Appendices a

for

Globalization, R&D and the iPod Cycle

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a Not to considered for publication. To be made available on the authors’ we sites and also upon request.
"GLOBALIZATION, R&D and the iPod CYCLE";

"APPENDIX A: ANALYTICAL DERIVATIONS OF COMPARATIVE STATICS
THE MODEL WITH EQUAL IMITATION RATES TARGETING BOTH NORTHERN AND OUTSOURCING INDUSTRIES;
Date: August 15, 2007";

"- This program requires Mathematica Version 5.0 or above. Before evaluating the cells, 'Math Econ' package written by Cliff Huang and Philip Crooke needs to be run. This package accompanies the book 'Mathematics and Mathematica for Economists', 1997, Blackwell Publishers: Oxford, written by the above authors.
- Notes: In this program, for convenience I enter the subscripts and superscripts of the main model in regular format."

"We consider the model with Southern imitation targeting both the Northern and Outsourcing firms. I first write the Steady State equations To get simpler terms I set $\Sigma_O = H_1 - \sigma_i O_L$ and $\Sigma_O = H_1 - \sigma_O L$ and also $dr = \rho - n$.

$\mu_O$: imitation targeting Outsourcing firms
$\mu_N$: imitation targeting Northern firms ";

Clear [FEIN, FEIO]; Clear [w, $\lambda$, $aN$, $aO$, $kN$, $kO$, c, $iN$, $iO$, $\Sigma_i O$, $\Sigma_O$, $mN$, $mO$, dr, $\eta_S$, $\mu_i$, $\mu_O$]; Clear[LN, LS]; Clear[nN, nO, nS];

"I first solve for the steady-state industry shares";

"nN flows in/out"; EQ1 = iN $nO$ + $nS$ == nN $\mu N$ + $iO$; "nO flows in/out"; EQ2 = iO $nN$ + $nS$ == nO $\mu O$ + $iN$; "mass of one of industries"; EQ3 = $nO + nN + nS == 1$;

Solve [{EQ1, EQ2, EQ3}, {$nO$, $nN$, $nS$}] // FullSimplify

FEIN = w $\star$ $\lambda$ $\star$ $aN$ $\star$ $kN$ == $c (\lambda - (mN \star w)) (dr + iN + iO + \mu_i)$;

FEIO = w $\star$ $\lambda$ $\star$ $aO$ $\star$ $\Sigma_i O \star kO$ == $c (\lambda - (\Sigma_O \star mO)) (dr + iN + iO + \mu_O)$;

"To capture the equal imitation rates, I set"; $\mu i = \mu$; $\mu O = \mu$;
2. For $w > 0$, we need
\[ aO \times kO \times \lambda \times \zetaO + aN \times kN \times (-\lambda + mO \times \SigmaO) > 0. \]
Put differently,
\[ aO \times kO \times \lambda \times \zetaO - aN \times kN \times (\lambda - mO \times \SigmaO) > 0. \]

3. The restriction that $\lambda > mN \times w$ (required for positive Northern profits), implies $w < \lambda / mN$. This automatically holds and can be seen by eye inspection of the $w$ expression. 

4. The restriction that $mN \times w > mO \times \zetaO$ implies $(\lambda - mO \times \SigmaO) \times (aO \times kO \times \lambda \times \zetaO - aN \times kN \times \nuO) > 0$.

This can be simplified to
\[ (aO \times kO \times \lambda \times \zetaO - aN \times kN \times \nuO) > 0; \]
This implies that the unit labor requirement in outsourcing-targeted R&D is larger that in local-sourcing-targeted R&D. This makes intuitive sense. 

"Now I set $c$ and $w$ equal to the solutions given above";

"Note the following restrictions and the implications for the parameters.

1. For $w > 0$, we need
\[ aO \times kO \times \lambda \times \zetaO + aN \times kN \times (-\lambda + mO \times \SigmaO) > 0. \]
Put differently,
\[ aO \times kO \times \lambda \times \zetaO - aN \times kN \times (\lambda - mO \times \SigmaO) > 0. \]

2. For $w > 1$, we need
\[ aO \times kO \times \lambda \times \zetaO + aN \times kN \times (-\lambda + mO \times \SigmaO) > aO \times kO \times mO \times \SigmaO. \]
This can be simplified to
\[ aO \times kO \times \lambda \times \zetaO(\lambda - mO \times \SigmaO > aN \times kN \times (\lambda - mO \times \SigmaO); \]

3. For $w > 1$, we need
\[ aO \times kO \times \lambda \times \zetaO + aN \times kN \times (-\lambda + mO \times \SigmaO) > aO \times kO \times mO \times \SigmaO. \]
This can be simplified to
\[ aO \times kO \times \lambda \times \zetaO(\lambda - mO \times \SigmaO > aN \times kN \times (\lambda - mO \times \SigmaO); \]

4. The restriction that $mN \times w > mO \times \zetaO$ implies $(\lambda - mO \times \SigmaO) \times (aO \times kO \times \lambda \times \zetaO - aN \times kN \times \nuO) > 0$.

This can be simplified to
\[ (aO \times kO \times \lambda \times \zetaO - aN \times kN \times \nuO) > 0; \]
This implies that the unit labor requirement in outsourcing-targeted R&D is larger that in local-sourcing-targeted R&D. This makes intuitive sense. 

