful innovator of a new variety can claim market share from existing producers—in addition to her own monopoly profits—by exploiting the technology link between the new variety and existing products. Jones and Williams also allow for a negative externality associated with R&D duplication, labeled as the "*stepping on toes effect.*" Incorporating the above mentioned mechanisms into the present paper's setting can be multiple directions for further research.[31]

6. CONCLUSION

This paper has constructed a scale-free quality-ladders endogenous growth model that combines two of the main approaches to removal of scale effects: the RPA approach (a la Dinopoulos and Syropoulos, 2006) and the DTO approach (a la Segerstrom, 1998). In addition, the model has allowed for inter-industry knowledge spillovers (a la Li, 2003) and inter-industry rent-protection spillovers (a new channel).

The steady-state equilibrium growth rate is a function of all of the model's parameters including the R&D subsidy rate. Hence, the model implies fully-endogenous growth. The presence of rent-protection activities augments the effective replacement rate faced by the incumbent firms. The magnitude of this augmentation is positively related to innovation-deterring elasticity, which is endogenously determined.

The optimal R&D policy exhibits an n-shaped relationship with respect to innovation size. When innovations are of very small and very large magnitudes, the optimal policy is a R&D tax, and for medium size innovations, the optimal policy is a R&D subsidy. The model also identifies the impact of various parameter changes on optimal R&D policy. The numerical simulations imply that the competitive markets typically underinvest in R&D and thus the optimal R&D policy is a subsidy.

[31] See also Li (2001) and O'Donoghue and Zweimuller (2004) for models that consider patent breadth in the context of optimal R&D policy.

The magnitude of the R&D subsidy lies between 5 to 25 for plausible parameter values and steady-state outcomes.

Several extensions of the model still remain to be explored. One can incorporate the variety expansion approach to this setting and study the implications for R&D policy. It is possible to incorporate human capital and physical capital accumulation and check the robustness of the main results. Finally, one can extend the model to a two-country setting and investigate the effects of intellectual property and tariff policies on economic growth.

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Table 1. The Response of Optimal R&D Subsidy ϕ_ι to Parameters

	s	δ	δA	μA	ρ	$a\iota$	n	μ	λ
This paper	–	–	–	+	–	–	+	–	+/–
Segerstrom (1998)	0	0	0	0	–	0	+	–	–
Dinopoulos and Syropoulos (2006)	–	–	0	0	–	–	+	0	+/–

Figure 1. Steady-State Equilibrium

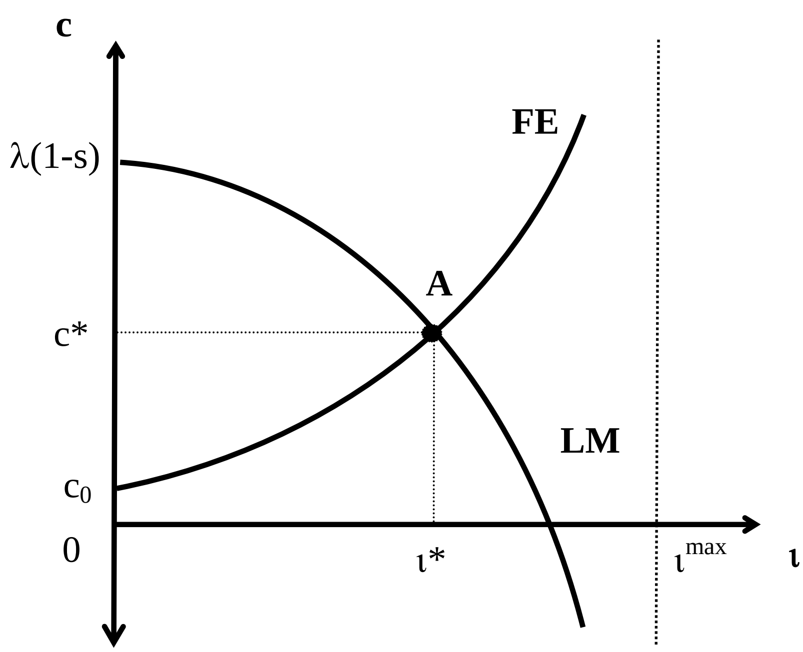
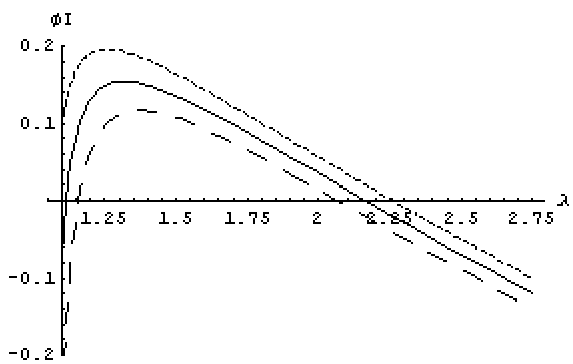
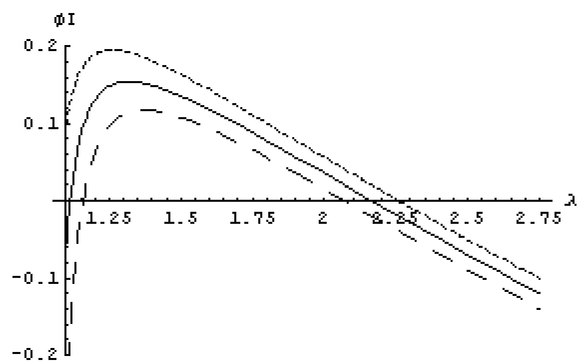


