APPENDICES[*]

for

LABOR MARKET RIGIDITIES AND R&D-BASED GROWTH IN THE GLOBAL ECONOMY

by

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Appendix A

(not to be considered for publication, to be made available on the author's web site and also upon request)

1. Existence and uniqueness of the steady-state equilibrium for the basic model with $h^E = h^A = 0$

To prove the existence and uniqueness of the steady-state equilibrium, I follow a graphical approach. Recall that in the basic model, $h^A = h^E = 0$ is assumed. To simplify notation let w* stand for the minimum wage w_L*, and let subscript EQM refer to steady-state equilibrium values. First, I analyze $(n^E)_{EQM}$. Substituting for $P = \lambda b^E w_L^*$ from SS(E) into SS(A) using $\eta^A = (1 - \eta^E)$ and $n^A = (1 - n^E)$ yields:

$$\left(\frac{\lambda-1}{\sigma}+\frac{\phi}{2}\right)=\frac{(1-\eta^E)}{(1-n^E)}\left(\frac{\lambda w^* b^E}{b^A}\right)^2\left((\rho-n)a^A k+\frac{\phi(1-\eta^E)}{2(1-n^E)}\right). \quad \mathbf{n^E\ (.)} \qquad \text{(A.1)}$$

Equation (A.1) implicitly defines n^E in terms of the parameters of the model. Note that the LHS of (A.1) is a constant, whereas the RHS is increasing in n^E for $n^E \in (0, 1)$. As $n^E \to 1$, the RHS $\to \infty$; and as $n^E \to 0$, the RHS $\to INT_0$, where $INT_0 = (1 - \eta^E)(\lambda w^* b^E/b^A)^2[(\rho - n)a^A k + (\phi(1 - \eta^E)/2)]$. Figure A.1 plots the RHS and the LHS of (A.1) with n^E on the horizontal axis. The necessary and sufficient condition for the existence and uniqueness of $(n^E)_{EQM}$ in the domain (0, 1) is

$$\left(\frac{\lambda-1}{\sigma}+\frac{\phi}{2}\right)>(1-\eta^E)\left(\frac{\lambda w^* b^E}{b^A}\right)^2\left((\rho-n)a^A k+\frac{\phi(1-\eta^E)}{2}\right). \qquad \text{(A.2)}$$

The next step is to investigate $(u^E)_{EQM}$, which is implicitly defined by SS(U) as a function of n^E and the parameters of the model:

$$\left(\frac{\lambda-1}{\sigma}+\frac{\phi}{2}\right)=\frac{\eta^E[\lambda w^*(1-u^E)]^2}{n^E}\left((\rho-n)a^E k+\frac{\phi\eta^E}{2n^E}\right) \qquad \mathbf{u^E(n^E;.)} \qquad \text{(A.3)}$$

Note that the LHS of (A.3) is a constant, whereas the RHS is decreasing in u^E for $u^E \in (0, 1)$. As $u^E \to 1$, the RHS $\to 0$, and as $u^E \to 0$, the RHS $\to INT_1$, where $INT_1 = \eta^E[\lambda w^*]^2[(\rho - n)ak + (\phi \eta^E/2n^E)]/n^E$. Figure A.2 plots the RHS and the LHS of (A.3) with u^E on the horizontal axis. Hence, given a unique $(n^E)_{EQM} \in (0, 1)$ exists, the necessary and sufficient condition for the existence and uniqueness of $(u^E)_{EQM}$ in the domain (0, 1) is

$$\left(\frac{\lambda-1}{\sigma}+\frac{\phi}{2}\right)<\frac{\eta^E[\lambda w^*]^2}{(n^E)_{EQM}}\left((\rho-n)a^E k+\frac{\phi\eta^E}{2(n^E)_{EQM}}\right). \qquad \text{(A.4)}$$

Figure A.1. The existence and uniqueness of n^E

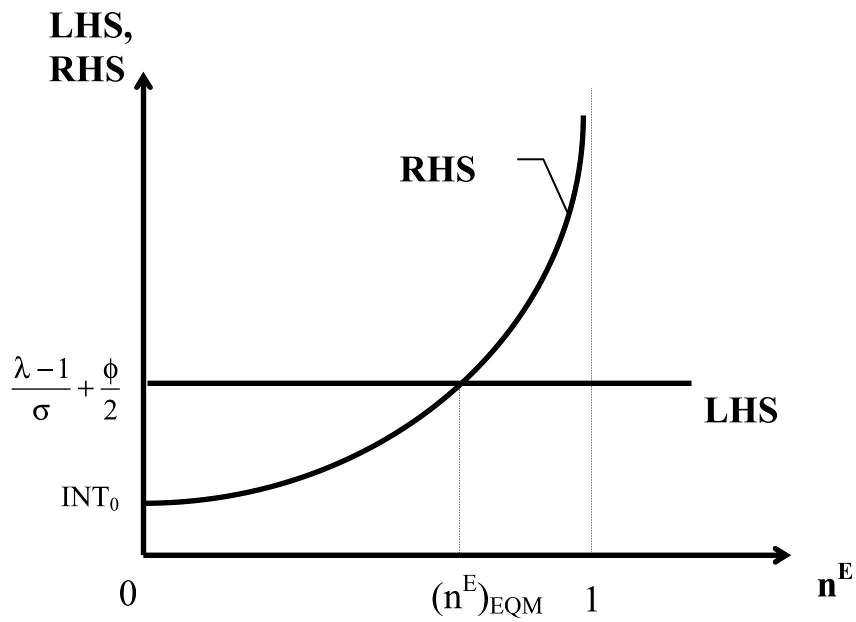
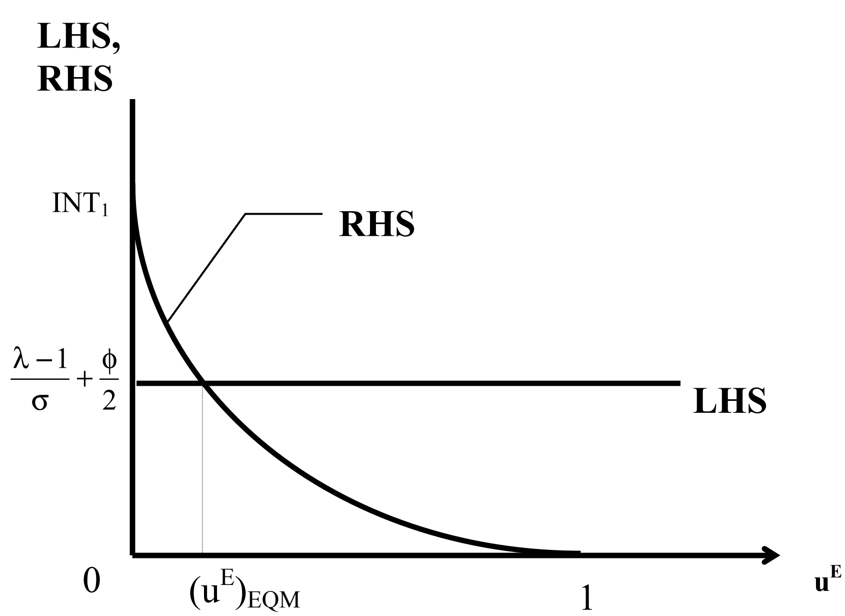