"Now I set the industry shares to the solutions";

\[ nO = \frac{iO}{iN + iO + \nuO}; \quad nN = \frac{iN}{iN + iO + \nuO}; \quad nS = \frac{\mu}{iN + iO + \mu}; \]

"Now I rewrite the Northern and Southern labor market equations";
In[133]:= \[LN = (iN * aN + kO) + (iO * aO + kO) + \left(\frac{c * mN}{\lambda} - \frac{1}{(1 + \eta S)}\right) // FullSimplify\]

Out[133]= \[aO \cdot iO \cdot kO - \frac{1}{1 + \eta S} \cdot aN \cdot dr \cdot iN \cdot kN + aO \cdot iN \cdot kO \cdot \lambda \cdot (dr + iN + iO + \mu) \cdot \Sigma iO \cdot (iN + iO + \mu) \cdot (\lambda - mO \cdot \Sigma O)\]

In[134]:= \[LS = \left(\frac{(nO) * c * mO}{\lambda} + (nS) * c - \frac{\eta S}{(1 + \eta S)}\right) // FullSimplify\]

Out[134]= \[-1 + \frac{1}{1 + \eta S} \cdot \left(\frac{(dr + iN + iO + \mu) \cdot (iO \cdot mO + \lambda \cdot \mu)}{mN \cdot (iN + iO + \mu) \cdot (\lambda - mO \cdot \Sigma O)}\right)\]

In[136]:= \[F1[iN_, iO_, \mu_, aO_, aN_, \etaS_, \etaN_, \lambda_, dr_, kN_, kO_, \SigmaO_, \SigmaiO_] := \[-1 + \frac{1}{1 + \eta S} \cdot aN \cdot dr \cdot iN \cdot kN + aO \cdot kO \cdot \left\{iO \cdot \frac{(dr + iN + iO + \mu) \cdot \Sigma iO \cdot (iN + iO + \mu) \cdot (\lambda - mO \cdot \Sigma O)}{\lambda \cdot (1 + \eta S)}\right\};\]

In[137]:= \[F2[iN_, iO_, \mu_, aO_, aN_, \etaS_, \etaN_, \lambda_, dr_, kN_, kO_, \SigmaO_, \SigmaiO_] := \[-1 + \frac{1}{1 + \eta S} \cdot \left(\frac{(dr + iN + iO + \mu) \cdot (iO \cdot mO + \lambda \cdot \mu)}{mN \cdot (iN + iO + \mu) \cdot (\lambda - mO \cdot \Sigma O)}\right);\]

In[138]:= \[Clear[J, B]\]

In[139]:= \[J = \{\text{grad}[F1[iN, iO, \mu, aO, aN, \etaS, \lambda, dr, kN, kO, \SigmaO, \SigmaiO], \{iN, iO\}], \text{grad}[F2[iN, iO, \mu, aO, aN, \etaS, \lambda, dr, kN, kO, \SigmaO, \SigmaiO], \{iN, iO\}]\};\]

In[140]:= \[\text{FullSimplify}[J] // \text{MatrixForm}\]

Out[140]//MatrixForm = \[
\begin{pmatrix}
\frac{\lambda \cdot (1 + \eta S)}{\lambda - mO \cdot \Sigma O} & aO \cdot kO \\
\frac{(aO \cdot kO \cdot \lambda \cdot \Sigma O \cdot aN \cdot kN \cdot (-\lambda - mO \cdot \Sigma O))}{mN \cdot (\eta N - 1) \cdot (\lambda - mO \cdot \Sigma O)}
\end{pmatrix}
\]

In[141]:= \"To see the slope clearly I set\"; \[dr = 0; \text{FullSimplify}[J] // \text{MatrixForm}\]

Out[141]//MatrixForm = \[
\begin{pmatrix}
aO \cdot kO \\
\frac{\lambda \cdot (1 + \eta S)}{\lambda - mO \cdot \Sigma O} \cdot \frac{(aO \cdot kO \cdot \lambda \cdot \Sigma O \cdot aN \cdot kN \cdot (-\lambda - mO \cdot \Sigma O))}{mN \cdot (\eta N - 1) \cdot (\lambda - mO \cdot \Sigma O)}
\end{pmatrix}
\]

In[142]= \"The LN curve is downward sloping, whereas the LS curve is vertical\";

In[143]= \"1. A FALL IN aO, BETTER OUTSOURCING TECHNOLOGY\";

In[144]= \[\text{Clear[B, dr]}\]

In[145]= \[B = \{\text{grad}[F1[iN, iO, \mu, aO, aN, \etaS, \lambda, dr, kN, kO, \SigmaO, \SigmaiO], \{aO\}], \text{grad}[F2[iN, iO, \mu, aO, aN, \etaS, \lambda, dr, kN, kO, \SigmaO, \SigmaiO], \{aO\}]\} // \text{FullSimplify}\]

Out[145]= \[
\left\{
\begin{pmatrix}
\frac{iO \cdot \lambda \cdot (dr + iN + iO + \mu) \cdot \Sigma iO \cdot (iN + iO + \mu) \cdot (\lambda - mO \cdot \Sigma O)}{mN \cdot (iN + iO + \mu) \cdot (\lambda - mO \cdot \Sigma O)}
\end{pmatrix}
\right\}
\]

In[146]= \[\text{impactaO = -Inverse}[J \cdot B \cdot \{\Delta aO\}] // \text{FullSimplify}\]
Out[147] = 
\[
\begin{align*}
&-\Delta aO (\lambda - \mu \Sigma O) \left( \frac{1}{\lambda \Sigma O} + \frac{1}{\lambda - \mu \Sigma O} \right) \left( \frac{\Delta aO \lambda \mu (\lambda - \mu \Sigma O)}{\Delta aO \lambda \mu (\lambda - \mu \Sigma O) - \frac{kO (iO mO + \lambda \mu)}{mO (aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O))}} \right) \\
&- \frac{kO \Delta aO \lambda (\lambda \Sigma O) \Sigma O}{mO (aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O))} \\
\end{align*}
\]

In[148] = \{\text{diN, diO}\} = \%

In[149] = "1.1. What happens to innovation rates: diN and diO?";

In[150] = "The above implies that \( \frac{\text{diN}}{\text{daO}} \) is ambiguous. To find an if and only if condition, I collect the iN and iO terms";

In[151] = \text{Collect}[\text{diN}, \{iO, iN\}]

Out[151] = \[
\begin{align*}
&-\frac{iO \Delta aO}{aO} + \frac{kO \Delta aO \lambda \mu (\lambda - \mu \Sigma O)}{mO (aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O))} \\
&\frac{iO \Delta aO (\lambda - \mu \Sigma O)}{aO \lambda \Sigma O} - \frac{kO}{aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O))} \\
\end{align*}
\]

In[152] = "--The first term in front of \( \text{iN} \) is negative. 
--The second term with \( \mu \) in it is positive. 
--The third term in front of \( iO \), which boils down to \( \frac{aN \Sigma O \Delta aO (\lambda - \mu \Sigma O)^2}{aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O)) \), is positive.";

