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Supplementary Appendix for

Private Patent Protection in the Theory of Schumpeterian Growth

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Abstract

The first section of this supplementary appendix presents further details on the numerical analysis: choice of benchmark parameter values, numerical analysis of R&D policy, and numerical analysis of joint R&D and RPA policies. The second section presents an extension of the main model where we permit firms to distinguish between innovation and imitation deterring RPAs and allow for varying degrees of spillovers between the two activities.

1. Numerical Analysis

1.1. Choice of Benchmark Parameters

The size of innovations, λ, corresponds to the gross mark up (the ratio of the price to the marginal cost) enjoyed by innovators and is estimated as ranging between 1.05 and 1.4 [see Basu, 1996, and Norrbin, 1993]. The population growth rate, n, is calculated as the annual rate of population growth in the US between 1975 and 1995 according to the World Development Indicators (World Bank, 2009). The subjective discount rate, ρ, is set at 0.07 to reflect a real interest rate of 7 percent, consistent with the average real return on the US stock market over the past century as calculated by Mehra and Prescott (1985). The relative RPA-R&D resource requirement is set at 1/7 to generate an innovation rate in the neighborhood of 6 percent. The resulting growth rate in utility $g = i \log\lambda$ is 1.5%, which is between 0.5% percent (Denison's (1985) estimate for long-term growth driven by knowledge advancements) and 2% (the average US GDP per capita growth rate from 1950 to 1994 as reported in Jones (2005)) .[1] We set the ease of imitation parameter μ at 0.02 such that patent holders command a sizeable 76 percent of the market and the rest is captured by the imitators. We set the subsidy rates for R&D and RPA at zero. We define the shares of production, R&D and RPA workers in the total labor force as sh_Q, sh_R and sh_X, respectively. The resulting benchmark LF equilibrium values are illustrated in column 2 of Table 1. We solve the social planner's problem numerically by imposing the condition $1\text{-}2m > 0$ which ensures that the SO solution can be replicated in a LF setting. We investigate three different cases.

[1] In general, the benchmark parameters and outcomes are in line with the recent theoretical growth papers that use numerical simulations [see Jones, 2002, Şener, 2008, Steger, 2003, and Segerstrom 2007]. We used Mathematica version 8 for our numerical simulations and conducted extensive robustness checks using a wide range of parameters. The Mathematica programs are available upon request.

1. 2. Numerical simulations of R&D Policy

First, we consider the case only with R&D policy and set $s_x = 0$. The social planner chooses the values shown in column 4 of Table A1. To generate this outcome, the planner would set the R&D subsidy rate $s_R \rightarrow 100\%$. This is also illustrated in (s_R, U) space in Figure A1. We find that this corner solution holds for all the parameters we considered that are consistent with an interior equilibrium. Intuitively, optimal s_R converging to 100% implies that the welfare gains from marginal innovation always dominate the welfare losses. We are keen to state that our corner solution simply strengthens the case for R&D subsidies and should not be viewed as the ultimate applicable R&D subsidy rate given the parsimonious nature of our model.[2] To understand the source of this corner solution, we consider two variants of our model. In the first one, we drop imitation but maintain the assumption of full labor mobility. We find that the optimal R&D subsidy rate again converges to 100%. In the second one we keep imitation but drop the assumption of full labor mobility. In particular, we assume as in Dinopoulos and Syropoulos (2007) that a certain portion of the labor force, classified as general-purpose labor, is mobile between R&D and production, whereas the remaining portion, classified as specialized labor, can only be employed in RPAs. In this case, we find that the optimal R&D policy indeed points to an interior RPA subsidy rate. We note that the Dinopoulos and Syropoulos (2007) setting, which does not include imitation and RPAs related with imitation deterrence, also implies an interior optimal R&D subsidy rate. Hence we conclude that our corner solution *is* exclusively linked to the assumption of full labor mobility between R&D, RPAs and production. The intuition is that