Figure A.2. The existence and uniqueness of u^E



I now analyze $(\theta_0^i)_{EQM}$ for i = A, E. Solving for w_H^A from (7), $\Pi^A(t)$ from (17), n^A from (22) and I^A from (23), and substituting the resulting expressions into (16) using (14), (15) and $P = \lambda b^E w_L^*$ gives:

$$\frac{(\lambda-1)}{\sigma} = \frac{1}{\theta_0^A}\left(\lambda(\rho - n)a^A kw^* \frac{b^E}{b^A} + \frac{\phi}{2\theta_0^A}(1-(\theta_0^A)^2)\right), \qquad \boldsymbol{\theta_0^A(.)} \qquad (A.5)$$

which implicitly defines θ_0^A as a function of the parameters of the model. Note that the LHS of (A.5) is a constant, whereas the RHS is decreasing in θ_0^A for $\theta_0^A \in (0, 1)$. As $\theta_0^A \to 0$, the RHS $\to \infty$; and as $\theta_0^A \to 1$, the RHS $\to$ INT_2, where $INT_2 = \lambda(\rho - n)a^A kw^*(b^E/b^A)$. Figure A.3 plots the RHS and the LHS of (A.5) with θ_0^A on the horizontal axis. The necessary and sufficient condition for the existence and uniqueness of $(\theta_0^A)_{EQM}$ in the domain (0, 1) is

$$\frac{\lambda-1}{\sigma} > \lambda(\rho - n)a^A kw^*\left(\frac{b^E}{b^A}\right). \qquad (A.6)$$

Solving for w_H^E from (8), Π^E from (17), u^E from (21) and I^E from (23), and substituting the resulting expressions into (16), using (14) and (15) yields and $P = \lambda b^E w_L^*$

$$\frac{\lambda-1}{\sigma} = \frac{1}{(\theta_0^E)^2}\left((\rho - n)a^E k\left(\frac{n^E}{\eta^E}\right) + \frac{\phi}{2}(1-(\theta_0^E)^2)\right) \qquad \boldsymbol{\theta_0^E(n^E;.)} \qquad (A.7)$$

which implicitly defines θ_0^E as an increasing function of n^E and the parameters of the model. Note that the LHS of (A.7) is a constant, whereas the RHS is decreasing in θ_0^E for $\theta_0^E \in (0, 1)$. As $\theta_0^E \to 0$, the RHS $\to$ ∞ ; and as $\theta_0^E \to 1$, the RHS $\to$ INT_3, where $INT_3 = (\rho - n)a^E kn^E/\eta^E$. Figure A.4 plots the RHS and the LHS of (A.7) with θ_0^A on the horizontal axis. Hence, given a unique $(n^E)_{EQM} \in (0, 1)$ exists, the necessary and sufficient condition for the existence and uniqueness of $(\theta_0^A)_{EQM}$ in the domain (0, 1) is

$$\frac{\lambda-1}{\sigma} > (\rho - n)a^E k\left(\frac{(n^E)_{EQM}}{\eta^E}\right). \qquad (A.8)$$

Finally, I analyze the existences and uniqueness of relative wages. Let $\omega^i = w_H^i/w^*$ represent the relative wage for i = A, E. With $(\theta_0^A)_{EQM} \in (0, 1)$ determined, a unique and positive $(\omega^A)_{EQM}$ can be instantly found from (7). To derive ω^E, solve for Π^E from (17), I^E from (23), and substitute these into (16) using (14) and (15). Dividing both sides of the resulting expression with w_L^* gives:

$$\omega^E = \frac{(\lambda-1)}{w_L^*\lambda\left((\rho - n)a^E k + (1-(\theta_0^E)^2)\frac{\phi}{2}\left(\frac{\eta^E}{n^E}\right)\right)}. \qquad (A.9)$$