In[153] = "Obviously there is an ambiguity. For \( \text{diN} < 0 \), \( \text{iN} \) needs to be larger than a critical level. To find this I use"; \text{Solve}[\text{diN} = 0, \text{iN}] \text{// FullSimplify}

Out[153] = \[
\begin{align*}
&\{ \text{iN} \rightarrow \frac{(\lambda - \mu \Sigma O) \left( aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O))}{mO \lambda \Sigma O (aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O))} \}
\end{align*}
\]

In[154] = "Thus we can state 
\[
\begin{align*}
&\text{diN} < 0 \quad \text{iff} \quad \text{iN} > \frac{(\lambda - \mu \Sigma O) \left( aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O))}{mO \lambda \Sigma O (aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O))} \}
\end{align*}
\]

In[155] = "1.2. What happens to aggregate innovation: diA= diN+diO?";

In[156] = \text{diA = diN + diO} \text{// FullSimplify}

Out[156] = \[
\begin{align*}
&\text{\Delta aO} \left( -aO \Sigma O (\lambda \Sigma O (\lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O)) + \\
&aN \Sigma O \lambda - \lambda + \mu \Sigma O)) \right) / (aO \Sigma O \lambda \Sigma O + aN \Sigma O \lambda - \lambda + \mu \Sigma O)) \}
\end{align*}
\]

In[157] = "Again, \( \frac{\text{diA}}{\text{daO}} \) looks ambiguous. To simplify, I collect the iN and iO terms";
Collect \([\text{diA, \{iO, iN\}}]\)

\[
\Delta aO \left\{ \left( -aO kO \lambda^2 \mu \Sigma - aO kO mO \lambda^2 \mu \Sigma \right) \right\} + \frac{aO mO \lambda \Sigma \left( aO kO \lambda \Sigma + aN kN \left( -\lambda + mO \Sigma \right) \right)}{aO mO \lambda \Sigma \left( aO kO \lambda \Sigma + aN kN \left( -\lambda + mO \Sigma \right) \right)}
\]

\[
\Delta aO \left\{ \left( -aO kO mO \lambda^2 \Sigma^2 + aN kN mO \lambda \Sigma \left( 1 - mO \Sigma \right) \right) \right\} + \frac{10 \Delta aO \left\{ \left( -aO kO mO \lambda^2 \Sigma^2 + aN kN mO \left( -\lambda - mO \Sigma \right)^2 \right) \right\}}{aO mO \lambda \Sigma \left( aO kO \lambda \Sigma + aN kN \left( -\lambda + mO \Sigma \right) \right)}
\]

--The sign of first term after simplifying is the same as that of \(\lambda(1-\Sigma \lambda - mO \Sigma)\). For this to be negative, we need \(\sigma \lambda < \frac{mO + \Sigma}{\lambda}\).

--The second term in front of \(iN\) is negative because of the restriction \(w>0\).

--The third term in front of \(iO\) is negative. To see this note that the first term in the numerator which is negative is multiplied by \(\lambda>1\), whereas the second term which is positive is multiplied by \((\lambda-mO \Sigma)\). Given \(\lambda > \lambda - mO \Sigma\) always holds, it follows that the entire term remains negative. The restriction \(w>0\) is also used here.

"Thus we conclude:

\[
\text{diA} < 0 \quad \text{if} \quad \sigma iO < \frac{mO + \Sigma}{\lambda}, \quad \text{a sufficient but hardly a necessary condition.}"

"1.3. What happens to the fraction of industries?"

Totally differentiating \(nO\) implies 

\[
\frac{dnO}{daO} = D[nO, iO] \frac{diO}{D[iO, iN] diN} / \text{FullSimplify}
\]

"Clearly, \(\frac{dnO}{daO} < 0\)

a. If \((\Sigma iO - \Sigma) > 0\).

b. Otherwise, that is for \((\Sigma iO - \Sigma) < 0\), we need \((\Sigma O - \Sigma iO) < \frac{\lambda}{mO}\), which means that the absolute value of \((\Sigma iO - \Sigma)\) needs to be sufficiently small.

The part \(a\) of the condition basically implies \(\sigma O > \sigma iO\), which is not a strict restriction";

Totally differentiating \(nN\) implies 

\[
\frac{dnN}{daO} = D[nN, iO] \frac{diO}{D[iO, iN] diN} / \text{FullSimplify}
\]

"Clearly, \(\frac{dnN}{daO} > 0\);"
In[168]:= "Totally differentiating nS implies:\n\[\text{dnS} = D[nS, i0] \text{di0} + D[nS, iN] \text{diN} // FullSimplify\n\]

Out[169]= \[\Delta aO \mu (aO kO \lambda^2 \Sigma iO (\lambda \mu (-1 + iO) + (iN + iO) mO \Sigma iO + mO \mu \Sigma 0)) -\n\ aN kN mO (\lambda - mO \Sigma 0) (iN \lambda \Sigma iO + iO (\lambda - mO \Sigma 0)) \]/\{aO mO \lambda (iN + iO + \mu)^2 \Sigma iO (aO kO \lambda \Sigma iO + aN kn (-\lambda + mO \Sigma 0))\}\]

In[170]:= "Clearly, \(\frac{\text{dnS}}{\text{daO}} \to 0\). Note that nS moves inversely with iA. Thus:\n\[\text{dc} = D[c, iN] \text{diN} + D[c, iO] \text{diO} + D[c, aO] \Delta aO // FullSimplify\n\]

Out[173]= \[\Delta aO \{aO kO \lambda^2 \Sigma iO (\lambda - \lambda \Sigma iO + mO (\Sigma iO - \Sigma 0)) + aN kN mO (\lambda - mO \Sigma 0) (iN \lambda \Sigma iO + iO (\lambda - mO \Sigma 0))\}\]

In[174]:= "So, \(\frac{\text{dc}}{\text{daO}} > 0\) if \(\Sigma iO - \Sigma 0 > 0\) or small,\nwhich alternatively can be stated as \(\sigma O > \sigma iO\).";

In[175]:= "2. A FALL IN \(\Sigma 0\), HIGHER MANUFACTURING SUBSIDY";