[2] We should note that our R&D subsidy recommendation, at least from a qualitative perspective, is in line with a large body of growth literature. See among others Segerstrom (2007), Alvarez-Pelaeza and Groth (2005), Li (2001), Jones and Williams (2000), Romer (1990), and the variety-expansion based model of Grossman and Helpman (1991). See Jones and Williams (1998) for an empirical paper that reports that actual R&D investment in the US is 25 to 50 percent of the optimal R&D investment. See Şener (2008) for a numerical simulation in a Schumpeterian growth setting with RPAs and diminishing technological opportunities where the optimal R&D subsidy rate is calculated to be between 5 and 25 percent.

when labor is perfectly mobile across activities, subsidizing R&D and thereby moving labor away from RPAs towards innovation leads to substantial increases in the rate of innovation such that the dynamic welfare gains always turn out to dominate the static welfare losses for all interior innovation rates considered.[3]

1. 3. Numerical simulations of joint R&D and RPA policies

Next we consider the case in which the social planner chooses the rates of both RPA and R&D subsidies. Numerical simulations imply that that the social planner chooses the values shown in column 5 of Table A1.To generate this outcome, the planner sets $s_R \to 100\%$ and $s_X \to 64.1\%$. This is also illustrated in (s_R, s_X ,U) space in Figure A2. The planner opts for a corner solution in R&D policy by maximizing s_R. On the RPA side, the optimal policy is a subsidy rate of 64.1%. With s_R set at its maximum level and resources being attracted to R&D, attaining an RPA level of $x \to 0.04$ turns out to require an RPA subsidy. This corner solution, setting $s_R \to 100\%$ and adjusting s_X such that $x \to 0.04$, is robust to a wide range of parameters.

2. Model with Variable Scope of RPA Spillovers

This section of the appendix addresses the role of economies of scope in innovation and imitation deterring RPAs. In the model presented in the body of the paper, we assume that any RPA contributes simultaneously to innovation and imitation deterrence. This assumption is embodied in the structures of equations (12) and (14). Here we relax this assumption to allow patent owners to direct RPAs toward either innovation deterrence or imitation deterrence. We

[3] In fact, in the setting with no imitation and full-labor mobility, $s_R \to 100\%$ implies that the innovation rate converges to infinity. In our model, $s_R \to 100\%$ generates a finite innovation rate albeit an unusually high rate of 214.3 percent for the benchmark case.

also permit some economies of scope, with innovation-deterring RPAs influencing the effectiveness of imitation deterring RPAs and vice versa. Using simulation exercises, we demonstrate that the inverted-U relationship between the RPA subsidy rate and the equilibrium innovation rate depends on the existence of economies of scope in RPAs.

Let X_R and X_M denote innovation deterring and imitation deterring RPAs, respectively. The total level of RPAs *X* then equals:

$$X = X_R + X_M \,. \tag{A.1}$$

Let $\hat{X}_R$ and $\hat{X}_M$ denote the *effective* levels of innovation and imitation deterrence RPAs. We specify these as:

$$\hat{X}_R = X_R(t)^{\alpha} X_M(t)^{1-\alpha} \tag{A.2}$$

and

$$\hat{X}_M = X_R(t)^{\beta} X_M(t)^{1-\beta} \,. \tag{A.3}$$

Here, we assume $\alpha > \beta$, so that innovation deterring RPAs contribute more to effective innovation deterrence than to effective imitation deterrence.

Depending on the values chosen for α and β, this model encompasses several potential relationships for the two forms of RPA. First, when $\alpha = \beta = 1$, we have $\hat{X}_R = \hat{X}_M = X$, and the model is identical to the one presented in the paper. Second, if $\alpha = 1$ and $\beta = 0$, economies of scope are absent, and we have $\hat{X}_R = X_R(t)$ and $\hat{X}_M = X_M(t)$. Finally, if $\alpha, \beta \in (0,1)$, then there are economies of scope in RPAs in that the effective levels of innovation and imitation deterrence depend on both underlying variables.