Figure A.3. The existence and uniqueness of θ_0^A

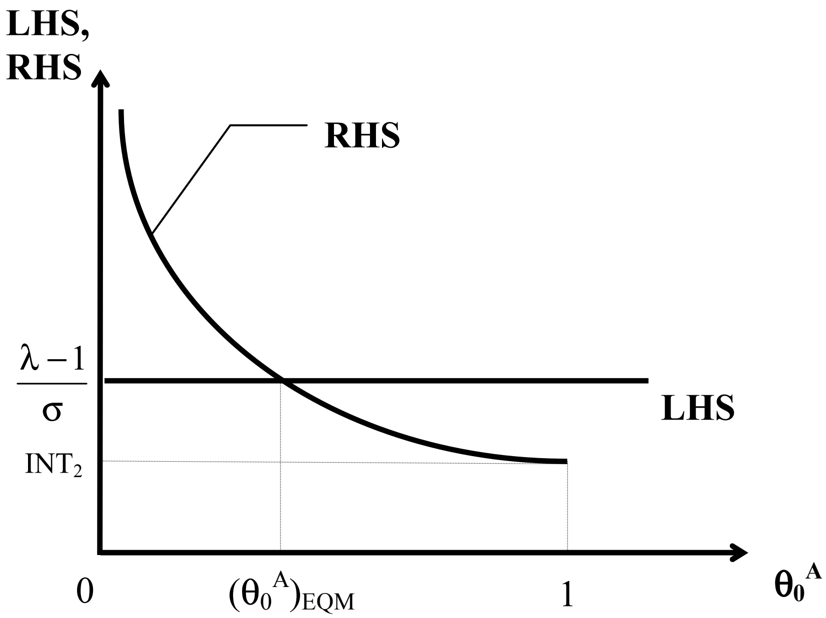
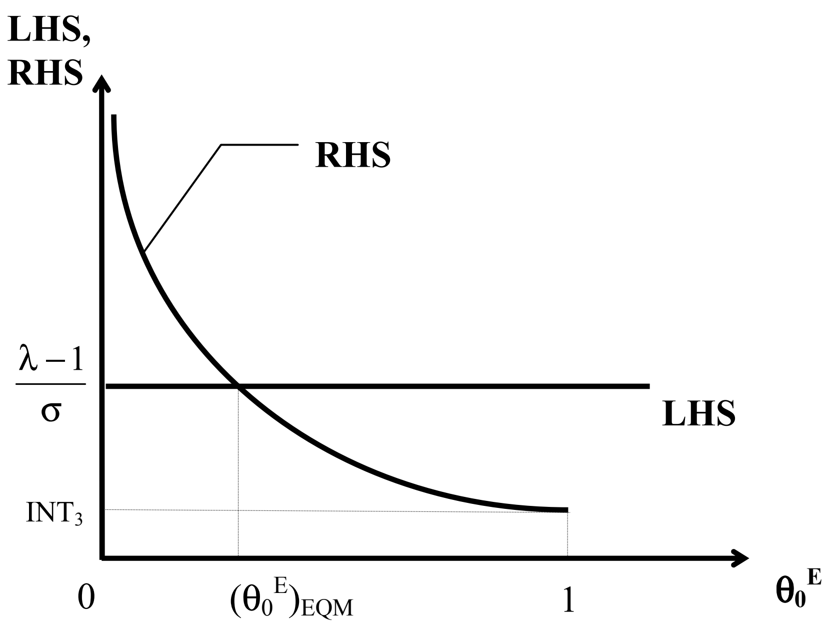


Figure A.4. The existence and uniqueness of θ_0^E



With $(\theta_0^E)_{EQM} \in (0, 1)$ and $(n^E)_{EQM} \in (0, 1)$, it follows from (A.9) that $(\omega^E)_{EQM}$ is unique and positive. For future use, one can simplify (A.9). First, find an expression for n^E using (8) and (21) as $n^E = (\theta_0^E)^2\omega^E\eta^E\lambda w_L^*/\sigma$, then substitute this back into (A.9) to obtain:

$$\omega^E = \frac{1}{\lambda w_L^*(\rho - n)a^E k}\left((\lambda - 1) - \frac{\phi\sigma(1-(\theta_0^E)^2}{2(\theta_0^E)^2}\right) \qquad \boldsymbol{\omega^E(\theta_0^E;.)} \qquad (A.10)$$

which defines the ω^E as an increasing function of θ_0^E and the parameters of the model.

To sum up, there exists a unique-steady-state equilibrium in which $(n^E)_{EQM}$, $(u^E)_{EQM}$, $(\theta_0^A)_{EQM}$, and $(\theta_0^E)_{EQM} \in (0, 1)$; and $(\omega^A)_{EQM} > 0$ and $(\omega^E)_{EQM} > 0$ if and only if (A.2), (A.4), (A.6) and (A.8) hold jointly. Since the parameters of the model enter the inequalities directly and indirectly via $(n^E)_{EQM}$, one cannot determine sufficient conditions on parameters that would satisfy all the inequalities. However, the simulations reveal that for a large set of reasonable parameter values a unique steady-state equilibrium with the above properties exists.

2. Proofs of comparative steady-state analysis

2.1. Proof of Proposition 1: Changes in the minimum wage rate

All derivatives are evaluated in the neighborhood of steady-state equilibrium. To simplify notation, I omit the subscripts EQM. Totally differentiating (A.1) gives $\frac{dn^E}{dw^*} < 0$. Given $n^E = n^E(w_L^*)$ by (A.1), totally differentiating (A.3) implies:

$$\frac{du^E}{dw^*} = \underbrace{\frac{\partial u^E}{\partial n^E}\frac{dn^E}{dw^*}}_{+} + \underbrace{\frac{\partial u^E}{\partial w^*}}_{+} > 0. \qquad (A.11)$$

To analyze the variation in θ_0^i, totally differentiate (A.5) and (A.7) to obtain:

$$\frac{d\theta_0^A}{dw^*} > 0, \qquad \frac{d\theta_0^E}{dw^*} = \underbrace{\frac{\partial\theta_0^E}{\partial n^E}}_{+}\underbrace{\frac{\partial n^E}{\partial w^*}}_{-} < 0. \qquad (A.12)$$

Using (A.12) and (23), one can obtain $\frac{d(I^A n^A)}{dw^*} < 0$ and $\frac{d(I^E n^E)}{dw^*} > 0$. In addition, using (A.12) and (7), one can show that $\frac{d\omega^A}{dw^*} < 0$. To analyze the change in ω^E, I totally differentiate (A.10) and derive:

$$\frac{d\omega^E}{dw^*} = \underbrace{\frac{\partial\omega^E}{\partial w^*}}_{-} + \underbrace{\frac{\partial\omega^E}{\partial\theta_0^E}}_{+}\underbrace{\frac{d\theta_0^E}{dw^*}}_{-} < 0. \qquad (A.13)$$

2.2. Proof of Proposition 2: Increased unemployment benefits

When unemployment benefits is introduced, the relevant training arbitrage condition TA(E)' becomes $\theta_0^E = \sigma(1 - u^E(1 - \alpha))w_L^*/w_H^E$, where $\alpha > 0$. Using this to derive the steady-state equations implies that (A.1), (A.5), and (A.9) remain the same, whereas (A.3) and (A.7) change as follows:

$$\frac{\lambda - 1}{\sigma} = \left[1 - u^E(1-\alpha)\right]\left[\frac{a^E k(w^*\lambda)^2(1-u^E)\eta^E}{n^E}\left(\rho - n + \frac{\phi\eta^E}{2ka^E n^E)}\right) - \frac{\phi}{2(1-u^E)}\right] \quad \mathbf{u^E(n^E;)}\text{(A.3)'}$$

$$\frac{\lambda - 1}{\lambda\sigma} = \frac{a^E k w_L{}^*}{\theta_0{}^E}\left[\alpha + \frac{(1-\alpha)n^E}{\lambda w^* \theta_0{}^E)\eta^E}\right]\left[\rho - n + \frac{\phi\eta^E(1-(\theta_0{}^E)^2)}{2ka^E n^E}\right] \qquad \boldsymbol{\theta_0^E(n^E;.)} \text{ (A.7)'}$$