In[177]:= B = {gradf[Fl[iN, iO, \mu, aO, aN, mN, mO, \etaS, \lambda, dr, kN, kO, \Sigma 0, \Sigma iO], {\Sigma 0}],
\quad \text{gradf}[F2[iN, iO, \mu, aO, aN, mN, mO, \etaS, \lambda, dr, kN, kO, \Sigma 0, \Sigma iO], {\Sigma 0}]\} // FullSimplify

Out[177]= \{\{aO kO kn \lambda (dr + iN + iO + \mu) \Sigma iO \}
\quad (iN + iO + \mu) (\lambda - mO \Sigma 0)\}, \{aO kO mO \lambda (dr + iN + iO + \mu) (iO mO + \lambda \mu) \Sigma iO
\quad (iN + iO + \mu) (\lambda - mO \Sigma 0)\}\}

In[178]:= B = {gradf[Fl[iN, iO, \mu, aO, aN, mN, mO, \etaS, \lambda, dr, kN, kO, \Sigma 0, \Sigma iO], {\Sigma 0}],
\quad \text{gradf}[F2[iN, iO, \mu, aO, aN, mN, mO, \etaS, \lambda, dr, kN, kO, \Sigma 0, \Sigma iO], {\Sigma 0}]\} // FullSimplify

Out[178]= \{\{aO kO kn \lambda (dr + iN + iO + \mu) \Sigma iO \}
\quad (iN + iO + \mu) (\lambda - mO \Sigma 0)\}, \{aO kO mO (dr + iN + iO + \mu) (iO mO + \lambda \mu)
\quad (iN + iO + \mu) (\lambda - mO \Sigma 0)\}
\quad mO (dr + iN + iO + \mu) (iO mO + \lambda \mu) (aO kO \lambda \Sigma iO + aN kn (-\lambda + mO \Sigma 0))\}
\quad mO (iN + iO + \mu) (\lambda - mO \Sigma 0)\}\}

In[179]:= impact\Sigma 0 = -\text{Inverse[J].B.}(\Delta \Sigma 0) // FullSimplify;

In[180]:= "The expressions are too complicated so I set dr=0.\"
In[181]:= \[\text{dr} = 0; \text{impact} \Sigma O = -\text{Inverse}[J].B.(\Sigma O) / \text{FullSimplify}\]

Out[181]= \[
\begin{align*}
\Delta \Sigma O & = \alpha O \Sigma O \lambda (i O m O + \lambda) \\
& + \alpha O \Sigma O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O) \\
& - \alpha O \Sigma O \lambda (\lambda - m O \Sigma O) \\
& + a O k O \Sigma I O + a N k N (\lambda - m O \Sigma O)
\end{align*}
\]

In[182]:= (diN, diO) = %;

In[183]:= "2.1. What happens to innovation rates: diN and diO?"

In[184]:= "The above implies that \[\frac{\text{diN}}{\Sigma O}\] is ambiguous. To find a necessary condition, I collect the iN and iO terms";

In[185]:= Collect[diN, \{iO, iN\}]

Out[185]= \[
\begin{align*}
\Delta \Sigma O & = \alpha O \lambda (i O m O + \lambda) \\
& + \alpha O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O) \\
& - \alpha O \lambda (\lambda - m O \Sigma O) \\
& + a O k O \Sigma I O + a N k N (\lambda - m O \Sigma O)
\end{align*}
\]

In[186]:= "-The first expression is negative
--The second expression is positive (note that the denominator is positive for \(w > 0\)).
--The third expression is positive.
Thus, \[\frac{\text{diN}}{\Sigma O}\] is ambiguous."

In[187]:= "For a necessary condition, I note that iN must be a larger than a critical level";

Solve[diN \[\to 0\], iN] / \text{FullSimplify}

Out[187]= \[
\begin{align*}
i N & \to -\frac{\alpha O \lambda (i O m O + \lambda) (\lambda - m O \Sigma O)}{m O (\alpha O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O))}
\end{align*}
\]

In[188]:= "Hence
\[\frac{\text{diN}}{\Sigma O}\] \[\to 0\] iff \[i N \to \frac{\alpha O \lambda (i O m O + \lambda) (\lambda - m O \Sigma O)}{m O (\alpha O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O))}\]"

In[189]:= "On the other hand, \[\frac{\text{diO}}{\Sigma O}\] \[\to 0\] unambiguously \!

In[190]:= "2.2. What happens to aggregate innovation: diA= diN+diO?"

In[191]:= diA = diN + diO / \text{FullSimplify}

Out[191]= \[
\begin{align*}
\Delta \Sigma O & = \frac{(i N + i O) m O \lambda + \lambda}{- \lambda + m O \Sigma O} \\
& - \frac{\alpha O k O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O)}{\alpha O k O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O)}
\end{align*}
\]

In[192]:= "Again, \[\frac{\text{diA}}{\Sigma O}\] is ambiguous. To simplify, I collect the iN and iO terms";

In[193]:= Collect[diA, \{iO, iN\}]

Out[193]= \[
\begin{align*}
\Delta \Sigma O & = \frac{i N m O \Delta \Sigma O + \Delta \Sigma O \lambda \Sigma I O + a O k O \Delta \Sigma O \lambda \mu}{- \lambda + m O \Sigma O} \\
& + \frac{\Delta \Sigma O \lambda \mu}{a O k O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O)} \\
& + \frac{i O m O \Delta \Sigma O}{\alpha O k O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O)} \\
& + \frac{(\alpha N k N + a O k O) \lambda \Sigma I O + a O k O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O)}{\alpha O k O \lambda \Sigma I O + a N k N (-\lambda + m O \Sigma O)}
\end{align*}
\]
-- The first term in front of iN is negative
-- The second term is negative.
-- The third term is positive given aN kN − aO kO < 0. This is due to the restriction for w that mN w > mO w.
-- The fourth term in front of iO can be positive or negative. Note that first term is negative and the second term is positive. In the second term, the numerator is negative aN kN − aO kO < 0 and the denominator is negative because of for w>0."

