



# Private patent protection in the theory of Schumpeterian growth

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## ABSTRACT

We develop a Schumpeterian growth model with privately optimal intellectual property rights (IPRs) enforcement and investigate the implications for intellectual property and R&D policies. In our setting, successful innovators undertake costly rent protection activities (RPAs) to enforce their patents. RPAs deter innovators who seek to discover higher quality products and thereby replace the patent holder. RPAs also deter imitators who seek to capture a portion of the monopoly market by imitating the patent holder's product. We investigate the role of private IPR protection by considering the impact of subsidies to RPAs on economic growth and welfare. We find that a larger RPA subsidy raises the innovation rate if and only if the ease of imitation is above a certain level. With regards to welfare, we find that depending on the parameters it may be optimal to tax or subsidize RPAs. Thus a prohibitively high taxation of RPAs is not necessarily optimal. We also show that the presence of imitation strengthens the case for subsidizing R&D.

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## 1. Introduction

By securing the rewards of successful innovators and thereby motivating R&D efforts, intellectual property rights (IPRs) protection plays a central role in endogenous growth theory. A potential shortcoming of the early endogenous growth literature is that it treats the strength of IPR protection as a policy parameter that is independent of the private actions of patent holders' efforts (see [Aghion and Howitt, 1998](#) for an overview). Yet, empirical evidence indicates that patent holders extensively engage in activities such as litigation and lobbying to protect the value of their intellectual property. For example, [Lerner \(1995\)](#) and [Bessen and Meurer \(2008\)](#) find that private litigation costs are a significant component of overall innovation costs, and [Chu \(2008\)](#) and [Lanjouw and Cockburn \(2000\)](#) find that private lobbying efforts appear to influence US patent policy and the promotion of IPRs in US trade negotiations, respectively.<sup>1</sup>

In keeping with this evidence, recent contributions by [Dinopoulos and Syropoulos \(2007\)](#) and [Eicher and Garcia-Peñalosa \(2008\)](#) endogenize the strength of IPRs by assuming that *de facto* IPRs depend on the level of private resources devoted to IPR enforcement.<sup>2</sup> Like much of the earlier literature on IPR protection, [Eicher and Garcia-Peñalosa \(2008\)](#) construct a variety-expansion model of growth in which imitation is the key threat to the profits of patent holders. In this framework, private resources devoted to IPR protection reduce the probability of imitation and, thereby, increase the expected return to innovation and the equilibrium rate of growth. In contrast, [Dinopoulos and Syropoulos \(2007\)](#) consider

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<sup>1</sup> See [Şener \(2008\)](#), [Dinopoulos and Syropoulos \(2007\)](#) and [Şener \(2006\)](#) for further empirical and anecdotal evidence on RPAs.

<sup>2</sup> Another contribution to this literature is by [Akiyama and Furukawa \(2009\)](#) where Northern patent holders can reduce the threat of Southern imitation by "masking" their innovations at the cost of facing higher marginal production costs. See also [Akiyama et al. \(2011\)](#) for a North-South model in which Northern patent holders hire Northern workers to reduce directly the imitation success rate of Southern firms.

private IPR protection in the context of a Schumpeterian growth model in which there is no possibility of imitation. Instead, current patent holders are threatened with replacement by innovators who develop a higher quality version of their patented good. To protect their rents, patent holders devote resources to litigation and lobbying, which reduce the instantaneous probability of further innovation. Because these rent protection activities (RPAs) use resources and serve to block the next round of innovation, any policy change that raises the efficiency of RPAs reduces innovation and growth.<sup>3</sup> Recent work by [Boldrin and Levine \(2004\)](#) raises similar concerns regarding the potential for rent-seeking by patent holders to reduce innovation rates.

We advance this literature by developing a Schumpeterian model that incorporates two types of threats faced by current patent holders: complete replacement due to further innovation and loss of current market due to imitation. We model privately optimal patent protection by quality leaders as activities that deter both innovation and imitation efforts of rival firms. Because our model incorporates both positive and negative effects of private IPR protection on the profitability of innovation, we are able to address the factors that determine the relative strengths of these two effects and, thus, the net effect of rent protection on the rate of economic growth. Our main IPR policy variable is an RPA subsidy, which we interpret as any policy change that facilitates the rent protection efforts of patent holders. Our policy parameter encompasses the broad nature of patent enforcement policies. In practice, policy makers may not make a clear distinction between patent protection policies against imitators versus future innovators.<sup>4</sup> Hence we depart from the literature where IPR policy is generally considered as a parameter that affects *only* patent enforcement against imitators, e.g., [Grossman and Lai \(2004\)](#), [Eicher and Garcia-Peñalosa \(2008\)](#), [Kwan and Lai \(2003\)](#).

This policy set-up allows us to address interdependencies between different dimensions of the IPRs framework, including RPA subsidies, R&D subsidies and the ease of imitation. Because these relationships are highly non-linear, we employ both analytical and numerical methods. In particular, we find that the impact of increased subsidies to RPAs is to increase the equilibrium rate of innovation *if and only if* the ease of imitation is above a certain threshold value. This result holds because when imitation is relatively easy, RPAs primarily deter imitators, which increases the patent holder's expected market share and thus the equilibrium return to innovation. In contrast, when imitation is relatively difficult, RPAs primarily deter potential innovators, which raise the cost of research and decreases economic growth. Using numerical simulations we also establish an inverted-U relationship between innovation and RPA subsidies. At low levels of RPA subsidy, promoting private patent protection raises growth. After a certain threshold subsidy level, further increases in RPA subsidies become counterproductive and reduce growth. The inverted-U relationship implies the existence of a growth-maximizing RPA policy. We show that depending on the parameters the growth-maximizing policy can be a tax or subsidy to RPAs. These results stand in contrast to [Dinopoulos and Syropoulos \(2007\)](#) whose model implies that private rent protection *always* retards innovation.

We also study the welfare implications of the model by considering a social planner who chooses the levels of R&D, RPA and consumption to maximize social welfare subject to the resource constraint. This analysis integrates the welfare distortions found in the two strands of the literature: quality-ladders Schumpeterian growth literature and the variety-expansion growth literature with imitation. In particular, we show that the addition of imitation and imitation-detering activities strengthens the case for R&D subsidies. In addition, we find that private and social returns to RPAs may differ due to a number of competing forces as detailed in [Section 4](#). Depending on the strength of these competing effects, it may be optimal to tax or subsidize RPAs. In particular, we find that RPA subsidies can be welfare maximizing in countries in which innovators have weak protection against imitators. Whereas RPA taxes tend to be welfare maximizing in countries with strong protection against imitation.<sup>5</sup> These findings differ from [Dinopoulos and Syropoulos \(2007\)](#) who find that it is *always* optimal to impose a prohibitively high tax on RPAs.

The paper is organized as follows. [Section 2](#) introduces the building blocks of the model. [Section 3](#) investigates the steady-state effects of policy changes. [Section 4](#) presents the welfare analysis and examines the optimal RPA and R&D policies. [Section 5](#) provides a numerical analysis and [Section 6](#) concludes the paper.

## 2. The model

The economy consists of a continuum of industries. In each industry, entrepreneurs hire scientists to innovate higher quality products. Successful innovators obtain patents to hold the exclusive legal right to use their technology. Consumers prefer higher quality products over lower quality ones by a certain margin. By engaging in limit pricing, patent holders can force the lower-quality producers out of the market. While enjoying monopoly power, patent holders face two threats: replacement due to further innovation and reduced expected market share due to imitation. Each patent holder hires lawyers to perform RPAs, which help the firm to fight against both innovation and imitation. Patent holders optimally

<sup>3</sup> See also [Şener \(2008\)](#) for a model that investigates optimal R&D policy in the presence of RPAs and diminishing technological opportunities. See [Grieben and Şener \(2009\)](#) and [Şener \(2006\)](#) for models that adopt the notion of RPAs in North-South settings and study trade liberalization and IPR policies, respectively.

<sup>4</sup> Our analysis also contributes to the recent literature on patent design and economic growth, in which lagging patent breadth is imitation deterring and leading breadth is innovation deterring. See recent contributions by [O'Donoghue and Zweimuller \(2004\)](#) and [Chu \(2009\)](#).