Observe that α does not enter (A.1) and (A.5); thus, $\frac{dn^E}{d\alpha} = 0$ and $\frac{d\theta_0{}^A}{d\alpha} = 0$, which, in turn, implies $\frac{d(I^A n^A)}{\alpha} = \frac{d\omega^A}{\alpha} = 0$ by (23) and (7).Totally differentiating (A.3)' and (A.7)'one can obtain $\frac{du^E}{d\alpha} > 0$, $\frac{d\theta_0{}^E}{d\alpha} > 0$, respectively. Using these and (23), one can derive $\frac{d(I^E n^E)}{d\alpha} < 0$. To determine the change in ω^E, totally differentiate (A.9). This yields

$$\frac{d\omega^E}{d\alpha} = \underbrace{\frac{\partial\omega^E}{\partial\theta_0{}^E}}_{+}\underbrace{\frac{\partial\theta_0{}^E}{\partial\alpha}}_{+} > 0. \qquad \text{(A.14)}$$

2.3. Proof of Proposition 3: Global and equi-proportionate technological change in R&D

Totally differentiating (A.1) gives $\frac{dn^E}{da^A} < 0$. Using (A.3) one can obtain

$$\frac{du^E}{da^A} = \underbrace{\frac{\partial u^E}{\partial n^E}}_{-}\underbrace{\frac{dn^E}{da^A}}_{-} + \underbrace{\frac{\partial u^E}{\partial a^E}}_{+} > 0. \qquad \text{(A.15)}$$

To analyze the change in $\theta_0{}^A$, I totally differentiate (A.5) and derive $\frac{d\theta_0{}^A}{da^A} > 0$, which, in turn, implies $\frac{d(I^A n^A)}{da^A} < 0$ by (23). To investigate the adjustment in $\theta_0{}^E$, totally differentiate (A.7), using $\frac{da^A}{a^A} = \frac{da^E}{a^E}$.

This implies:

$$\text{sign}\frac{d\theta_0{}^E}{da^E} = \text{sign}\frac{d(a^E n^E)}{da^E} = \text{sign } (1 - \varepsilon(n^E, a^A)), \qquad \text{(A.16)}$$

where $\varepsilon(n^E, a^A) = -(\partial n^E/\partial a^A)(a^A/n^E)$. Thus, $\frac{d\theta_0^E}{da^E} > 0$ if and only if $\varepsilon(n^E, a^A) \leq 1$. By (23), this, in turn,

implies $\frac{d(I^E n^E)}{da^E} = \underbrace{\frac{\partial(I^E n^E)}{\partial a^E}}_{-} + \underbrace{\frac{\partial(I^E n^E)}{\partial\theta_0^E}}_{-}\underbrace{\frac{\partial\theta_0^E}{\partial a^E}}_{+} < 0$ if $\varepsilon(n^E, a^A) \leq 1$. To analyze the variation in ω^E, totally

differentiate (A.10). For the elastic case $\varepsilon(n^E, a^A) > 1$, this yields:

$$\frac{d\omega^E}{da^E} = \underbrace{\frac{\partial\omega^E}{\partial a^E}}_{-} + \underbrace{\frac{\partial\omega^E}{\partial\theta_0^E}}_{+}\underbrace{\frac{d\theta_0^E}{da^E}}_{-} < 0. \tag{A.17}$$

For the inelastic case $\varepsilon(n^E, a^A) < 1$, instead of (A.10), consider $\omega^E = \sigma(1 - u^E)/\theta_0^E$, which is implied by (8). Totally differentiating this particular expression for ω^E implies:

$$\frac{d\omega^E}{da^E} = \underbrace{\frac{\partial\omega^E}{\partial u^E}}_{-}\underbrace{\frac{du^E}{da^E}}_{+} + \underbrace{\frac{\partial\omega^E}{\partial\theta_0^E}}_{-}\underbrace{\frac{d\theta_0^E}{da^E}}_{+} < 0. \tag{A.18}$$

Thus, $d\omega^E/da^E$ is negative regardless of the value of $\varepsilon(n^E, a^A)$.

2.4. Proof of Proposition 4: Changes in the distribution of global population

Consider a change in η^E and note that $d\eta^E = -d\eta^A$. For (A.1) to hold, the ratio $\frac{(1-\eta^E)}{(1-n^E)}$ must remain constant. Thus $\frac{d(1-n^E)}{(1-n^E)} = \frac{d(1-\eta^E)}{(1-\eta^E)}$, which in turn implies $\frac{dn^E}{d\eta^E} = \frac{(1-n^E)}{(1-\eta^E)} > 0$ and $\frac{dn^E}{d\eta^A} < 0$.

Since η^E does not enter (A.5), θ_0^A and therefore ω^A remain constant. With θ_0^A constant and $dn^A/d\eta^A > 0$, it follows from (23) that $d(I^A n^A)/d\eta^A > 0$. In Europe, the effects depend on the variation in $(n^E)^* = n^E/\eta^E$. Totally differentiating $(n^E)^*$ with respect to η^E and appropriately substituting the American levels using $d\eta^E = -d\eta^A$, $n^A = (1 - n^E)$ and $\eta^A = (1 - \eta^E)$ gives

$\frac{d(n^E)^*}{d\eta^A} = \frac{\eta^A - n^A}{\eta^A(1-\eta^A)^2}$. Hence,

$$\frac{d(u^E)}{d\eta^A} = \underbrace{\frac{d(u^E)}{d(n^E)^*}}_{-}\underbrace{\frac{d(n^E)^*}{d\eta^A}}_{-} \geq 0 \qquad \text{iff} \qquad n^A \geq \eta^A. \tag{A.19}$$

Totally differentiating (A.7) gives

$$\frac{d\theta_0^E}{d\eta^A} = \underbrace{\frac{\partial\theta_0^E}{\partial(n^E)^*}}_{+}\underbrace{\frac{d(n^E)^*}{d\eta^A}}_{-} \leq 0, \qquad \text{iff} \qquad n^A \geq \eta^A. \tag{A.20}$$