"To simplify the fourth term, I cross-multiply terms and obtain the new numerator. This gives";

\[
\frac{mO \Delta \Sigma O \ast (aO kO \lambda \Sigma iO + aN kN (-\lambda + mO \Sigma O)) + (-aN kN + aO kO) mO \Delta \Sigma O (-\lambda + mO \Sigma O)}{\text{FullSimplify}}
\]

\[
\text{Out[196]} = \frac{aO kO mO \Delta \Sigma O (\lambda (-1 + \Sigma iO) + mO \Sigma O)}{\text{FullSimplify}}
\]

"Note that the denominator is negative (one negative and one positive multiplied). The above will be positive iff \( \sigma_{iO} < mO \ast \Sigma O \lambda \), where \( \sigma_{iO} \) is the subsidy rate to outsourcing technology (recall that \( \Sigma iO = 1 - \sigma_{iO} \)). With \( \Sigma iO \) close to 1, that is, with \( \sigma_{iO} \) going to zero, we establish that the term in front of iO is negative. Hence, the only term that is positive is associated with the imitation rate. We can identify this but we cannot sign \( dA \) unambiguously." ;

"To find a necessary condition"; Solve[diA == 0, iN] // FullSimplify

\[
\text{Out[198]} = \frac{\lambda \mu}{\Sigma iO + mO \Sigma O} - \frac{aO kO \lambda \Sigma iO + aN kN (-\lambda + mO \Sigma O)}{\text{FullSimplify}}
\]

"Hence if \( \lambda (-1+\Sigma iO) + mO \Sigma O > 0 \), which holds when \( \sigma_{iO} < mO \Sigma O / \lambda \), the above will also automatically hold. Thus,

\[
\frac{diA}{d\Sigma O} = \frac{mO + \Sigma O}{\lambda}
\]

"2.3. What happens to the fraction of industries?";

"Totally differentiating nO implies ";

\[
dnO = D[nO, iO] diO + D[nO, iN] diN // FullSimplify
\]

\[
\Delta \Sigma O \left( aN iN iO kN mO (\lambda - mO \Sigma O) + aO kO \left( \lambda^2 \mu (iN + \mu) \Sigma iO + iO \lambda \mu (\lambda + mO (\Sigma iO - \Sigma O)) + iO^2 mO (\lambda - mO \Sigma O) \right) \right)
\]

\[
\text{Out[203]} = \frac{(iN + iO + \mu)^2 (\lambda - mO \Sigma O) (-aO kO \lambda \Sigma iO + aN kN (\lambda - mO \Sigma O))}{\text{FullSimplify}}
\]

"Clearly, \( \frac{dnO}{d\Sigma O} < 0 \) if \( (\Sigma iO - \Sigma O) > 0 \) or small,

a. If \( (\Sigma iO - \Sigma O) > 0 \).

b. Otherwise, that is for \( (\Sigma iO - \Sigma O) < 0 \), we need \( (\Sigma O - \Sigma iO) < \frac{\lambda}{mO} \), which means that the absolute value of \( (\Sigma O - \Sigma iO) \) needs to be sufficiently small

The part 'a' of the condition basically implies \( \sigma O > \sigma iO \), which is not a strict restriction";
In[205]:= "Totally differentiating nN implies ";
\[dnN = D\[nN, iO\] diO + D\[nN, iN\] diN\]  // FullSimplify

Out[206]= \[
\frac{\Delta \Sigma \ O \ N \ iN \ kN \ mO \ (i0 + \mu) \ (-\lambda + mO \ \Sigma O) + \ a0 \ kO \ (-\lambda (i0 + \mu) (i0 \ mO + \lambda \mu) + iN \ (mO - \lambda) \ \lambda \mu \ \Sigma iO + mO \ (i0 + \mu) (i0 \ mO + \lambda \mu) \ \Sigma O)) \)}{((iN + i0 + \mu)^2 (\lambda - mO \ \Sigma O) (-a0 \ kO \ \lambda \ \Sigma iO + aN \ kN \ (\lambda - mO \ \Sigma O)))}\]

In[207]:= "Clearly, \[\frac{dnN}{d\Sigma O}\] > 0";

In[208]:= "Totally differentiating nS implies";
\[dnS = D\[nS, iO\] diO + D\[nS, iN\] diN\]  // FullSimplify

Out[209]= \[
\frac{-\Delta \Sigma \ O \ \mu \ (aN \ \Sigma iO \ mO \ (\lambda - mO \ \Sigma O) + a0 \ kO \ (i0 \ mO \ (\lambda (-1 + \Sigma iO) + mO \ \Sigma O) + \lambda (\lambda \mu (-1 + \Sigma iO) + mO \ (iN \ \Sigma iO + \mu \ \Sigma O)))\)}{((iN + i0 + \mu)^2 (\lambda - mO \ \Sigma O) (-a0 \ kO \ \lambda \ \Sigma iO + aN \ kN \ (\lambda - mO \ \Sigma O)))}\]

In[210]:= "Clearly, \[\frac{dnS}{d\Sigma O}\] > 0. However, note that nS moves inversely with iA. Thus:
\[\frac{dnS}{daO} \rightarrow 0 \ if \ \sigma i0 < \frac{mO \ \Sigma O}{\lambda}";

In[211]:= "2.4. What happens to expenditure c?"

In[212]:= "Let's totally differentiate";
\[dc = D[c, iN] diN + D[c, iO] diO + D[c, aO] \Delta \Sigma O\]  // FullSimplify

Out[213]= \[
\frac{1}{\Delta \Sigma O \ \lambda \ (aN \ \Sigma iO \ mO \ (\lambda - mO \ \Sigma O) - kO \ (-\lambda (iN + i0 + \mu) \ \Sigma iO (\lambda - mO \ \Sigma O) + a0 \ (i0 \ mO (\lambda (-1 + \Sigma iO) + mO \ \Sigma O) + \lambda (\lambda \mu (-1 + \Sigma iO) + mO \ (iN \ \Sigma iO + \mu \ \Sigma O)))\)} (\frac{-\lambda (iN + i0 + \mu) (i0 \ mO + \lambda \mu)}{mN (iN + i0 + \mu) (\lambda - mO \ \Sigma O))})\]

In[214]:= "So, \[\frac{dc}{d\Sigma O}\] > 0";

In[215]:= "3. A FALL IN \Sigma iO, HIGHER OUTSOURCING SUBSIDY"