Figure 2: Simulation Results for Optimal Subsidy ϕ_ι^{SO}
(λ unrestricted, range for each parameter +/- 30%)

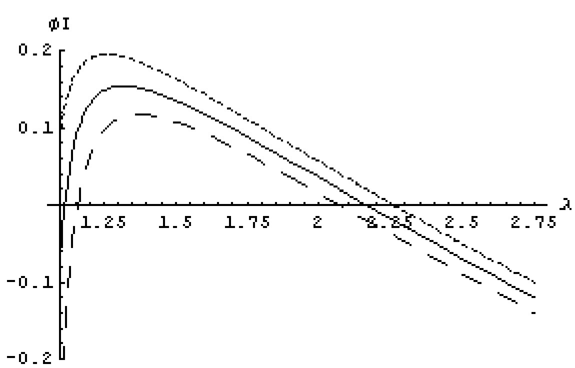
a) $s = 0.00023$ (Δs = +/– 30 %)



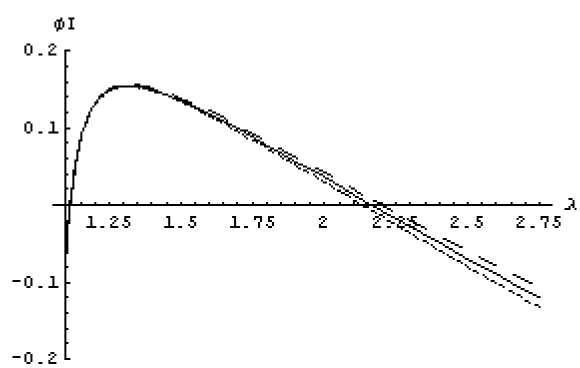
b) $\delta = 1$ ($\Delta\delta$ = +/– 30 %)



c) $\delta_A = 0.01$ ($\Delta\delta_A$ = +/– 2000%)



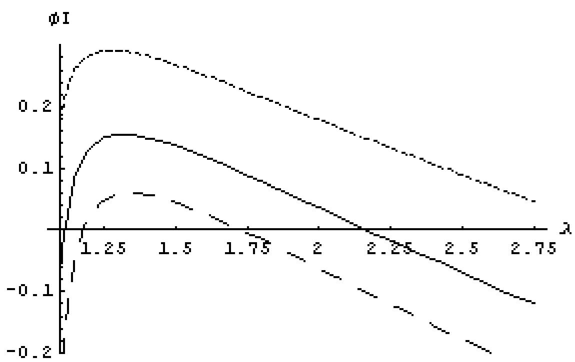
d) $\sigma = 0.01$($\Delta\sigma$ = +/– 30 %)



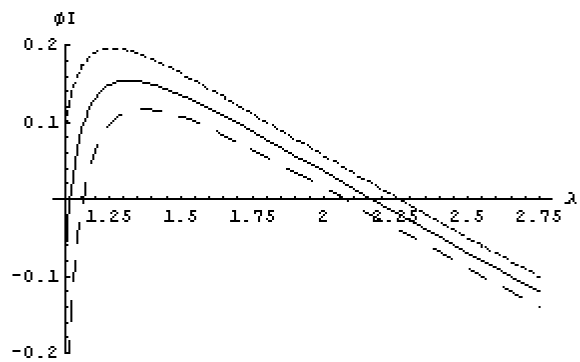
Note: Long-dashing for higher parameter value, short-dashing for lower parameter value.