Using (23) and (A.20), one can obtain $\frac{d(I^E n^E)}{d\eta^A} = \underbrace{\frac{\partial(I^E n^E)}{\partial \eta^A}}_{-} + \underbrace{\frac{\partial(I^E n^E)}{\partial \theta_0^{\ E}}}_{-}\underbrace{\frac{\partial \theta_0^{\ E}}{\partial \eta^A}}_{+} < 0$ if $\eta^A \geq n^A$ and ambiguous otherwise. To analyze ω^E, totally differentiate (A.10) to obtain

$$\frac{d\omega^E}{d\eta^A} = \underbrace{\frac{\partial \omega^E}{\partial \theta_0^{\ E}}}_{+}\underbrace{\frac{d\theta_0^{\ E}}{d\eta^A}}_{-} \leq 0, \qquad \text{iff} \qquad n^A \geq \eta^A. \qquad \text{(A.21)}$$

Appendix B

(not to be considered for publication, to be made available on the author's web site and also upon request)

1. Derivation of the steady-state equilibrium for the general model with $h^E > 0$ and $h^A > 0$

I first derive the SS(A) equation. Solving w_H^A from Lemma (1) using TA(A) and LM(A) gives

$$w_H^A = \frac{P}{\lambda\left(\dfrac{n^A(b^A)^2}{P\eta^A\sigma} + h^A\right)}. \qquad \mathbf{w_H^A(P,n^A;.)}$$

Substituting $w_H^A(P,n^A;.)$ from above into FE(A) gives

$$I^A = \frac{(\lambda-1)}{Pa^Ak}\left(\frac{n^A(b^A)^2}{P\eta^A\sigma} + h^A\right) - (\rho - n). \qquad \mathbf{I^A(P,n^A;.)}$$

Substituting $I^A(P,n^A;.)$ from above into SM(A) and solving for θ_0^A yields

$$(\theta_0^A)^2(\overset{+}{P}, \overset{-}{n^A}; \overset{-}{b^A}, \overset{+}{a^A}, \overset{+}{\eta^A}) = 1 - \frac{2n^A}{\phi\eta^A}\left[\left(\frac{n^A(b^A)^2}{P\eta^A\sigma} + h^A\right)\frac{(\lambda-1)}{P} - (\rho - n)a^Ak + \frac{h^A}{P}\right], \quad \boldsymbol{\theta_0^A}\mathbf{(P,n^A;.)}$$

where the signs above the variables/parameters stand for the signs of partial derivatives. Substituting the above into LM(A) implies

$$\theta_0^A(\overset{+}{P}, \overset{-}{n^A}; \overset{-}{b^A}, \overset{+}{a^A}, \overset{+}{\eta^A}) = \frac{b^An^A}{\eta^AP}, \qquad \mathbf{SS(A)\ (P,n^A;.)}$$

where $dP/dn^A|_{SS(A)} > 0$ and hence the upward sloping SS(A) curve as shown in Figure B.1. For future use, note that for a given n^A, $dP/w_L^*|_{SS(A)} = 0$, $dP/da^A|_{SS(A)} < 0$, $dP/d\eta^A|_{SS(A)} < 0$, $dP/db^A|_{SS(A)} > 0$.

Second, I derive the SS(U) equation. Solving for w_H^E from Lemma (1) gives:

$$w_H^E = \left(\frac{P}{\lambda} - b^Ew_L^*\right)\frac{1}{h^E}. \qquad \mathbf{w_H^E\ (P;.)}$$

Substituting w_H^E (P;.) from above into FE(E) gives:

$$I^E = \frac{(\lambda-1)h^E}{a^Ek(P - \lambda b^Ew_L^*)} - (\rho - n). \qquad \mathbf{I^E\ (P;.)}$$

Substituting I^E (P;.) from above into SM(E) gives

$$(\theta_0^E)^2(\overset{+}{P}, \overset{-}{n^E}; \overset{-}{b^E}, \overset{+}{\eta^E}, \overset{-}{w_L^*}, \overset{+}{a^E}) = 1 - \frac{2n^E}{\phi\eta^E}\left[\frac{(\lambda-1)h^E}{a^Ek(P - \lambda b^Ew_L^*)} - (\rho - n)a^Ek + \frac{h^E}{P}\right], \boldsymbol{\theta_0^E}\mathbf{(P,n^E;.)}$$

Finally, substitute the above into LM(E) to find:

$$(1-u^E)\eta^E\theta_0^E(\overset{+}{P},\overset{-}{n^E};\overset{-}{b^E},\overset{+}{\eta^E},\overset{-}{w_L}*,\overset{+}{a^E}) = \frac{n^E b^E}{P}, \qquad \textbf{SS(U) (P,}\mathbf{n^E}\textbf{, }\mathbf{u^E}\textbf{;.)}$$

where $du^E/dn^E|_{SS(U)} < 0$ and hence the downward sloping SS(U) curve as illustrated in Figure B.1. For future use, note that for a given n^E, $du^E/dw_L*|_{SS(U)} < 0$, $du^E/da^E|_{SS(U)} > 0$, $du^E/d\eta^E|_{SS(U)} > 0$, $du^E/db^E|_{SS(U)} < 0$, $du^E/dP|_{SS(U)} > 0$.

Third, I derive the SS(E) equation. Combining Lemma (1) and TA(E) gives $(1 - u^E) = \theta_0^E/[\sigma h^E[(P/(\lambda w_L*)) - b^E]]$. Substituting this into LM(E) immediately gives $(\theta_0^E)^2 = n^E\sigma h^E/[\eta^E P[(P/(\lambda w_L*b^E)) - 1)]]$. Finally combining this with the above $\theta_0^E(P,n^E;.)$ expression gives:

$$(\theta_0^E)^2(\overset{+}{P},\overset{-}{n^E};\overset{-}{b^E},\overset{+}{\eta^E},\overset{-}{w_L}*,\overset{+}{a^E}) = \frac{n^E\sigma h^E}{\eta^E P\left[\dfrac{P}{\lambda w_L * b^E} - 1\right]}, \qquad \textbf{SS(E) (P,}\mathbf{n^E}\textbf{;.)}$$

where $dP/dn^E|_{SS(E)} > 0$, which implies $dP/dn^A|_{SS(E)} < 0$ and hence the downward sloping SS(E) curve as illustrated in Figure B.1. For future use, note that for a given n^E, $dP/dw_L*|_{SS(E)} > 0$, $dP/da^E|_{SS(E)} < 0$, $dP/d\eta^E|_{SS(E)} < 0$, $dP/db^E|_{SS(E)} > 0$.