Using effective RPA inputs, we may rewrite the free entry, innovation and imitation equations, respectively (11), (12) and (14), as

$$V(t) = (1 - s_R) a_R \left[X_R(t)^{\alpha} X_M(t)^{1-\alpha} \right], \tag{A.4}$$

$$i = \frac{R(t)}{X_R(t)^{\alpha} X_M(t)^{1-\alpha}}, \tag{A.5}$$

and

$$m = \frac{\mu N(t)}{X_R(t)^{\beta} X_M(t)^{1-\beta}}. \tag{A.6}$$

Substituting (A.1), (A.4), (A.5) and (A.6) into the arbitrage condition (15) and maximizing the expected instantaneous return to patent ownership with respect to $X_R(t)$ and $X_M(t)$, respectively, we have the following FOCs:

$$(1 - s_x) a_x X_R = \beta m \pi(t) + \alpha i V \tag{A.7}$$

and

$$(1 - s_x) a_x X_M = (1 - \beta) m \pi(t) + (1 - \alpha) i V. \tag{A.8}$$

Summing these FOCs, we find an expression for the optimal level of total RPA as $(1 - s_x) a_x X(t) = m\pi(t) + iV(t)$, which is identical to (16). Similarly, substituting (16) into the arbitrage condition, we recover the expressions for the relationship of profit to value and for the effective discount rate that were given in (17) and (18).

Taking the ratio of (A.7) and (A.8), we have

$$\left[\beta m \pi(t) + \alpha i V(t) \right] X_M(t) = \left[(1 - \beta) m \pi(t) + (1 - \alpha) i V(t) \right] X_R(t). \tag{A.9}$$

Substituting (17) and (18) into this expression provides expressions for the share of total RPA resources devoted to innovation deterrence:

$$q(t) \equiv \frac{X_R(t)}{X(t)} = \frac{\beta m \rho^* + \alpha i}{m\rho^* + i}. \tag{A.10}$$

The share of RPA resources allocated to innovation deterrence depends positively on the contribution of X_R to effective innovation deterrence, α, times the probability of replacement, i, and on its contribution to effective imitation deterrence, β, times the probability of imitation, m, and the effective discount rate ρ^*. With $\alpha > \beta$, it follows that the larger the discount rate ρ^*, the greater the importance of current relative to future losses, and the greater the importance of effective imitation deterrence in the allocation of RPA resources (1 - q).

Taking the ratio of (A.4) and (16), utilizing the definition of $q(t)$, and combining the resulting expression with (17), we obtain revised expressions for the *relative profitability condition*, SS1,

$$i = \frac{(1-s_x)a_x}{(1-s_R)a_R q(t)^{\alpha}(1-q(t))^{1-\alpha}} - \left[\rho - n + 2\frac{(1-s_x)a_x}{(1-s_R)a_R q(t)^{\alpha}(1-q(t))^{1-\alpha}}\right]m, \tag{A.11}$$

and for the effective discount rate:

$$\rho^* = \rho - n + 2\frac{(1-s_x)a_x}{(1-s_R)a_R q(t)^{\alpha}(1-q(t))^{1-\alpha}}. \tag{A.12}$$

We derive an expression for optimal consumption by substituting (5) into (17) and substituting the resulting expression for $V(t)$ into (A.4), and using (A.6) to eliminate X, which provides

$$c=\left[\frac{\lambda}{\lambda-1}\right]\left[\frac{(\rho-n)(1-s_R)a_R q^{\alpha}(1-q)^{1-\alpha}+2(1-s_x)a_x}{q^{\beta}(1-q)^{1-\beta}}\right]\frac{\mu}{m}. \tag{A.13}$$