<sup>5</sup> We should note that [Eicher and Garcia-Peñalosa \(2008\)](#) identify the welfare distortions associated with IPR policy only when imitation is exogenous but they do not conduct a formal welfare analysis when imitation responds to private enforcement of IPRs.

choose the level of their RPAs, which in turn sets the level of employment for lawyers. Hence, the extent of IPR enforcement is endogenously determined by the amount of lawyers devoted to RPAs. There exists a financial market which channels the savings of households to R&D firms. In the labor market, workers are mobile between R&D, RPA and goods production.

### 2.1. Households

There is a unit continuum of infinitely-lived dynastic households. Each family starts at time  $t=0$  with a single member and grows at an exogenous rate  $n > 0$ , such that population size at time  $t$  is given by  $N(t) = e^{nt}$ . Households discount future utility at a rate  $\rho > 0$ , so that dynastic utility is given by

$$U \equiv \int_0^{\infty} e^{-(\rho-n)t} \log u(t) dt, \quad (1)$$

where we assume  $\rho - n > 0$ . Households consume goods from a unit continuum of industries indexed by  $\theta \in [0,1]$ . Instantaneous per capita utility,  $u(t)$ , is defined as follows:

$$\log u(t) \equiv \int_0^1 \log \left[ \sum_k \lambda^k Z(k, \theta, t) \right] d\theta, \quad (2)$$

The variable  $Z(k, \theta, t)$  is the consumption of final good in industry  $\theta \in [0,1]$  of quality  $k$  at time  $t$ . The parameter  $\lambda > 1$  defines the size of quality improvements. Each household allocates its per capita consumption expenditure  $c$  to maximize  $\log u(t)$  given prices at time  $t$ . Adjusted for quality, goods in an industry are perfect substitutes, so that households purchase only the good with the lowest quality-adjusted price. Moreover, products enter the utility function symmetrically, thus households spread their consumption expenditure evenly across the continuum of industries. The resulting demand functions are identical across industries, with

$$Z(k, \theta, t) = cN(t)/p, \quad (3)$$

where  $p$  is the price of the purchased good.

Maximizing (1) subject to the standard intertemporal budget constraint and considering (3) produces the familiar equation of motion for  $c$ :

$$\frac{\dot{c}}{c} = r(t) - \rho, \quad (4)$$

where  $r(t)$  is the market rate of interest. In the steady state,  $r(t) = \rho$  such that consumption expenditure is constant. Economic growth takes the form of increases in utility due to the introduction of higher quality goods available at constant prices.

### 2.2. Market structure and production

In each industry along the continuum, there exists a successful innovator who has the exclusive legal right via a patent to produce the highest quality good. Patents, however, are imperfectly enforced. At each point in time, there exists a positive probability  $m$  that the incumbent's patents will not be enforced. This modeling follows from Grossman and Lai (2004), and Eicher and Garcia-Peñalosa (2008) who consider imperfect IPR enforcement by assuming that patent holders face a positive probability of losing their market to imitators.

With probability  $(1 - m)$ , a patent is enforced and the quality leader competes with followers who can produce a lower quality product. Production of one unit of good requires one unit of labor regardless of the quality level. Thus the marginal cost of production equals the wage, which is taken to be the *numeraire*:  $w = 1$ . Given the equal production costs, the patent holder can drive the followers out of the market by engaging in limit pricing. More specifically, the leader offers the lowest quality-adjusted price by charging  $p = \lambda$ . The followers cannot do better than break even and exit the market. Instantaneous monopoly profit is given by:

$$\pi(t) = \frac{\lambda - 1}{\lambda} cN(t) \quad (5)$$

where  $\lambda - 1$  is the profit margin and  $cN(t)/\lambda$  is the total output sold.

With probability  $m$ , the incumbent's patent is not enforced and the quality leader competes with a large number of imitators who can produce the same quality level product. In this case, production takes place under competitive conditions, which implies marginal cost pricing,  $p = 1$ , and zero profits. Combining the levels of production under a competitive market  $mcN(t)$  and the monopoly market  $(1 - m)[cN(t)/\lambda]$ , the expected production of a representative good at each moment in time is given by

$$Z^e(t) = \frac{1 + (\lambda - 1)m}{\lambda} cN(t). \quad (6)$$

Eq. (6) shows that the expected consumption of each good is increasing in the share of the competitive market  $m$ , since in this market goods are offered at a lower price compared to the monopolized market. Exploiting symmetry across industries, we derive employment in the production sector as:

$$N_Q(t) = \frac{1 + (\lambda - 1)m}{\lambda} cN(t). \quad (7)$$

Incumbent firms can engage in costly RPAs that take the form of lobbying and litigation in defense of their IPRs. Accounting for RPAs, which we denote by  $X$ , and the probability of patent enforcement  $(1 - m)$ , the incumbent's net instantaneous profit is given by

$$\hat{\pi}(t) = (1 - m)\pi(t) - (1 - s_X)a_X X(t), \quad (8)$$

where  $0 < s_X < 1$  is the subsidy rate (or the tax rate if  $s_X < 0$ ) on RPAs and  $a_X > 0$  is the unit labor requirement in RPAs. Following the literature, we assume that all subsidies are financed by non-distortionary lump sum taxes.<sup>6</sup> Employment in rent protection is given by

$$N_X(t) = a_X X(t). \quad (9)$$

### 2.3. Innovation

Entrepreneurs in each industry participate in R&D races to innovate higher quality products. Thus, at any point in time, the current patent holder faces multiple entrepreneurs who seek to replace her as the market leader by discovering the good one step up the quality ladder. The patent holder may seek to deter these entrepreneurs by engaging in rent protection and thus prolong her market dominance. These activities are assumed to be non-rival within the firm in that they simultaneously deter innovators who aim to discover the next-generation product and imitators who seek to claim a portion of the leader's market.

An entrepreneur  $j$  that invests  $R_j(t)$  in research at time  $t$  discovers the next higher-quality product with instantaneous probability  $i_j = R_j(t)/X(t)$ , where  $R_j(t)$  is the entrepreneur's investment in R&D and  $X(t)$  is the incumbent's intensity of RPAs. Let  $V(t)$  denote the value of a successful innovation,  $a_R > 0$  the unit labor requirement in R&D and  $0 < s_R < 1$  is the subsidy rate (or the tax rate if  $s_R < 0$ ) for research expenditures. Free-entry into R&D implies:

$$\begin{cases} i_j V(t) - (1 - s_R)a_R R_j(t) \leq 0, \\ R_j(t) \geq 0, \end{cases} \quad (10)$$

where strict equality must hold in exactly one of the lines of (10).  $i_j V(t)$  is entrepreneur  $j$ 's expected reward from undertaking R&D and  $(1 - s_R)a_R R_j(t)$  is her expenditure on R&D. Free entry in R&D implies that in every industry with positive research expenditure, the cost of research must equal the expected gain:

$$V(t) = (1 - s_R)a_R X(t). \quad (11)$$

Similarly, the absence of R&D in an industry indicates that the costs exceed the expected gains.

The probability of successful innovation is assumed to be independently distributed over entrepreneurs, industries and over time. For a representative industry, the rate of innovation is found by summing over the probabilities for each entrepreneur:

$$i = \sum_j \frac{R_j(t)}{X(t)} = \frac{R(t)}{X(t)}. \quad (12)$$

In each industry, the arrival of innovations follows a Poisson process with intensity  $i$ , which we call the rate of innovation. Total employment in research is given by:

$$N_R(t) = a_R R(t). \quad (13)$$

In equilibrium,  $R(t)$  is proportional to the size of the population  $N(t)$ , a relationship which gives rise to a *scale* effect. However, this scale effect is counteracted by RPAs  $X(t)$ , which, in equilibrium, will also be proportional to the size of population. Thus, the equilibrium rate of innovation  $i$  will remain constant and is independent of population size.