Figure 2: Simulation Results for Optimal Subsidy ϕ_ι^{SO} (continued)
(λ unrestricted, range for each parameter +/- 30%)

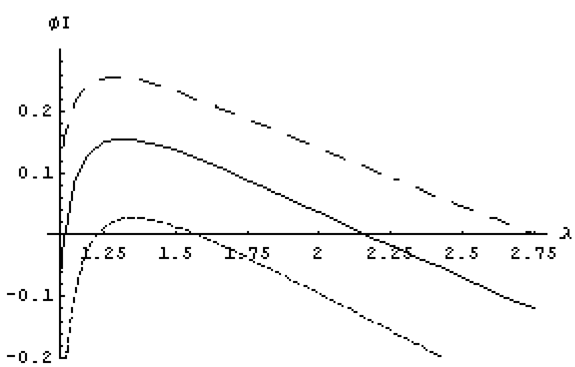
e) ρ = 0.07 ($\Delta\rho$ = +/– 30 %)



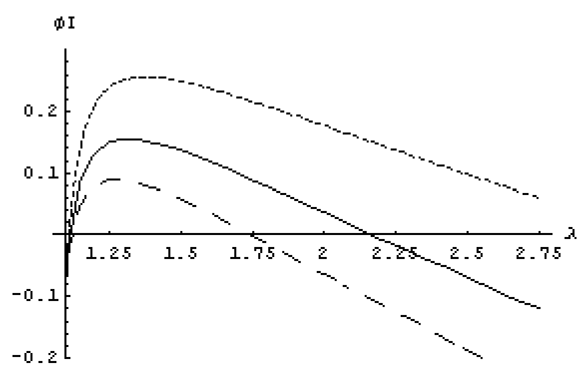
f) a_ι = 70 (Δa_ι = +/– 30 %)



g) n = 0.01 (Δn = +/– 30 %)



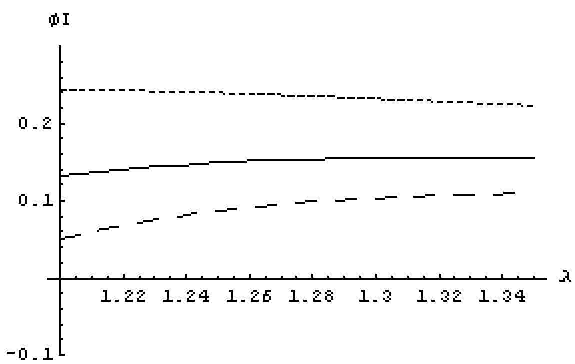
h) μ = 0.01 ($\Delta\mu$ = +/– 30 %)



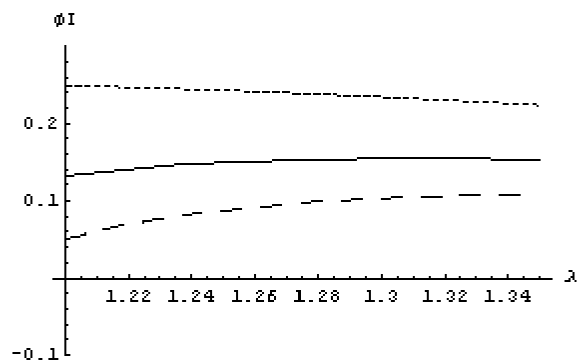
Note: Long-dashing for higher parameter value, short-dashing for lower parameter value.

Figure 3: Simulation Results for Optimal Subsidy ϕ_ι^{SO}
(λ restricted, parameter range contingent on ι impact)