In[216]:= Clear[B, dr, \Sigma iO]

In[217]= B = \{gradf[F1[in, iO, \mu, aO, aN, mN, mO, \eta S, \lambda, dr, kN, kO, \Sigma O, \Sigma iO], {\Sigma iO}],
gradf[F2[in, iO, \mu, aO, aN, mN, mO, \eta S, \lambda, dr, kN, kO, \Sigma O, \Sigma iO], {\Sigma iO}]\}  // FullSimplify

Out[217]= \[
\left\{\frac{a0 \ kO \ \lambda \ (dr + iN + i0 + \mu) (i0 \ mO + \lambda \mu)}{(iN + i0 + \mu) (\lambda - mO \ \Sigma O)}\right\}, \left\{\frac{a0 \ kO \ \lambda \ (dr + iN + i0 + \mu) (i0 \ mO + \lambda \mu)}{mN (iN + i0 + \mu) (\lambda - mO \ \Sigma O)}\right\}\]

In[218]= impact\[\Sigma iO\] = -Inverse[J].B.\[\Delta \Sigma iO\]  // FullSimplify;
In[219]:= "The expressions are too complicated, thus, I set":

dr = 0; impact dio = -Inverse[J].B.{ΔΣio} // FullSimplify

Out[219]= 

In[220]= (dIN, dio) = %;

In[221]= "3.1. What happens to innovation rates: dIN and dio?";

In[222]= "It looks like \( \frac{dIN}{daO} \) is ambiguous, so I collect the iN and iO terms";

In[223]= Collect [dIN, {iO, iN}] 

Out[223]= 

In[224]= "--The first expression is negative.
--The second expression is positive (note that the denominator is positive for w > 0).
--The third expression is positive.

Hence \( \frac{dIN}{dΣio} \) is ambiguous";

In[225]= "I now look for a critical condition which will require that iN be above a critical level";

Solve [dIN = 0, iN] // FullSimplify

Out[225]= 

In[226]= "Hence a cleaner expression is
\( \frac{dIN}{dΣio} < 0 \) iff \( iN > \frac{aO kO (iO mO + λ μ) (λ - mO ΣO)}{mO (aO kO λ Σio + aN kN (-λ + mO ΣO))} \)

Observe that this is the same condition that is required for \( \frac{dIN}{dΣio} < 0 \);";

In[227]= "On the other hand, \( \frac{dio}{dΣio} < 0 \) unambiguously ";

In[228]= "3.2. What happens to aggregate innovation: diA = dIN+dio?";

In[229]= diA = dIN + dIO // FullSimplify

Out[229]= (-ΔΣio (aN kN mO (-λ + mO ΣO) +

aO kO (iO mO (λ (-1 + ΣO) + mO ΣO) + λ (λ μ (-1 + ΣO) + mO (iN Σio + μ ΣO)))) /

(mO Σio (aO kO λ Σio + aN kN (-λ + mO ΣO))))

In[230]= "Again, \( \frac{diA}{dΣio} \) is ambiguous. To simplify, I collect the iN and iO terms";
In[231] := Collect [diA, {iO, iN}]

Out[231] = aO iO kO \[Delta] \Sigma iO (\lambda (-1 + \Sigma iO) + mO \Sigma O) \\
     \[Sigma] iO (aO kO \lambda \Sigma iO + aN kN (-\lambda + mO \Sigma O)) \\
     \[Delta] \Sigma iO \{aO kO \lambda^2 \mu (-1 + \Sigma iO) + aO kO mO \lambda \mu \Sigma O\} \\
     mO \Sigma iO (aO kO \lambda \Sigma iO + aN kN (-\lambda + mO \Sigma O)) \\
     iN \[Delta] \Sigma iO (aO kO mO \lambda \Sigma iO + aN kN mO (-\lambda + mO \Sigma O)) \\
     mO \Sigma iO (aO kO \lambda \Sigma iO + aN kN (-\lambda + mO \Sigma O))

In[232] := "The coefficient in front of \[iN\] is negative, so a necessary condition would 
require that \[iN\] be above a critical level"; Solve[diA == 0, iN] // FullSimplify

Out[232] = \[
\begin{aligned}
&\{\{iN \to - aO kO \{iO mO + \lambda \mu\} \{\lambda (-1 + \Sigma iO) + mO \Sigma O\}\} \\
&\{iN \to - aO kO \lambda \Sigma iO + aN kN (-\lambda + mO \Sigma O)\}\}
\end{aligned}

In[233] := "diA \[lt] 0 \text{ iff } iN \[lt] - aO kO \{iO mO + \lambda \mu\} \{\lambda (-1 + \Sigma iO) - mO \Sigma O\}," \\
\text{iO} \Sigma iO \\
In[234] := "Hence if \(\lambda \{1 - \Sigma iO\} - mO \Sigma O \[lt] 0,\) which holds 
when \(\sigma iO < mO \Sigma O / \lambda,\) then the above will automatically hold. Thus,

\[
diA \[lt] 0 \text{ if } \sigma iO < \frac{mO \Sigma O}{\lambda}.
\]

Observe that this is the same condition as observed in the case of \(\Sigma O\);"

In[235] := "3.3. What happens to the fraction of industries?";

In[236] := "Totally differentiating \(nO\) implies ";
\text{nO} = D[nO, iO] diO + D[nO, iN] diN // FullSimplify

Out[237] = \[
\begin{aligned}
&\Delta \Sigma iO \{aO iN iO kN mO (-\lambda + mO \Sigma O) + \\
&aO kO \{\lambda (-1 + \Sigma iO) - iO \lambda \mu (\lambda + mO (\Sigma iO - \Sigma O)) + iO^2 mO (-\lambda + mO \Sigma O)\}\} \\
&mO (iN + iO + \mu)^2 \Sigma iO (aO kO \Sigma iO + aN kN (-\lambda + mO \Sigma O)) \\
&\Delta \Sigma iO \{aO IN iN kN mO (iO + \mu) (\lambda - mO \Sigma O) + \\
&aO kO \{\lambda (iO + \mu) (iO mO + \lambda \mu) + iN \lambda (-mO + \lambda) \mu \Sigma iO - mO (iO + \mu) (iO mO + \lambda \mu \Sigma O)\} \\
&mO (iN + iO + \mu)^2 \Sigma iO (aO kO \lambda \Sigma iO + aN kN (-\lambda + mO \Sigma O)) \\
\end{aligned}