### 2.4. Imitation

We model the probability of imitation  $m$  as an endogenous variable determined by

$$m = \frac{\mu N(t)}{X(t)}, \quad (14)$$

<sup>6</sup> If lump-sum taxes are not feasible, then distortionary taxation can alter some of our results.

which is in the same spirit as the innovation probability specification (12). In this equation, the exogenous parameter  $\mu > 0$  represents the degree of *ease of imitation* that is linked to technological and institutional constraints. In addition, for a given  $X(t)$ , the probability of imitation increases with the size of the population, an assumption that reflects the idea that it arguably requires more resources to protect innovations against imitation in China or India than in Singapore or Mauritius. Thus, as the population size  $N(t)$  expands, the number of potential imitators increases and the RPA effort required to provide a given level of imitation deterrence rises as well. Similar to Eq. (12), this scale effect in the probability of imitation is counteracted by RPAs  $X(t)$ , which, in equilibrium, will also be proportional to the size of population. Thus, the equilibrium probability of imitation  $m$  remains constant.<sup>7</sup>

We note that the same RPA term,  $X(t)$ , enters the denominator of both innovation and imitation specifications, (12) and (14), respectively. We thus assume the presence of economies of scope between innovation- and imitation-detering RPAs.<sup>8</sup> For example, the innovator can engage in patent fencing to protect its core intellectual property and use these same set of patents to deter both innovation and imitation. Furthermore, patent holders may use the same information and arguments when litigating against innovators and imitators.

## 2.5. Stock market valuation and optimal RPAs

In equilibrium, the value of the patent holder  $V(t)$  is determined by an arbitrage condition that holds that at each point in time and equates the expected returns on the stocks issued by the incumbent firm to the risk-free market interest rate  $r(t)$ . This condition takes the form:

$$rV(t)dt = \hat{\pi}(t)dt - V(t)idt + \dot{V}(1-idt)dt. \quad (15)$$

On the right-hand side, the first term captures the profits net of RPAs and accounting for probability of patent enforcement. The second term is the capital loss incurred if the firm is replaced by a successful innovator. The final term captures the change in the value of the firm if no further innovation takes place.

Incumbent firms choose the level of RPAs to maximize the expected rate of return on their stocks. Substituting (12), (14) and (8) into the arbitrage condition (15) and maximizing the right hand side of (15) with respect to  $X(t)$ , we find the optimality condition for RPAs:

$$(1-s_X)a_X X(t) = m\pi(t) + iV(t). \quad (16)$$

Eq. (16) implies that when  $X(t)$  is at the optimal level, the expenditure on rent protection equals the reduction in profits due to imitation  $m\pi(t)$  plus the instantaneous expected capital loss due to replacement  $iV(t)$ .

Substituting (16) into the stock market arbitrage condition (15), noting that  $\dot{V}/V = n$  and  $r = \rho$  at the steady-state, using (8) and taking limits as  $dt \rightarrow 0$ , we find an expression for  $V(t)$

$$V(t) = \frac{1-2m}{\rho-n+2i} \pi(t), \quad (17)$$

which implies that the value of the firm  $V(t)$  equals potential instantaneous profits  $\pi(t)$  discounted by the *effective discount rate*:

$$\rho^* \equiv \frac{\rho-n+2i}{1-2m}. \quad (18)$$

The effective discount rate is the firm's discount rate adjusted for the risk of replacement, losses in profits due to imitation, and the cost of rent protection. Note from (16) that the impact of optimally chosen rent protection is effectively to double the capital losses from imitation and replacement in the profits flow expression; hence the doubling of  $i$  and  $m$  in (17).

## 2.6. The first steady state condition: Relative profitability of rent protection and R&D

We establish the steady-state equilibrium by reducing the model to two equations in  $i$  and  $m$ . The first steady state equation is derived from taking the ratio of free entry and optimal rent protection conditions, (11) and (16):

$$\frac{(1-s_X)a_X}{(1-s_R)a_R} = \frac{m\pi(t) + iV(t)}{V(t)}, \quad (19)$$

<sup>7</sup> Our symmetric treatment of innovation and imitation is in line with the endogenous growth literature where the probabilities of innovation and imitation have analogous specifications [see for example Glass and Saggi, 2002 and Grossman and Helpman, 1991, Chs. 11 and 12].

<sup>8</sup> We checked the robustness of our results by developing an extension of this model in which we permit firms to distinguish between innovation and imitation deterring RPAs and allow for varying degrees of spillovers between the two activities. Using numerical simulations we find that the results of this extended model are qualitatively the same as the model presented here provided that there is *some* degree of spillover from one activity to the other. The details of this model and the relevant numerical simulations are relegated to the [Supplementary appendix](#). See Grieben and Şener (2009, p.1048) who also model RPAs as simultaneously deterring both innovation and imitation in a North–South setting.

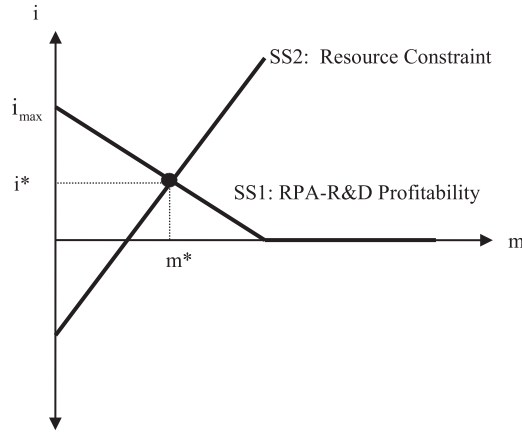


Fig. 1. Steady-state equilibrium.

In (19), the left-hand side is the cost of rent protection relative to the cost of research. The right-hand side is the return to rent protection relative to the return to research. Substituting  $\pi(t)/V(t)$  from (17) into (19) and solving for  $i$ , provides our first steady state condition, SS1, which we will refer to as the *relative profitability condition*:

$$i = \frac{(1-s_X)a_X}{(1-s_R)a_R} \left[ \rho - n + 2 \frac{(1-s_X)a_X}{(1-s_R)a_R} \right] m. \tag{20}$$

Eq. (20) establishes a negative relationship between  $i$  and  $m$ , as shown in Fig. 1.

The vertical intercept of the SS1 curve indicates the maximum rate of innovation,  $i_{max} \equiv [(1-s_X)a_X]/[(1-s_R)a_R]$ , which obtains when the probability of imitation is zero. From that point, innovation decreases in the probability of imitation. Intuitively, the larger the threat of imitation  $m$ , the higher the relative expected returns from engaging in RPA. Restoring equilibrium requires a fall in  $i$ , which decreases the threat coming from rival researchers and thus reduces the relative returns from RPA. Substituting (20) into (18) gives a parsimonious expression for the effective discount rate  $\rho^*$  in terms of the model's parameters:

$$\rho^* = \rho - n + 2 \frac{(1-s_X)a_X}{(1-s_R)a_R}. \tag{21}$$

For future use, note that  $\rho^*$  is increasing in  $i_{max}$ . Recall that (18) equals (21); thus, the maximum feasible innovation rate in (21) captures the entire set of changes in  $\rho^*$  due to replacement risk, imitation related loss, and rent protection costs. We thus refer to  $2i_{max}$  as the *expected value loss rate*.

2.7. The second steady state condition: The resource constraint

Workers may engage in any of three activities, production, R&D or rent protection. We can write the labor market equilibrium condition as

$$N(t) = N_Q(t) + N_X(t) + N_R(t). \tag{22}$$

In equilibrium, the amount of labor allocated to each activity will increase at the same rate as the population, so that the share of labor in each activity remains constant.