The steady-state equilibrium is fully illustrated in Figure B.1. It appears to be analytically infeasible to derive parametric conditions that guarantee the existence of equilibrium; however, numerical simulations suggest that for a wide range of reasonable parameters a unique steady-state equilibrium exists in which all endogenous variables attain nonnegative values in a relevant range.

2. Comparative Steady-State Analysis

For the sake of conciseness, I provide a full account of the comparative steady-state analysis only for the case of a minimum wage hike. For the rest of the parameter changes, I outline the resulting shifts using Figure B.1. as a template and report the effects on the endogenous variables, which remain ambiguous in most cases.

2.1. An increase in the minimum wage w_L* (Figure B.2.)

We know that for a given n^A, $dP/w_L*|_{SS(A)} = 0$ and $dP/dw_L*|_{SS(E)} > 0$; thus, when w_L* increases the SS(A) curve remains intact whereas the SS(E) curve shifts up. Hence, the equilibrium levels of P and n^A both increase. To determine the change in θ_0^A, substitute for $I^A(.)$ from above into SM(A) using $n^A = \theta_0^A\eta^A P/b^A$ (which is from LM(A)) to obtain:

$$\left[1-(\theta_0^{\ A})^2\right]\frac{\phi}{2}=\frac{\theta_0^{\ A}}{b^A}\left[\left(\frac{\theta_0^{\ A}(b^A)^2}{\sigma}+h^A\right)(\lambda-1)-(\rho-n)a^AkP+h^A\right], \qquad \boldsymbol{\theta_0^A(.)}$$

which defines θ_0^A in terms of the parameters of the model. It follows from the above that the rise in P triggers an increase in the equilibrium level of θ_0^A. The TA(A) equation then implies that w_H^A/w_L^A decreases. Using $I^A(.)$, it follows that the change in the equilibrium level of I^An^A is ambiguous.

I now turn to Europe. Since $du^E/dP|_{SS(U)} > 0$, the higher P shifts the SS(U) curve to the left. On the other hand, since $du^E/dw_L^*|_{SS(U)} < 0$ the higher w_L^* shifts the SS(U) line to the right. Consequently, the change in u^E remains ambiguous. To analyze the change in θ_0^E, first substitute $(1 - u^E) = \theta_0^E/[\sigma h^E[(P/(\lambda w_L^*)) - b^E]]$ into the LM(E) equation and obtain $n^E = (\theta_0^E)^2\eta^E P(P - \lambda w_L^* b^E)/[\sigma h^E c\lambda w_L^* b^E]$. Substituting this expression for n^E into the SM(E) equation, one can obtain:

$$\frac{(1-(\theta_0^{\ E})^2)}{(\theta_0^{\ E})^2}=\frac{2}{\sigma\phi}\left[\left(\frac{P}{w_L^*b^E}-1\right)-P\left(\frac{P}{\lambda w_L^*b^E}-1\right)\frac{(\rho-n)a^Ek}{h^E}\right], \qquad \boldsymbol{\theta_0^E(.)}$$

which defines θ_0^E in terms of the parameters of the model. It follows from the above that the rise in w_L^* and the endogenous increase in P generates an ambiguous change in θ_0^E. With changes in both u^E and θ_0^E being ambiguous, the changes in w_H^E/w_L^* and I^En^E (as can be observed from the TA(E) and $I^E(.)$ equations) also remain indeterminate.

2.2. An increase in unemployment benefit rate α (Figure B.3.)

To study the effects of an increase in α, we need to rework the equations for Europe. Using the TA′(E) equation and Lemma 1, one can derive an expression for $(1 - u^E)$. Substituting this $(1 - u^E)$ expression and using $\theta_0^E(P,n^E;.)$ from above yields the new SS(E) curve as:[1]

$$\frac{\dfrac{\theta_0^{\ E}(P,n^E;.)}{\sigma h^E}\left(\dfrac{P}{\lambda w_L^*}-b^E\right)-\alpha}{(1-\alpha)}=\frac{n^Eb^E}{P\theta_0^{\ E}(P,n^E;.)\eta^E}. \qquad \textbf{SS′(E)}$$

It is straightforward to show that $dP/dn^A|_{SS'(E)} < 0$ and for a given P, $dn^A/d\alpha|_{SS'(E)} > 0$. As a result an increase in α shifts the SS′(E) curve up, leading to a rise in P and n^A. Using the $\theta_0^A(.)$ equation, one can

[1] Since the TA(E) equation is not utilized in the derivation of $\theta_0^E(P,n^E;.)$, the introduction of $\alpha > 0$ does not change the $\theta_0^E(P,n^E;.)$ expression.

show that θ_0^A increases and w_H^A/w_L^A decreases. It follows from $I^A(.)$ that the change in the equilibrium level of $I^A n^A$ is ambiguous.

Since $du^E/dP|_{SS(U)} > 0$ for a given n^E, the rise in P shifts the SS(U) curve to the left.[2] With n^E falling and SS(U) shifting to the left, it follows that u^E unambiguously increases. Using the $\theta_0^E(.)$ equation, one can show that the change in $\theta_0^E(.)$ is ambiguous. It then follows from the TA(E) and $I^E(.)$ equations that the changes in w_H^E/w_L^* and $I^E n^E$ also remain ambiguous.

2.3. Global technological change in R&D in the form of a decline in both a^A and a^E (Figure B.4.)

For a given n^A, $dP/da^A|_{SS(A)} < 0$ and $dP/da^E|_{SS(E)} < 0$; thus, a decline in both a^A and a^E shifts the SS(A) and SS(E) curves up. Consequently, P increases whereas the change in n^A remains ambiguous. Since $du^E/dP|_{SS(U)} > 0$ for a given n^E, the rise in P forces the SS(U) curve to the left. On the other hand, with $du^E/da^E|_{SS(U)} > 0$ for a given n^E, the decline in a^E forces the SS(U) curve to the right. The direction of the shift in the SS(U) curve remains indeterminate. With these findings, the changes in the rest of the endogenous variables also remain ambiguous.