Now substituting (7), (13), (9) and (A.5) into the resource constraint and using (A.6) to eliminate *X* and (A.13) to eliminate *c*, we have

$$\begin{aligned}1=&\left[\frac{1+(\lambda-1)m}{\lambda}\right]\left[\frac{\lambda}{\lambda-1}\right]\left[\frac{(\rho-n)(1-s_R)a_R q^{\alpha}(1-q)^{1-\alpha}+2(1-s_x)a_x}{q^{\beta}(1-q)^{1-\beta}}\right]\frac{\mu}{m}\\&+\left[\frac{a_R q^{\alpha}(1-q)^{1-\alpha}i+a_X}{q^{\beta}(1-q)^{1-\beta}}\right]\frac{\mu}{m},\end{aligned} \tag{A.14}$$

which is a revised version of the resource constraint condition, SS2. In the revised model, equilibrium values for the innovation rate *i*, the probability of imitation *m*, and the share of RPA resources devoted to innovation deterrence *q* are determined by a system of three equations. These are the condition for the optimal allocation of RPAs (A.10), the relative probability condition (A.11), and the resource constraint condition (A.14).

We use these equations to conduct numerical simulations to illustrate the role of economies of scope for the two RPAs in the model. In our first exercise, we use the benchmark parameterization as in the body of the paper: λ = 1.25, ρ = 0.07, n = 0.01, a_X = 1, a_R = 7, μ = 0.02 and consider combinations of values for α and β such that $\alpha,\beta\in\{0,0.2,0.4,0.6,0.8,1.0\}$. We find the following results. First, when $\alpha=\beta=0$ or $\alpha=\beta=1$, the revised model is identical to the original model, and we obtain results for innovation rate and probability of imitation that are identical to those in the paper.[4] Second, when $\alpha=1$ and $\beta=0$ or when $\alpha=0$ and $\beta=1$ there are no economies of scope between the two forms of RPA. In this case, we find that the

[4] Indeed, given $\alpha=\beta\in(0,1)$ the revised model is isomorphic to the original model, up to scalars multiples of two parameters: $\mu'=\mu\alpha^{-\alpha}(1-\alpha)^{\alpha-1}$ and $a_R'=\alpha^{\alpha}(1-\alpha)^{1-\alpha}a_R$.

equilibrium innovation rate is decreasing in the RPA subsidy. That is, in the absence of economies of scope for RPAs, we do not obtain an inverted-U shaped relationship between these variables. Third, for all other combinations of α and β within the set above, we consistently find the inverted-U shaped relationship reported in the paper. Finally, we consider a case with minimal economies of scope between RPAs. In particular, we consider a slight deviation from $\alpha = 1$ and $\beta = 0$ (the case of no economies of scope) by setting $\alpha = .99$, $\beta = .01$. This case too implies an inverted-U shaped relationship between the RPA subsidy and the equilibrium innovation rate. The numerical simulations use *Mathematica* version 8 and are available from the authors upon request.

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Table A1: Optimal RPA and R&D Policies

	Benchmark	Optimal RPA	Optimal R&D	Both RPA and R&D Optimal
i	0.062	0.061	2.143	1.731
c	1.036	1.051	0.400	0.143
x	0.086	0.068	0.040	0.040
m	0.234	0.294	0.500	0.500
sh_Q	0.877	0.903	0.360	0.129
sh_R	0.037	0.029	0.600	0.831
sh_X	0.086	0.068	0.040	0.040
Optimal s_X	0	-33.1%	0	64.1%
Optimal s_R	0	0	100%	100%

Figure A1: Optimal R&D Policy s_R

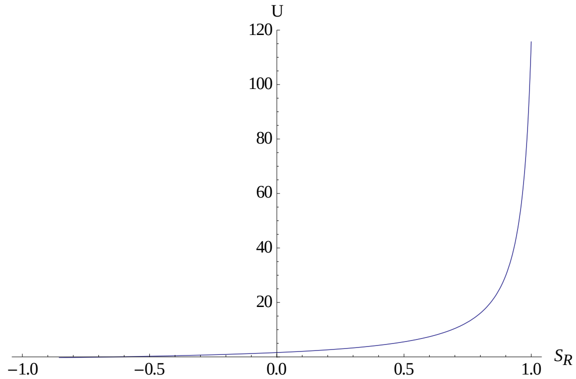

Figure A2: Optimal RPA and R&D Policy