We first find an expression that relates  $c$  to  $m$  by substituting  $\pi(t)$  from Eq. (5) into (17). We then plug the resulting expression into (11), and use (14) and (21) to eliminate  $X(t)$  and  $\rho^*$ , respectively. This gives

$$c = \frac{[(\rho-n)(1-s_R)a_R + 2(1-s_X)a_X]}{(\lambda-1)m} \frac{\lambda\mu}{m}. \tag{23}$$

Substituting  $N_Q(t)$  from (7),  $N_R(t)$  from (13),  $N_X(t)$  from (9) into (22), using (12) for  $R(t)$ , (14) for  $X(t)$  and (23) for  $c$  provides an expression for the resource constraint in terms of  $i$  and  $m$

$$1 = \frac{\mu}{m} \left\{ [(\rho-n)(1-s_R)a_R + 2(1-s_X)a_X] \left[ \frac{1}{\lambda-1} + m \right] + a_X + a_R i \right\}, \tag{24}$$

which may also be expressed as

$$i = - \frac{(\rho-n)(1-s_R)a_R + (\lambda + 1 - 2s_X)a_X}{(\lambda-1)a_R} + \left[ \frac{1}{\mu} - (\rho-n)(1-s_R)a_R - 2(1-s_X)a_X \right] \frac{m}{a_R}. \tag{25}$$



Eq. (25) provides our second steady state condition, SS2. As shown in Fig. 1, SS2 has a positive slope if the bracketed part of the second term is positive. The intuition is that an increase in the intensity of R&D activity  $i$  leaves fewer resources for production and RPAs and leads to a decline in  $X(t)$ .<sup>9</sup> This in turn translates into a larger  $m$  and thus a larger competitive market share via (14). For the SS2 curve to be upward sloping we need  $\mu(1-s_R)a_R\rho^* < 1$ , which is a necessary condition for an equilibrium with  $i > 0$ . We assume this condition to hold.

## 2.8. Steady-state equilibrium

In an interior equilibrium, the equilibrium values for  $i$  and  $m$  are jointly determined by the relative profitability condition (20) and the resource constraint (25).<sup>10</sup> As illustrated in Fig. 1 above, Eq. (20) has a positive intercept and negative slope, while Eq. (25) has a negative intercept and positive slope. These characteristics guarantee that any interior equilibrium will be unique. The equilibrium probability of imitation and innovation rates are given by

$$m^* = \frac{(\rho-n)(1-s_R)^2 a_R + [2\lambda - (1+\lambda)(s_R+s_X) + 2s_R s_X] a_X}{(\lambda-1)[(1-s_R)/\mu] + (\rho-n)s_R(1-s_R)a_R + 2s_R(1-s_X)a_X}, \quad (26)$$

and

$$i^* = \frac{(1-s_X)a_X}{(1-s_R)a_R} - \left[ \rho - n + 2 \frac{(1-s_X)a_X}{(1-s_R)a_R} \right] m^*. \quad (27)$$

Substituting (26) into (23) gives the equilibrium level of per capita consumption expenditure:

$$c^* = \frac{\lambda\mu[(\rho-n)(1-s_R)a_R + 2(1-s_X)a_X]}{(\lambda-1)m^*}, \quad (28)$$

which is strictly positive under the parametric restrictions of the model.

We may also write (27) as  $i^* = i_{max}^* - \rho^* m^*$  where the first term,  $i_{max}^*$ , is the relative RPA/R&D cost component and  $\rho^* m^*$  is the imitation component of equilibrium innovation. Eqs. (26) and (27) imply that the equilibrium rate of innovation is independent of population size and thus the model is free of scale effects. We also note that  $i^*$  responds to a large set of parameters including the R&D and RPA subsidy rates. Hence our model belongs to the class of fully-endogenous growth models.<sup>11</sup> Our model may also be viewed as a generalization of Dinopoulos and Syropoulos (2007). By incorporating imitation, our model expands the sets of parameters that influence growth. In particular, their innovation rate is  $i^* = i_{max}^*$ , which follows from setting  $m^* = 0$  in (27).<sup>12</sup> Thus, we have the following result:

**Proposition 1.** *The presence of imitation  $\mu > 0$  expands the set of parameters that affects growth and implies a lower growth rate in comparison to the case with  $\mu = 0$ .*

## 3. Policy changes

We consider the impact of incremental changes in  $s_X$ ,  $s_R$  and  $\mu$ . We consider  $s_X$ , the subsidy rate to RPAs, as the primary IPR policy parameter as it affects the private patent enforcement efforts of firms. We note that the results are qualitatively the same if one considers instead an increase in the efficiency of lawyers (i.e., a fall in  $\alpha_X$ ). We note that  $\mu$  is another possible IPR policy parameter as it is influenced by the extent of patent enforcement against imitators. However, in practice, policy makers cannot necessarily distinguish between patent enforcement against imitators and that against further innovators. We report the results on  $\mu$  for comparability with the literature which focuses exclusively on IPR policy changes aimed at reducing imitation.

### 3.1. Rent protection subsidy

An incremental increase in rent protection subsidy  $s_X$  has two competing effects on the relative profitability condition (20). First, it reduces the relative costs of rent protection. Second, it implies a decrease in the effective replacement risk  $i_{max}$  and thus

<sup>9</sup> A lower  $X(t)$  implies a fall in the demand for R&D labor  $\alpha_X i X(t)$  and RPA labor  $a_R \mu X(t)$ . It also implies a decline in production labor demand, which involves two competing effects. First, a lower  $X(t)$  decreases R&D costs and forces resources to move from production to R&D. This leads to a fall in  $c$ , which in turn decreases the demand for production labor. Second, a lower  $X(t)$  increases the share of industries with competitive pricing and thereby raises the demand for production labor. The net effect is a decline in the share of production labor demand, which is captured in (24) by the first two terms in the main brackets multiplied by  $\mu/m$ .

<sup>10</sup> Appendix A provides the necessary and sufficient conditions that guarantee the existence and uniqueness of an interior equilibrium. The model also supports boundary equilibria associated with constrained outcomes in the Kuhn-Tucker equations for optimal R&D and RPA. See Davis and Şener (2012) for a complete boundary equilibria analysis in a similar setting.

<sup>11</sup> Fully-endogenous growth models have recently received more empirical support than semi-endogenous growth models, see e.g., Madsen (2007, 2008) and Ang and Madsen (2011). See Jones (2005) for counter evidence on this. See Dinopoulos and Şener (2007) for further discussion.

<sup>12</sup> We note though that  $i_{max}^*$  is the equilibrium level that would be obtained in Dinopoulos and Syropoulos (2007) when labor is of one type and perfectly mobile between three activities R&D, production, and rent protection. As their main model, they consider two types of labor: general purpose labor (employable in either production or R&D), and specialized labor (employable exclusively in RPAs).

the effective discount rate  $\rho^*$ . This in turn increases the relative rewards from R&D. In an interior equilibrium  $m < 1/2$ , which implies that the former effect dominates the latter. For a given  $m$ , restoring the relative profitability condition requires a decrease in innovation intensity  $i$  and therefore the SS1 curve shifts down.

We now consider the impact on resource allocation. The decline in  $\rho^*$  increases the rewards from R&D, forcing resources to move into R&D.<sup>13</sup> For a given  $m$ , restoring the resource constraint requires an increase in innovation intensity  $i$  and hence the SS2 curve shifts up. A graphical analysis using Fig. 1 implies that the equilibrium probability of imitation  $m^*$  increases and the change in rate of innovation  $i^*$  is ambiguous.

We address this ambiguity by differentiating (26) and (27) with respect to  $s_X$ . To simplify presentation, we evaluate our comparative static results at  $s_X = s_R = 0$ . We have

$$\frac{dm^*}{ds_X} = -\frac{\lambda+1}{\lambda-1} a_X \mu < 0, \quad (29)$$

and

$$\frac{di^*}{ds_X} = -\frac{a_X}{a_R} + m^* \frac{2a_X}{a_R} - \rho^* \frac{dm^*}{ds_X} > < 0. \quad (30)$$

The result reported in (30) implies that the ambiguity surrounding the impact of  $s_X$  on  $i$  prevails. The first term in (30) captures the impact that works through the *relative RPA–R&D cost* channel. An increase in rent protection subsidy rate  $s_X$  reduces the cost of rent protection relative to cost of research. This puts downward pressure on the equilibrium innovation rate  $i^*$ . The second and third terms capture the effects that work through the *imitation* channel  $\rho^* m^*$ . A higher  $s_X$  reduces the effective replacement risk faced by the patent holder  $i_{max}$  and thus the effective discount rate  $\rho^*$ . With  $m^* > 0$ , this translates into a fall in the rewards from blocking imitation relative to the rewards from R&D. This effect is captured by the second term and works to increase  $i^*$ . Finally, it follows from Eq. (29) that a higher  $s_X$  reduces the threat of imitation  $m^*$  and decreases the relative returns from blocking imitation. This effect is captured by the third term and works to increase  $i^*$ .