In[238] := "Clearly, \[\frac{\text{nO}}{\Sigma iO} \[lt] 0 \text{ if } (\Sigma iO - \Sigma O) > 0 \text{ or small,}

which can also be stated as \(\sigma O > \sigma iO.\) Note that the denominator is negative";

In[239] := "Totally differentiating \(nN\) implies ";
\text{nN} = D[nN, iO] diO + D[nN, iN] diN // FullSimplify

Out[240] = \[
\begin{aligned}
&\Delta \Sigma iO \{aO iN iN kN mO (iO + \mu) (\lambda - mO \Sigma O) + \\
&aO kO \{\lambda (iO + \mu) (iO mO + \lambda \mu) + iN \lambda (-mO + \lambda) \mu \Sigma iO - mO (iO + \mu) (iO mO + \lambda \mu \Sigma O)\} \\
&mO (iN + iO + \mu)^2 \Sigma iO (aO kO \lambda \Sigma iO + aN kN (-\lambda + mO \Sigma O)) \\
\end{aligned}

In[241] := "Clearly, \[\frac{\text{nN}}{\Sigma iO} \[gt] 0\;\text{;}

which can also be stated as \(\sigma O > \sigma iO.\) Note that the denominator is negative";
In[242]:= "Totally differentiating nS implies;
\[\text{\texttt{\textbf{d}}nS = \text{\texttt{\textbf{D}}}[\text{\texttt{nS}}, \text{\texttt{iO}}] \text{\texttt{d}}\text{\texttt{iO}} + \text{\texttt{D}}[\text{\texttt{nS}}, \text{\texttt{iN}}] \text{\texttt{d}}\text{\texttt{iN}}]//\text{\texttt{FullSimplify}}\]

Out[243]= \(\text{\texttt{\textbf{d}}nS} = \frac{\partial}{\partial nS} (\partial nS, iO) diO + \frac{\partial}{\partial nS} (\partial nS, iN) diN\)

In[244]:= "Clearly, \(\text{\texttt{d}}nS\) looks ambiguous. However, note that nS moves inversely with iA. Thus:

\[\text{\texttt{\textbf{d}}nS > 0} \ \text{\texttt{if \ \sigma_{iO}} < \lambda}\];

In[245]:= "3.4. What happens to expenditure c?"

In[246]:= "Let's totally differentiate";

In[247]:= \(\text{\texttt{dc = \text{\texttt{D}}[\text{\texttt{c}}, \text{\texttt{iN}}] \text{\texttt{d}}\text{\texttt{iN}} + \text{\texttt{D}}[\text{\texttt{c}}, \text{\texttt{iO}}] \text{\texttt{d}}\text{\texttt{iO}} + \text{\texttt{D}}[\text{\texttt{c}}, \text{\texttt{aO}}] \text{\texttt{D}}\text{\texttt{diO}} + \text{\texttt{D}}[\text{\texttt{c}}, \text{\texttt{aN}}] \text{\texttt{D}}\text{\texttt{diN}}]//\text{\texttt{FullSimplify}}\)

Out[247]= \(\text{\texttt{dc}} = \frac{\text{\texttt{\textbf{d}}c}}{\text{\texttt{d}}\text{\texttt{iO}}} = \frac{\partial}{\partial c} (\partial c, iO) diO + \frac{\partial}{\partial c} (\partial c, iN) diN\)

In[248]:= "So, \(\text{\texttt{dc}} > 0\);"

In[249]:= "4. A FALL IN \(\mu\), STRONGER IPRs";

In[250]:= Clear[B, dr]

In[251]:= \(B = \{\text{\texttt{\textbf{gradf}}}[\text{\texttt{F1[\text{\texttt{in}}, \text{\texttt{iO}}, \text{\texttt{\mu}}, \text{\texttt{aO}}, \text{\texttt{aN}}, \text{\texttt{mN}}, \text{\texttt{mO}}, \text{\texttt{\etaS}}, \text{\texttt{\lambda}}, \text{\texttt{dr}}, \text{\texttt{kN}}, \text{\texttt{kO}}, \text{\texttt{\Sigma}}, \text{\texttt{\SigmaIO}}], \{\text{\texttt{\mu}}]\}],
\text{\texttt{\textbf{gradf}}}[\text{\texttt{F2[\text{\texttt{in}}, \text{\texttt{iO}}, \text{\texttt{\mu}}, \text{\texttt{aO}}, \text{\texttt{aN}}, \text{\texttt{mN}}, \text{\texttt{mO}}, \text{\texttt{\etaS}}, \text{\texttt{\lambda}}, \text{\texttt{dr}}, \text{\texttt{kN}}, \text{\texttt{kO}}, \text{\texttt{\Sigma}}, \text{\texttt{\SigmaIO}}], \{\text{\texttt{\mu}}]\}]\} //\text{\texttt{FullSimplify}}\)

Out[251]= \(B = \{\text{\texttt{\textbf{dr}}} \text{\texttt{\textbf{in}}} (\partial aO kO \lambda \Sigma IO + aN kN (\lambda - mO \Sigma)) \},
\text{\texttt{\textbf{dr}}} \text{\texttt{\textbf{in}}} (\partial aO kO \lambda \Sigma IO + aN kN (\lambda - mO \Sigma)) \},
\text{\texttt{\textbf{dr}}} \text{\texttt{\textbf{in}}} (\partial aO kO \lambda \Sigma IO + aN kN (\lambda - mO \Sigma)) \}

In[252]:= \(\text{\texttt{impact}} \mu = \text{\texttt{-Inverse\{\text{\texttt{J}}\}.B.\{\Delta\mu\]} //\text{\texttt{FullSimplify}}\);
In[253] := "The expressions are too complicated, thus, I set";
\[ dr = 0; \] impact\[\mu = -\text{Inverse}[J].B.\{\Delta\mu} // \text{FullSimplify} \]

Out[253] = \[
\frac{\Delta\mu (\lambda - mO \Sigma O)}{mO \Sigma I O}, \quad -\frac{\Delta\mu \lambda}{mO}
\]