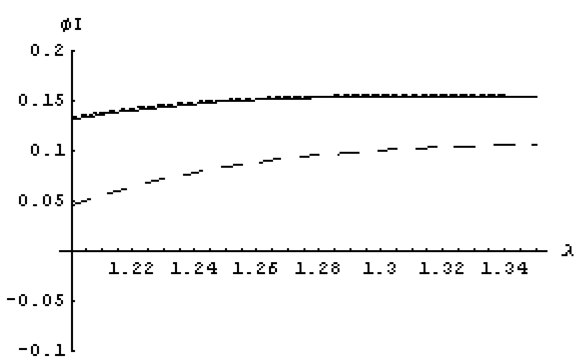
a) $s^L = 0.000092$, $s^H = 0.00031$



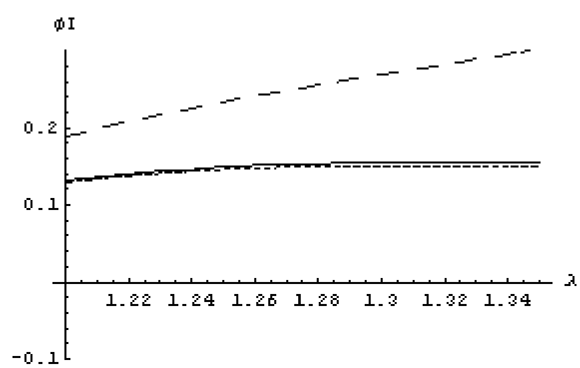
b) $\delta^L = 0.37$, $\delta^H = 1.35$



c) $\delta_A{}^L = 0.00$, $\delta_A{}^H = 0.38$



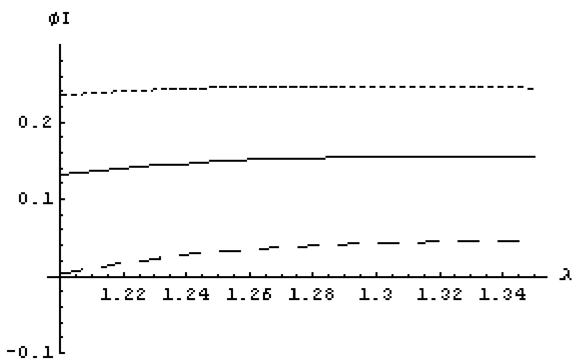
d) $\sigma^L = 0.00$, $\sigma^H = 0.24$



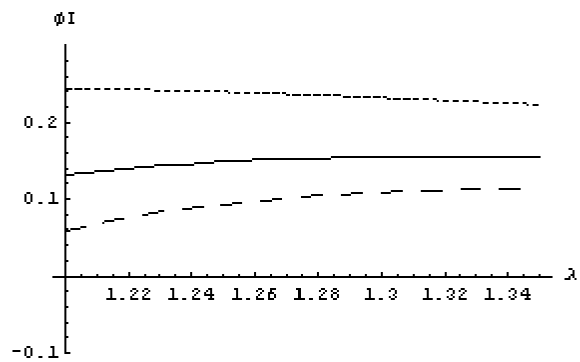
Note: Long-dashing for higher parameter value, short-dashing for lower parameter value.

Figure 3: Simulation Results for Optimal Subsidy ϕ_ι^{SO} (continued)
(λ restricted, parameter range contingent on ι impact)

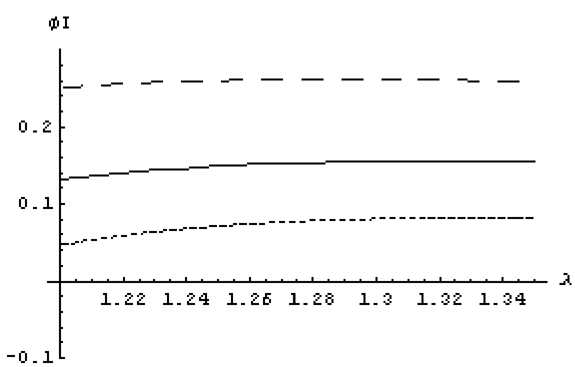
e) $\rho^L = 0.055$, $\rho^H = 0.095$



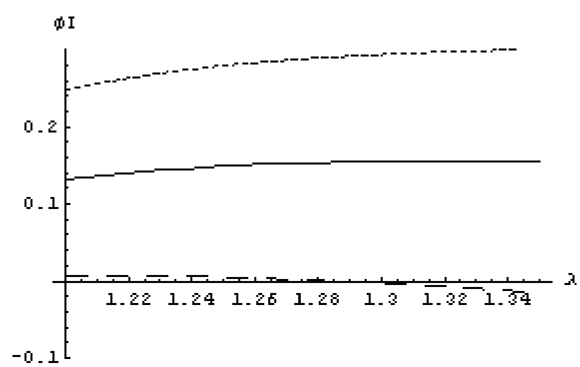
f) $a_\iota^L = 28$, $a_\iota^H = 92$



g) $n^L = 0.0082$, $n^H = 0.0132$



h) $\mu^L = 0.12$, $\mu^H = 0.40$



Note: Long-dashing for higher parameter value, short-dashing for lower parameter value.