With  $m^* < 1/2$  the first and the second terms combined points to a decrease in  $i^*$ . The third term points to an increase in  $i^*$ . Substituting  $m^*$  from (26) and  $dm^*/ds_X$  from (29) into (30) implies that  $di^*/ds_X > 0$  if and only if  $\mu^c \equiv (\lambda-1)/[(\lambda+3)(\rho-n)a_R + (6\lambda+2)a_X]$ .<sup>14</sup> Observe that  $\mu^c$  is increasing in  $\lambda$ , and decreasing in  $a_R$ ,  $a_X$  and  $(\rho-n)$ . Thus, the growth-promoting effect of  $s_X$  is more likely to be observed the higher the imitation ease parameter  $\mu$ , the higher the resource requirements in R&D and RPA,  $a_R$  and  $a_X$ , the higher the population-adjusted discount rate  $(\rho-n)$ , and the lower the innovation size  $\lambda$ . We summarize our findings below.

**Proposition 2.** *An increase in the RPA subsidy rate  $s_X$  unambiguously reduces the probability of imitation  $m^*$  but increases the rate of innovation  $i^*$  if and only if the ease of imitation parameter  $\mu$  is above the critical value  $\mu^c \equiv (\lambda-1)/[(\lambda+3)(\rho-n)a_R + (6\lambda+2)a_X]$ .*

This result stands in contrast to Dinopoulos and Syropoulos (2007) who find that more rent protection *always* retards growth. This is not surprising because they consider only the R&D-difficulty-building impact of RPAs without modeling the market-expanding impact of RPAs. Our result also shed light on the interaction between existing IPR regime and private protection of IPRs from a cross-country perspective. Since we interpret  $\mu$  as an indicator of patent enforcement against imitators, our results may seem surprising at first. They suggest that subsidizing private rent protection (or making lawyers more efficient at patent enforcement) is good for growth in countries that have relatively underdeveloped IPR regimes, and bad for growth in countries with more developed IPR regimes. The logic of this outcome hinges on a key distinction between private enforcement of IPRs and the level of  $\mu$ , which captures the technological and institutional constraints on imitation. Whereas private enforcement deters both imitation and innovation, patent enforcement via reducing  $\mu$  deters *only* imitation. As a result, in countries with strong patent protection (low  $\mu$  case), where the threat to patent holders posed by imitation is low, there is little growth-promoting role for private IPR protection. In contrast, in countries with weak patent protection (high  $\mu$  case), private efforts to protect IPRs have a much stronger growth-promoting role since they effectively substitute for the absence of adequate institutional IPR enforcement.

We also use numerical simulations to plot the innovation rate  $i^*$  against the main IPR policy  $s_X$  (See Section 5 for the choice of benchmark parameters). As shown in Fig. 2 the relationship between  $i^*$  and  $s_X$  implies an inverted U-shaped curve and remains robust to a wide range of parameter choices. At low levels of  $s_X$  raising RPA subsidy promotes growth whereas at high levels of  $s_X$  raising RPA subsidy reduces growth. At the benchmark level of  $\mu=0.02$ , the growth maximizing level of  $s_X$  equals  $-0.135$ , that is a tax on RPA. Consistent with the analytical results the innovation-maximizing policy switches from a tax to a subsidy as the ease of imitation parameter  $\mu$  increases. At high levels of  $\mu$ , subsidizing RPAs maximizes growth, whereas at low levels of  $\mu$ , taxing RPAs maximizes growth. We should note that our results are driven by the assumption that RPAs simultaneously deter both innovation and imitation. This is due to the

<sup>13</sup> To see this, rewrite the free entry condition (11) by using Eq. (5) for  $\pi(t)$ , (17) for  $V(t)$ , (14) for  $X(t)$ . Noting (18) this implies  $c(\lambda-1)/\lambda\rho^* = a_R(1-s_R)\mu/m$ , where  $\rho^*$  is increasing in  $s_R$  as shown in (21).

<sup>14</sup> In the context of an interior equilibrium this necessary condition holds if and only if  $\mu \in (\mu^c, \bar{\mu})$ , an interval that has positive measure provided  $(2a_X/a_R) > \rho-n$ . Note that  $\bar{\mu} \equiv (\lambda-1)/\{[(\rho-n)a_R + 2\lambda a_X][(\rho-n)(a_R/a_X) + 2]\}$  is the upper bound for  $\mu$  in an interior equilibrium [see Appendix A for details].



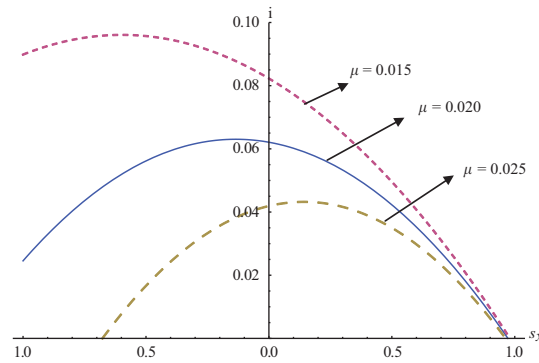


Fig. 2. Plot of  $i^*$  against  $s_X$ .

inherent economies of scope in the context of private RPAs. For our results to hold, only some portion of  $X$  should play a role in both imitation deterrence and innovation deterrence [see footnote 8 for further details].

The inverted-U relationship has found some empirical support in the recent papers. Lerner (2009) investigates the impact of patent reform in a sample of 60 countries over the past 150 years by using patent filings in Great Britain from each country. He finds that following positive patent policy change, patent applications increase. However, the interaction between policy change and existence of a stronger IPR of protection turns out to be negative. Focusing on innovation in the pharmaceutical sector, Qian (2007) concludes that “there appears to be an optimal level of intellectual property rights regulation above which further enhancement reduces innovative activities.”<sup>15</sup>

The inverted U-shaped relationship between innovation and IPR protection is also identified in the theoretical work of Furukawa (2010) and (2007), albeit through a substantially different mechanism. Furukawa constructs a Romer-type variety-expansion based model with technological change tied to cumulative experience. He first identifies the standard growth boosting effect: IPRs raise innovation incentives by limiting imitation and extending the duration of monopoly power. Second, Furukawa shows that higher IPR protection can reduce growth by raising the fraction of monopolized goods and thus leads to a fall in cumulative production, which in turns hinders the process of learning by experience.<sup>16</sup>

### 3.2. Research subsidy

An increase in R&D subsidy  $s_R$  has two competing effects on the relative profitability condition (20). First, it reduces the relative costs of R&D. Second, it implies an increase in the effective discount rate  $\rho^*$  by increasing the expected value loss rate  $2l_{max}$ . This in turn reduces the expected discounted rewards from R&D relative to RPA. In an interior equilibrium with  $m < 1/2$ , the former effect dominates the latter. For a given  $m$ , restoring the relative profitability condition requires an increase in innovation intensity  $i$  and therefore the SS1 curve shifts up.

Similarly, a higher  $s_R$  also exerts two competing effects on the free-entry condition in R&D. It reduces R&D costs and at the same increases the effective discount rate  $\rho^*$ . The net effect is an increase in the returns from R&D, which forces resources to move out of production.<sup>17</sup> For a given  $m$ , restoring the resource constraint requires an increase in innovation intensity  $i$  and hence the SS2 curve also shifts up. The graphical analysis implies that  $i^*$  increases but the change in  $m^*$  appears ambiguous. To resolve the ambiguity on  $dm^*/ds_R$ , we differentiate (26) and (27) with respect to  $s_R$  and evaluate at  $s_X = s_R = 0$ . This gives

$$\frac{dm^*}{ds_R} = \frac{\{(\lambda-1)a_X - [(\rho-n)a_R + 2\lambda a_X][(\rho-n)a_R + 2a_X]\mu - (\rho-n)a_R\}\mu}{(\lambda-1)} > < 0, \tag{31}$$

and

$$\frac{di^*}{ds_R} = \frac{a_X}{a_R} - m^* \frac{2a_X}{a_R} - \rho^* \frac{dm^*}{ds_R} > 0, \tag{32}$$

<sup>15</sup> We should note though that the broad empirical literature offers a variety of results on the link between innovation and IPRs. See Park (2008) for an extensive overview of the empirical literature.