2.4. An increase in the population share of America η^A (Figure B.5.)

For a given n^A, $dP/d\eta^A|_{SS(A)} < 0$ and $dP/d\eta^E|_{SS(E)} < 0$; thus an increase in η^A shifts the SS(A) curve down and the SS(E) curve up. Consequently, n^A increases whereas the change in P remains ambiguous. With $du^E/dP|_{SS(U)} > 0$ for a given n^E, the ambiguous change in P forces the SS(U) curve in an ambiguous direction. On the other hand, with $du^E/d\eta^E|_{SS(U)} > 0$, the fall in η^E forces the SS(U) curve to the right. Obviously, the net impact on the SS(U) curve remains indeterminate. With these findings, the changes in the rest of the endogenous variables also remain ambiguous.

2.5. Global sbtc in manufacturing in the form of a decline in both b^A and b^E (Figure B.6.)

For a given n^A, $dP/db^A|_{SS(A)} > 0$ and $dP/db^E|_{SS(E)} > 0$; thus, a decline in both b^A and b^E shifts both the SS(A) and the SS(E) curves down. Consequently, P decreases whereas the change in n^A remains ambiguous. Since $du^E/dP|_{SS(U)} > 0$ for a given n^E, the fall in P forces the SS(U) curve to the right. On the other hand, with $du^E/db^E|_{SS(U)} < 0$, the fall in b^E forces the SS(U) curve to the left. Obviously, the direction of the shift in the SS(U) curve remains indeterminate. With these findings, the changes in the rest of the endogenous variables also remain ambiguous.

[2] Since the TA(E) equation is not utilized in the derivation of the SS(U) equation, the introduction of $\alpha > 0$ does not affect the SS(U) curve.