<sup>16</sup> See O'Donoghue and Zweimuller (2004), Akiyama and Furukawa (2009) and Horii and Iwaisako (2007) for models that can generate the inverted U-shaped relationship. See also the recent studies by Chen and Iyigun (2011), Iwaisako and Futagami (2011) and Chu and Pan (2011), who also show that patent length, patent breadth and blocking patents respectively generate an inverted-U effect of IPRs on innovation.

<sup>17</sup> To see this, note again the modified free-entry in R&D condition  $c(\lambda-1)/\lambda\rho^* = a_R(1-s_R)\mu/m$ , where  $\rho^*$  is decreasing in  $s_X$  as shown in (21).

Thus, the ambiguity on  $dm^*/ds_R$  prevails. We confirmed the existence of the ambiguity by running numerical simulations in the interior equilibrium. We summarize our results below:

**Proposition 3.** *An increase in the R&D subsidy rate  $s_R$  increases the rate of innovation  $i^*$  but has an ambiguous impact on the probability of imitation  $m^*$ .*

We can decompose the effects of an increase in R&D subsidy  $s_R$  similar to the case of an RPA subsidy  $s_R$ . The first term in (32) captures the impact that works through the relative RPA–R&D cost channel. An increase in the R&D subsidy rate  $s_R$  reduces the cost of R&D relative to RPA. This puts upward pressure on the equilibrium innovation rate  $i^*$ . The second and third terms capture the impact that works through the imitation channel  $\rho^*m^*$ . A higher  $s_R$  increases the effective replacement risk faced by the patent holder  $i_{max}$  and thus the effective discount rate  $\rho^*$ . With  $m^* > 0$ , this translates into a rise in the rewards from blocking imitation relative to the rewards from R&D. This effect is captured by the second term and works to reduce  $i^*$ . With  $m^* < 1/2$ , the first and the second effects combined point to an increase in  $i^*$ . Finally, from Eq. (31) it follows that a higher  $s_R$  has an ambiguous effect on  $m^*$  and thus an ambiguous effect on  $i^*$  through this channel. Unlike the indeterminate result on  $dm^*/ds_R$ , we conclude from the graphical analysis that the first two effects prevail and thus a higher R&D subsidy boosts the equilibrium rate of innovation  $i^*$ .

### 3.3. Patent enforcement against imitators

A decrease in the extent of patent enforcement against imitators  $\mu$  implies a lower level of rent protection  $X$  through (14). This in turn reduces the demand for labor in all three activities – R&D, RPA and production (see footnote 9 for details) – and thereby relaxes the resource constraint. Consequently, more resource can be allocated to R&D activity, and for a given  $m$ , this implies an upward shift of the SS2 curve in Fig. 1. In contrast, the profitability of innovation relative to imitation remains unaffected and so does the SS1 curve. At the new equilibrium, the rate of innovation  $i^*$  is higher whereas the probability of imitation  $m^*$  is lower. Differentiating (26) and (27) with respect to  $\mu$  and evaluating at  $s_X=s_R=0$ , we find:

$$\frac{dm^*}{d\mu} = \frac{a_R(\rho-n) + 2\lambda a_X}{\lambda-1} > 0 \tag{33}$$

and

$$\frac{di^*}{d\mu} = -\rho^* \frac{dm^*}{d\mu} < 0. \tag{34}$$

We summarize our findings below.

**Proposition 4.** *An increase in the intensity of patent enforcement against imitators through a lower  $\mu$  increases the rate of innovation  $i^*$  and lowers the probability of imitation  $m^*$ .*

## 4. Welfare analysis

We now derive an expression for steady-state welfare. As in Dinopoulos and Syropoulos (2007), the economy immediately jumps to the steady-state; thus the model is free of transitional dynamics.<sup>18</sup> We first express instantaneous utility as a function of the model's endogenous variables. At any instant in time, a fraction  $m$  of goods is available at competitive prices  $p=1$ , while the remaining  $(1-m)$  goods are produced by patent holders and sold at the monopoly price  $p=\lambda$ . Instantaneous utility is given by  $\log u_t = \int_0^{m^*} \log(\sum_{k(\theta,t)} \lambda^{k(\theta,t)} c^*) d\theta + \int_{m^*}^1 \log(\sum_{k(\theta,t)} \lambda^{k(\theta,t)} (c^*/\lambda)) d\theta$ . Substituting this expression into (1), we have

$$(\rho-n)U(i,m,c) = \frac{\log \lambda}{\rho-n} i + \log c - (1-m)\log \lambda. \tag{35}$$

Eq. (35) indicates that steady-state welfare is increasing in the rate of innovation, consumption expenditure and the imitation rate. The third term captures the utility gains from having access to goods at competitive prices rather than monopoly prices.

We consider a social planner who chooses the levels of R&D, RPA and consumption to maximize social welfare  $U$  in (35) subject to the resource constraint. We then compare the resulting Socially Optimal (SO) outcome to the Laissez-Faire (LF) solution. We define per-capita R&D difficulty as  $x=X/N$  and then note  $m=\mu/x$ . The Lagrangian for the social planner is

$$L(i,c,x,k) = \frac{i \log \lambda}{\rho-n} + \log c + \frac{\mu \log \lambda}{x} - \log \lambda + k \left\{ 1 - \left[ 1 + \frac{(\lambda-1)\mu}{x} \right] \frac{c}{\lambda} - a_X x - a_R i x \right\}$$

<sup>18</sup> The intuition is that  $c$ ,  $i$  and  $X$  are all choice variables and thus other endogenous variables reach their steady-state levels at time zero. See Appendix, part (c) of Dinopoulos and Syropoulos (2007) for further details.

where  $k$  is the Lagrange multiplier.<sup>19</sup> The first-order conditions are:

$$\frac{dL}{di} = -a_R k x + \frac{\log \lambda}{\rho - n} = 0, \quad (36)$$

$$\frac{dL}{dc} = \frac{1}{c} - \frac{k}{\lambda} \left[ 1 + \frac{(\lambda - 1)\mu}{x} \right] = 0, \quad (37)$$

$$\frac{dL}{dx} = k \left[ -a_X - a_{Ri} + \frac{\mu c (\lambda - 1)}{x^2 \lambda} \right] - \frac{\mu \log \lambda}{x^2} = 0, \quad (38)$$

and

$$\frac{dL}{dk} = 1 - \left[ 1 + \frac{(\lambda - 1)\mu}{x} \right] \frac{c}{\lambda} - a_{Xx} - a_{Rix} = 0. \quad (39)$$

#### 4.1. Optimal RPA policy

We first compare the RPA levels between the SO and LF solutions. Substituting  $k$  from (37) to (38) and solving for  $x$  gives:

$$a_{Xx} = \frac{[c(\lambda - 1)/\lambda][(\mu/x)(\rho - n)] - a_{Rix}(\rho - n) - \frac{\mu c \log \lambda}{x \lambda} [1 + (\lambda - 1)\mu/x](\rho - n)}{\rho - n} \quad \mathbf{SO}_{RPA} \quad (40)$$

This expression equates the social cost of RPAs (LHS) to the social return from RPAs (RHS). We obtain an analogous expression for the LF level of RPA by substituting  $V(t)$  from (17) into the optimal RPA condition (16), using  $\pi(t)$  from Eq. (5). This yields:

$$a_{Xx} = \frac{[c(\lambda - 1)/\lambda][(\mu/x)(\rho - n) + i]}{\rho - n + 2i} \quad \mathbf{LF}_{RPA} \quad (41)$$

Comparing these two expressions, we see four reasons for the laissez-faire and socially-optimal levels of RPA to differ.

- The *replacement avoidance effect*. Patent holders realize higher returns from RPAs since they use RPAs not only to deter imitation but also to reduce the threat of replacement coming from rival R&D firms. The gains from avoiding replacement do not enter social planner's welfare problem. This effect is captured by the difference in the RPA returns:  $(\mu/x)(\rho - n) + i$  in Eq. (41) versus  $(\mu/x)(\rho - n)$  in Eq. (40). Thus, private firms conduct too much RPA, and this calls for a tax on RPAs.
- The *resource-wasting effect*. Patent holders do not consider the negative externalities created by their innovation-detering activities, which increase the input requirement to for a given level of R&D intensity  $i$ . This effect is captured by the  $-a_{Rix}(\rho - n)$  term in Eq. (40), which follows from the  $-a_{Rik}$  term in Eq. (38). Thus private firms conduct too much RPAs, and this calls for a tax on RPAs.
- The *monopoly-pervasiveness effect*. Patent holders do not take into account the negative externalities created by their imitation-detering activities, which reduces the share of competitive markets  $m$  and thereby increase the prevalence of monopoly pricing. This effect is captured by the term  $-[\mu c \log \lambda / x \lambda] [1 + (\lambda - 1)\mu/x](\rho - n)$  in Eq. (40), which follows from the  $-(\mu \log \lambda) / x^2$  term in (38). Thus private firms conduct too much RPA, and this calls for a tax on RPAs.
- The *intertemporal spillover effect* of further innovation on RPAs. Patent holders discount the returns from RPAs by more than the social planner due to the threat of replacement and the associated deterrence costs. These two effects adversely affect the valuation of patent holders. This effect is captured by the difference in the discount rates:  $\rho - n + 2i$  in Eq. (41) versus  $\rho - n$  in Eq. (40). Thus private firms conduct too little RPA and this calls for a subsidy on RPAs.

Our results on the welfare effects of RPAs contrast strongly with those of [Dinopoulos and Syropoulos \(2007\)](#). In their setting RPAs generate no positive social returns but consume resources. Thus they generate a pure resource-wasting externality, which calls for a prohibitively high tax on these activities. In our model the presence of competitive markets and the imitation-detering effect of RPAs give rise to additional welfare externalities, both positive and negative. In particular, we identify two additional effects on the *returns* from RPAs that materialize due to our modeling of imitation  $\mu > 0$ . First, RPAs generate *positive* returns both at the social and private level by suppressing imitation. For private firms, a lower imitation rate  $m$  implies an increase in the share of the monopolized market. This generates additional private returns given by  $[c(\lambda - 1)/\lambda][(\mu/x)[(\rho - n)]/(\rho - n + 2i)]$  in Eq. (41). For the social planner, a lower imitation rate implies a

<sup>19</sup> The resource constraint in  $L$  is derived by substituting  $N_Q$  from (7),  $N_R$  from (13), and  $N_X$  from (9) into (22), and by using  $R = ixN$  and  $m = \mu/x$ .

reduction in production labor demand and thus relaxes the resource constraint. This generates additional social returns given by  $[c(\lambda-1)/\lambda](\mu/x)$  in Eq. (40).<sup>20</sup> Since  $(\rho-n)/(\rho-n+2i) < 1$ , private returns fall short of social returns at the margin and this calls for subsidizing RPAs. In addition, RPAs generate *negative* social returns because they reduce the scope of the competitive market and thereby increase the pervasiveness of monopoly pricing. This is captured by the  $-\left[(\mu c \log \lambda)/(x\lambda)\right][1+(\lambda-1)\mu/x]$  term in Eq. (40) and calls for taxing RPAs. We have the following result.

**Proposition 5.** *The optimal RPA policy may be either a tax or subsidy depending on the parameters of the model.*

#### 4.2. Optimal R&D policy

We turn next to the comparison of R&D levels between the SO and LF solutions. Substituting  $k$  from (37) into (36) gives an expression for the SO research condition:

$$a_{R^X} = \frac{c \log \lambda}{\lambda \rho - n} [1 + (\lambda - 1)m]. \quad \mathbf{SO}_{RD} \quad (42)$$

This expression equates the social cost of research (LHS) with the social return from research (RHS). We obtain the analogous LF solution by substituting  $V$  from (17) and  $\pi$  from Eq. (5) into the free-entry in R&D condition (11):

$$a_{R^X}(1-s_R) = \frac{c}{\lambda} \frac{\lambda-1}{\rho-n+2i} (1-2m). \quad \mathbf{LF}_{RD} \quad (43)$$

Comparing these two expressions, we see three reasons for the laissez-faire and socially optimal levels of R&D to differ.

- The *monopoly distortion effect*. Private researchers consider the monopoly the mark-up rate on the newly-innovated products,  $\lambda - 1$ , whereas the social planner is concerned about the impact on consumer utility,  $\log \lambda$ . Since  $\lambda - 1 > \log \lambda$ , this effect implies that the private level of R&D is too high, so the optimal R&D policy points towards a tax.
- The *intertemporal spillover effect* of further innovation. Private researchers account for the expected capital loss due possible replacement by a rival innovator and the costs associated with deterring innovators. Thus the discount rate they consider is  $\rho - n + 2i$ , which is greater than the social planner's discount rate  $\rho - n$ . This implies that the private level of R&D is too low, so the optimal R&D policy points towards a subsidy.
- Two new effects due to imitation. One is the *profit stealing effect*. Private researchers consider the fact that successful innovators realize only a fraction  $(1-2m)$  of the potential monopoly profits. This is due to expected loss in market share due to imitation as well as costs incurred to deter imitators. The other is the *competitive pricing effect*. The social planner takes into account the reduction in the average price level faced by consumers due to imitation and the resulting competitive markets, which imply a higher level of consumption. This effect is valued by the planner as a return to R&D and is captured by the term  $1 + (\lambda - 1)m$ ; however, it is not considered by private researchers. To sum up, the effects that are linked to imperfect IPR enforcement against imitators induce private firms to undertake too little R&D; hence, the optimal R&D policy points toward a subsidy.

We summarize our findings below.

**Proposition 6.** *The optimal R&D policy can be either a tax or subsidy depending on the parameters of the model. There are two new effects due to the presence of imitation, profit stealing effect and the competitive pricing effect, which work to strengthen the case for R&D subsidies.*

#### 4.3. Optimal patent enforcement against imitators

Differentiating social welfare (35) with respect to  $\mu$  using (27) to substitute for  $i$  and evaluating at  $s_X = s_R = 0$  gives:<sup>21</sup>

$$(\rho - n) \frac{dU}{d\mu} = \frac{\partial U}{\partial i} \frac{di^*}{d\mu} + \frac{\partial U}{\partial m^*} \frac{dm^*}{d\mu} = -\frac{2a_X}{a_R} \log \lambda \frac{dm^*}{d\mu} < 0. \quad (44)$$

A lower  $\mu$  increases the arrival rate of innovations  $i^*$  and thus increases the dynamic welfare gains associated with sustained improvements in the quality of goods. It also decreases the share of competitive markets  $m^*$  and thus the range of high-quality goods available at competitive prices. This generates static welfare losses by amplifying the price distortions associated with monopoly markets, *monopoly price distortion effect*. Eq. (44) implies that the dynamic gains through higher innovation dominate the static losses stemming from higher prices, thus the welfare maximizing level of  $\mu$  equals zero. With  $\mu = 0$ , the model produces an equilibrium in which imitation is absent. Patent holders capture the entire market, and innovation occurs at its maximum rate,  $i^* = i_{max}$ . We should note that optimal  $\mu = 0$  is specific to our model.

<sup>20</sup> To see this, focus on (38) and consider the impact of an incremental increase in  $x$  on the resource constraint, noting  $m = \mu/x$ .

<sup>21</sup> Note that when evaluated at  $s_X = s_R = 0$ , it follows that  $dc^*/d\mu = 0$ . Thus changes in  $c^*$  drop out of welfare considerations. See Appendix A for details.