Figure B.1. Steady-state equilibrium in the global Economy

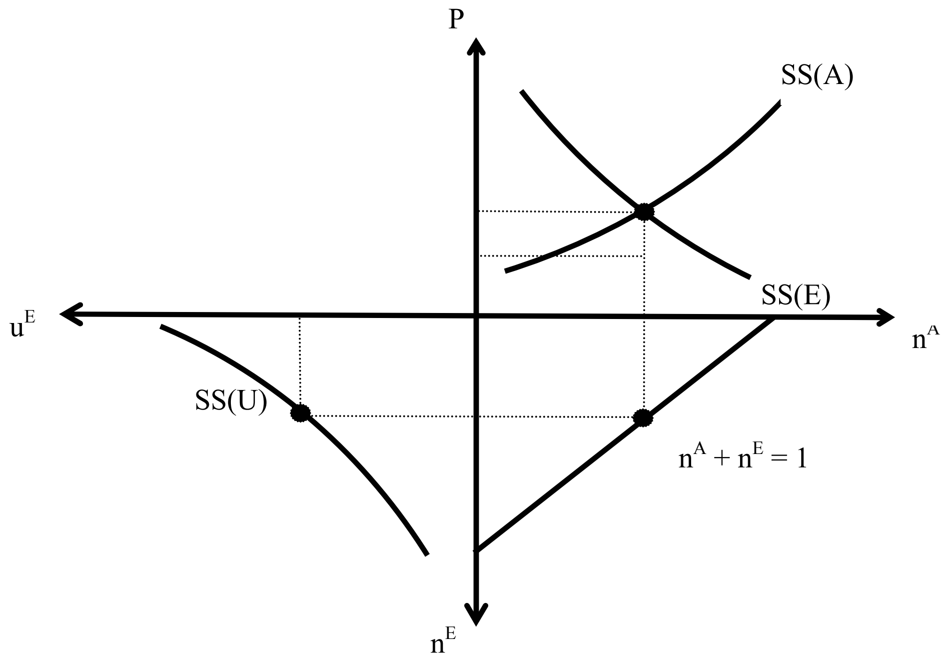


Figure B.2. An increase in the European minimum wage: $w_L*\uparrow$

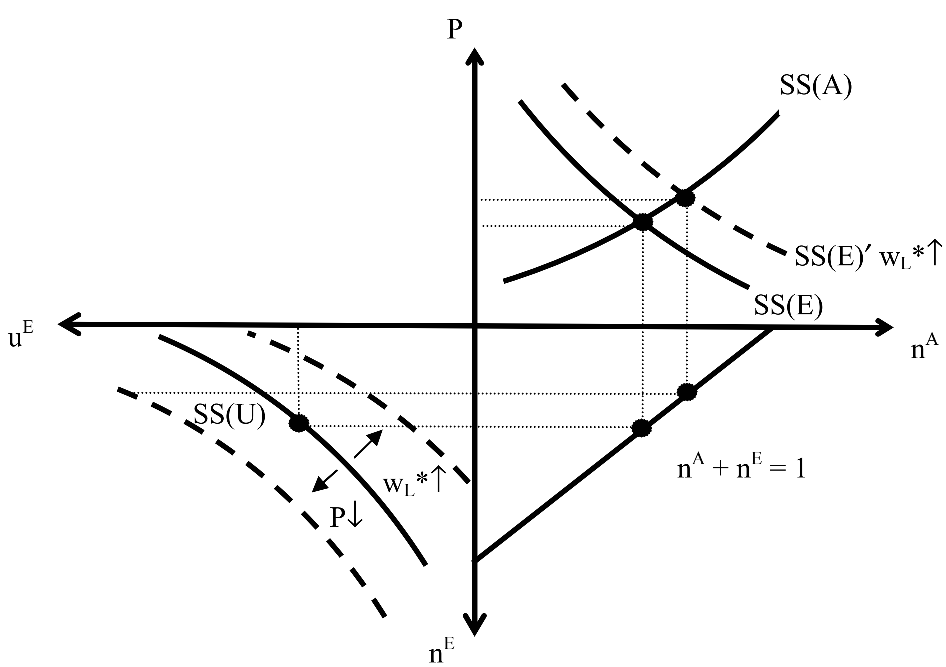


Figure B.3. An increase in unemployment benefit rate: α↑

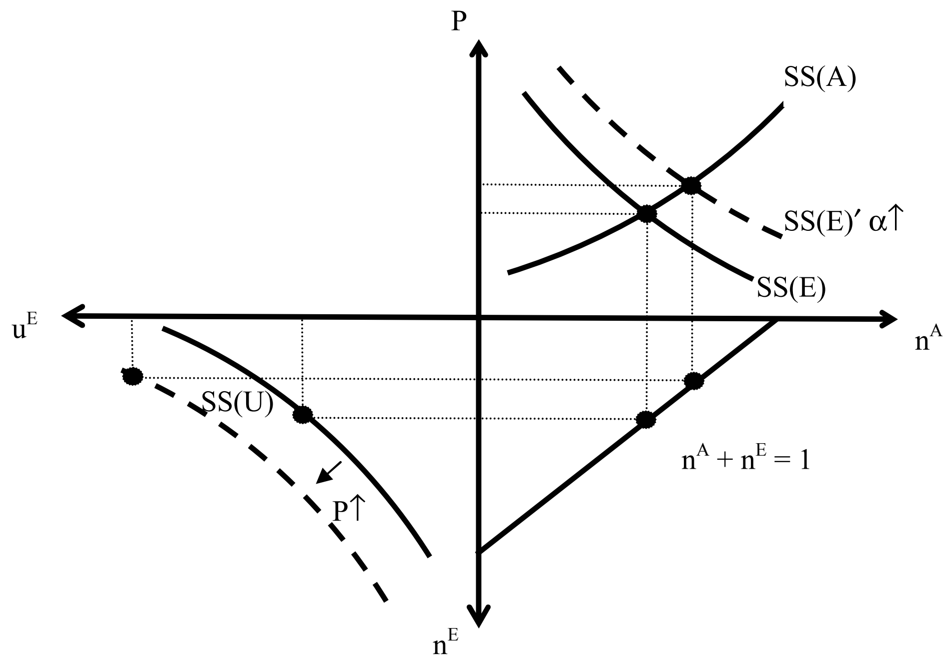


Figure B.4. Global technological change in R&D: a^E and a^A both ↓

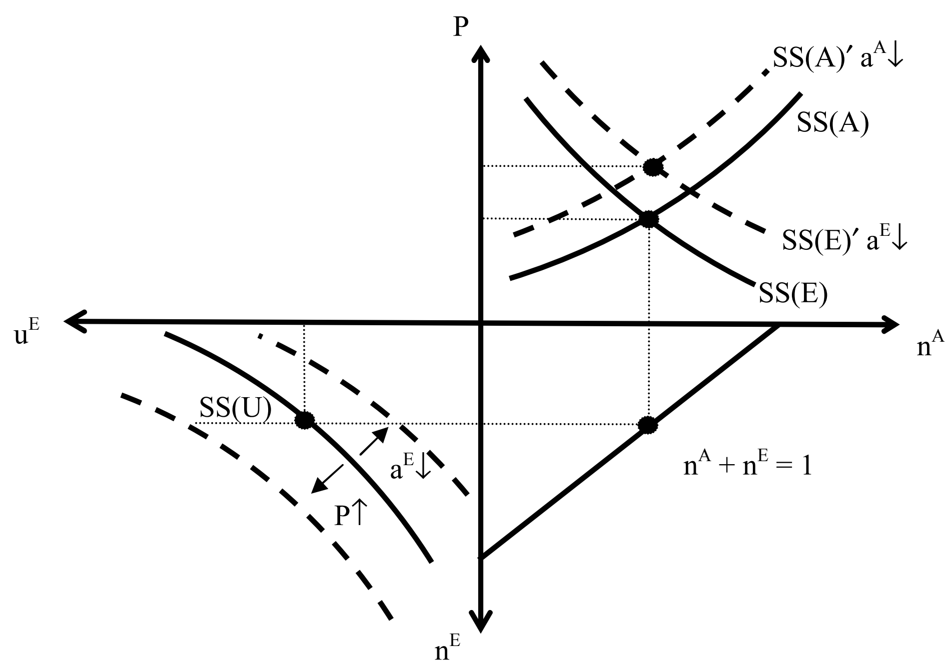


Figure B.5. An increase in the world population share of America: $\eta^A\uparrow$

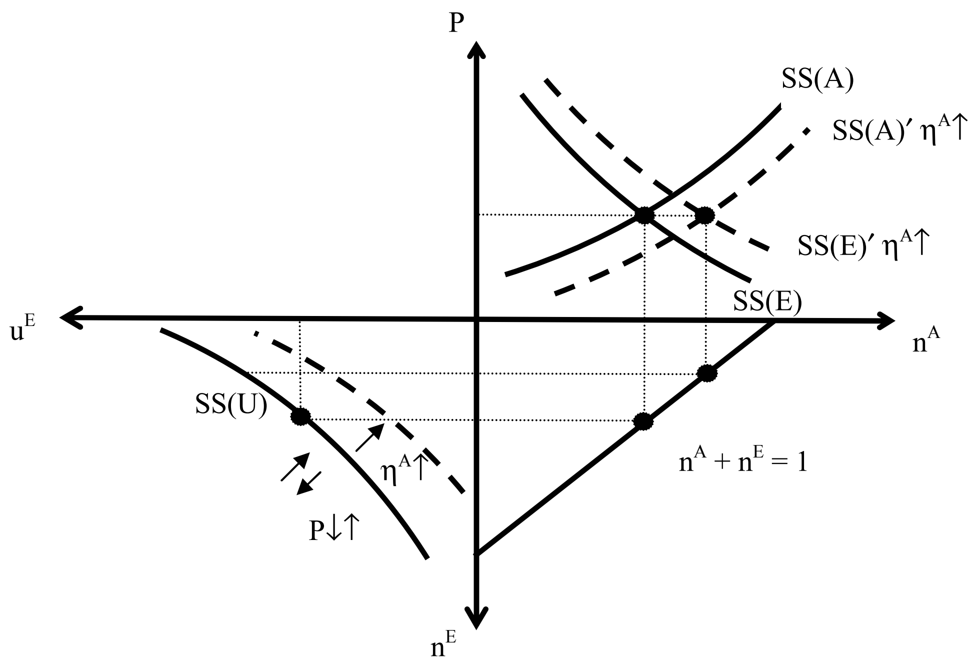


Figure B.6. Global skill-biased technological change in manufacturing: b^A and b^E both $\downarrow$