Kwan and Lai (2003) identify a positive optimal  $\mu$  in a Romer type variety-expansion growth model with exogenous imitation. More specifically, they identify two competing effects of lower imitation on welfare: one is the instantaneous loss in consumption and the subsequent transitional impact, the other is the increased rate of growth in consumption due to increased R&D investment. In their model, these competing effects give rise to a positive value for optimal  $\mu$ . In our model, we also identify competing effects due to lower  $\mu$  (dynamic gains vs. static losses); however, the dynamic effect always dominates the static effect, implying welfare maximization occurs at  $\mu=0$ .

## 5. Numerical analysis

We run numerical simulations of the model to determine the socially-optimal levels of  $i$ ,  $x$  and  $c$ , and the corresponding RPA and R&D subsidy/tax rates. We choose the following benchmark parameters:  $\lambda=1.25$ ,  $\rho=0.07$ ,  $n=0.01$ ,  $a_X=1$ ,  $a_R=7$ ,  $\mu=0.02$ . We assume that the only available policy tool is RPA subsidies/taxes and thus set  $s_R=0$ .<sup>22</sup> The benchmark outcomes for the endogenous variables when  $s_X$  is set to zero are shown in column 2 of Table 1. The benchmark outcomes when  $s_X$  is optimally chosen are shown in column 3. We also calculate the shares of production, R&D and RPA workers in the total labor force, which we denote as  $sh_Q$ ,  $sh_R$  and  $sh_X$ , respectively. At the benchmark parameters, the planner's optimal policy is to set  $s_X \rightarrow -33.1\%$  and hence to tax RPAs at this rate. We should note that the optimal RPA policy can be a tax or a subsidy depending on the parameters of the model. In particular, we find that at low levels of  $\mu$ , it is optimal to tax RPAs whereas at high levels of  $\mu$ , it is optimal to subsidize RPAs. Hence, in economies with strong IPR protection against imitation, there is less scope for raising welfare by promoting RPAs. Conversely, in economies with weak IPR protection against imitators, there is more scope for improving welfare through promoting RPAs. This suggests that private patent protection in the form of RPAs can indeed substitute for effective public protection against imitation and lead to welfare gains. The relationship between the RPA subsidy and welfare is illustrated for in  $(s_X, U)$  space of Fig. 3 for several values of  $\mu$ . As seen in this figure, the optimal RPA subsidy is increasing in the degree of imitation ease  $\mu$ . We note that the growth effects of  $s_X$  also depend on  $\mu$  in the same fashion.

## 6. Conclusion

We construct a Schumpeterian growth model in which patent holders engage in RPAs to deter further innovation and imitation. In our model, RPAs have two first order effects. First, they raise the difficulty of further research and thus suppress growth. Second, they expand the expected market share of patent holders and thus promote growth. We investigate the implications of policy changes for growth and welfare. We find that a more stringent IPR protection that subsidizes patent holders' RPAs has an ambiguous impact on innovation rate. Strengthening IPRs in this fashion promotes growth if and only if patent enforcement against imitators is below a certain level. In effect, this implies that lawyers are good for growth in countries with weak IPR enforcement as private enforcement acts as a substitute for inadequate IPR institutions.

Our welfare analysis implies that it may be optimal to tax or subsidize RPAs depending on the parameters of the model. Thus, pulling RPAs down to zero via a prohibitively high tax rate may not be optimal given the competing effects of RPAs on welfare. We also find that the presence of imitation gives rise to additional positive externalities associated with R&D and thus our model strengthens the case for subsidizing R&D. We should note that our model can be extended in various directions. One could consider a North–South setting with Northern innovation and Southern imitation and fully incorporate the profit maximizing behavior of imitators. One could model patent litigation as a two-way interaction between innovators and imitators and study the implications for patent policy and growth.

Our paper is related to work on the consequences of private legal activity for economic growth. For the most part, this literature views litigation and lobbying in a negative light, stressing that rent-seeking distorts relative prices and reduces the resources available for production. We break from this line of research by recognizing a growth-promoting role for private legal activity. Because the rents accruing to patent holders are in part the legitimate rewards to research and development, private RPAs may serve to align the private and social returns to innovation and, thereby raise social welfare.

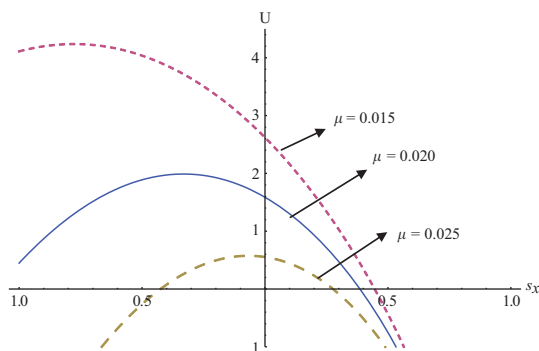
Finally, our paper addresses what we view as the central gap in the literature on economic growth. While models of endogenous technical progress have been at the center of theorizing on economic growth, the dominant theme in the empirical growth literature has been the importance of institutions, and in particular, the protection of property. Closing this gap will require a much better understanding of how different institutional structures affect the incentives to engage in innovation. In the current paper, we do this by modeling the private patent protection efforts of firms in response to IPR policies.<sup>23</sup> In our companion paper Davis and Şener (2012), we take a different approach by introducing an institutional quality measure to captures the IPR regime's effectiveness in distinguishing between illegitimate imitation and legitimate innovation and channeling the economy's resources accordingly. We believe that addressing the empirics-theory gap in the context of institutions, IPRs and growth should be high on the agenda of growth economists.

<sup>22</sup> See the Supplementary Appendix for a discussion of benchmark parameter values and the case where both RPA and R&D policies are available.

<sup>23</sup> Indeed, a number of recent papers have also started filling this gap. Cozzi and Galli (2011) construct a Schumpeterian growth model to study the evolution of jurisprudential changes in IPR protection and its implications for wage inequality. Chu et al. (2012) consider innovation-blocking patents in an endogenous growth model with horizontal and vertical innovation.

**Table 1**  
Optimal RPA policy.

	$s_X=0$	Optimal $s_X$
$s_X$	0	−33.1%
$i$	0.062	0.061
$c$	1.036	1.051
$x$	0.086	0.068
$m$	0.234	0.294
$sh_Q$	0.877	0.903
$sh_R$	0.037	0.029
$sh_X$	0.086	0.068



**Fig. 3.** Optimal RPA policy  $s_X$ .

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## Appendix A

We derive the necessary and sufficient conditions for an interior equilibrium. To simplify the analysis, we set the subsidy rates equal to zero in (26), (27) and (28). This yields:

$$m_0^* = \frac{a_R(\rho-n) + 2\lambda a_X}{\lambda-1} \mu,$$

$$i_0^* = \frac{a_X}{a_R} - \left[ 2 \frac{a_X}{a_R} + (\rho-n) \right] \left[ \frac{a_R(\rho-n) + 2\lambda a_X}{\lambda-1} \right] \mu,$$

and

$$c_0^* = \frac{2\lambda a_X + (\rho-n)\lambda a_R}{2\lambda a_X + (\rho-n)a_R}.$$

Here we adopt the notational convention that the subscript “0” denotes the value of a variable in the case in which  $s_X = s_R = 0$ . Given the model’s parametric restrictions, it immediately follows that  $m_0^* \geq 0$  and  $c_0^* > 0$ . For  $i_0^* > 0$ , we need  $m_0^* < i_{max}/\rho^*$ . In other words, the competitive market share in  $m^*$  equilibrium should be lower than a certain level. Using  $m_0^*$  from above, we can express this condition as  $\mu < \bar{\mu}$ , where  $\bar{\mu} \equiv (\lambda-1) / \{[(\rho-n)a_R + 2\lambda a_X][(\rho-n)(a_R/a_X) + 2]\}$ . Observe that  $\bar{\mu}$  is increasing in  $\lambda$ , decreasing in  $a_R$  and  $(\rho-n)$ , and decreasing in  $a_X$  iff  $(2a_X/a_R)\lambda^{1/2} > \rho-n$ .



## Appendix B. Supplementary appendix

Supplementary data associated with this article can be found in the online version at <http://dx.doi.org/10.1016/j.eurocorev.2012.07.002>.

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