In[254] := \{(diN, diO) = \%; \}

In[255] := "3.1. What happens to innovation rates: diN and diO?";

In[256] = "Clearly, \[\frac{\text{diN}}{d\mu} > 0 \] and \[\frac{\text{diO}}{d\mu} < 0."; \]

In[257] := "3.2. What happens to aggregate innovation: diA = diN + diO?";

In[258] := diA = diN + diO // \text{FullSimplify} \]

Out[258] = \[
\frac{\Delta\mu (\lambda - \lambda \Sigma I O - mO \Sigma O)}{mO \Sigma I O}
\]

In[259] = "Note that \((\lambda - \lambda \Sigma I O - mO \Sigma O) < 0\) holds when \(oI0 < mO \Sigma O / \lambda\), which is a reasonable assumption. Thus,
\[
\frac{\text{diA}}{d\mu} < 0 \quad \text{if and only if} \quad \frac{mO \Sigma O}{\lambda} < \frac{mO \Sigma O}{\lambda};
\]

In[260] := "3.3. What happens to the fraction of industries?";

In[261] = "Totally differentiating no implies ";
\[ dnO = D[nO, iO] \text{diO} + D[nO, iN] \text{diN} // \text{FullSimplify} \]

Out[262] = \[
-\Delta\mu \lambda (iO + (iN + \mu) \Sigma I O) + iO mO \Delta\mu \Sigma O
\]

Out[262] = \[
\frac{mO (iN + iO + \mu)^2 \Sigma I O}{mO \Sigma I O}
\]

In[263] = "Clearly, \[\frac{\text{dnO}}{d\mu} < 0."; \]

In[264] = "Totally differentiating nN implies ";
\[ dnN = D[nN, iO] \text{diO} + D[nN, iN] \text{diN} // \text{FullSimplify} \]

Out[265] = \[
\frac{\Delta\mu \lambda (iO + \mu + iN \Sigma I O) - mO \Delta\mu (iO + \mu) \Sigma O}{mO (iN + iO + \mu)^2 \Sigma I O}
\]

Out[265] = \[
\frac{\text{dnN}}{\Sigma \mu} > 0;
\]

In[266] = "Clearly, \[\frac{\text{dnN}}{\Sigma \mu} > 0.";
In[267]:= "Totally differentiating nS implies";
dnS = D[nS, iO] diO + D[nS, iN] diN + D[nS, μ] Δμ // FullSimplify

Out[268]= Δμ (λ μ (-1 + Σi0) + (iN + iO) mO Σi0 + mO μ ΣO)
          mO (iN + iO + μ) Σi0

In[269]:= "Clearly, \(\frac{dnS}{dμ}\) > 0 if \(\frac{mO + ΣO}{λ}\) < ”;

In[270]:= "3.4. What happens to expenditure c?";

In[271]:= "Let’s totally differentiate”;

In[272]:= dc = D[c, iN] diN + D[c, iO] diO + D[c, aO] Δμ // FullSimplify

Out[272]= \(\frac{1}{mN mO ΣiO (-λ + mO ΣO)}\)
          \(Δμ (λ (aN kN (λ - mO ΣO) (λ (l + Σi0) + mO ΣO) + kO λ Σi0 (-mO (iN + iO + μ) Σi0 + aO (λ (l + Σi0) + mO ΣO))))\}

In[273]:= "So, \(\frac{dc}{dμ}\) < 0";
**APPENDIX B: NUMERICAL SIMULATIONS TO SIGN COMPARATIVE STATICS**

**THE MODEL WITH UNEQUAL IMITATION RATES TARGETING BOTH NORTHERN AND OUTSOURCING INDUSTRIES;**

Date: August 24, 2007;

**CONSIDERATIONS IN BENCHMARK PARAMETER CHOICES:**

1. Note that \( m_O > 1 \) must hold. Otherwise the current-quality leader has to compete the previous Northern outsourcer and not the Southern follower. This is not going to be compatible with the pricing scheme in the paper.

2. I impose \( a_O + k_O > a_N + k_N \) so that the value of an outsourcing firm is larger than that of a local-sourcing firm.

3. Choose \( \lambda \) such that the mark up over marginal cost \( \lambda - m_N w \) is around 25% in the North. Choosing a high \( \lambda \) also creates some room for the North-South relative wage to diverge significantly.

4. Choose \( a_N \) and \( a_O \) such that the aggregate rate of innovation is in the neighborhood of 2%.

5. Choose \( \mu \) such that it is in the neighborhood of the aggregate innovation rate and it leads to a somewhat equal configuration of industries.

6. The \( m_N \) and \( m_O \) parameters below may look close to each other but actually the marginal costs in the North and the South differ substantially when the relative wage \( w \) is taken into account.

7. \( k_N \) and \( k_O \) are normalized to one because they always enter as multiplicative terms attached to \( a_N \) and \( a_O \); hence variations in \( a_N \) and \( a_O \) are sufficient.

8. The terms \( REPO = i_O + i_N + \mu_O \) and \( REPN = i_O + i_N + \mu_N \) capture the threat of replacement faced by Outsourcing and Northern industries.

9. Further details on the choice of parameters with specific references to empirical literature can be found in my previous papers (such as Sener (2006), IPRs and Rent Protection in a North-South Product Cycle Model, manuscript, Sayek and Sener (2006), Outsourcing and Wage Inequality in a Dynamic Product Cycle Model, Appendix)."

```
Clear [FEIN, FEIO]; Clear [\lambda, aN, aO, kN, kO, \lambda, \Sigma iO, \Sigma O, mN, mO, \eta S, dr, \mu N, \mu O];
Clear [LN, LS]; Clear [c, w, iN, iO, nN, nO, nS];
```

```
THE BENCHMARK PARAMETERS
```

dr = 0.07 - 0.014; \( \mu N = 0.003; \mu O = 0.011; \lambda = 1.5; \)
\( \eta S = 2; \)
\( k_N = 1; k_O = 1; \)
\( a_N = 1.4; a_O = 3.8; \)
\( \Sigma O = 1; \Sigma iO = 1; \)
\( m_N = 1.05; m_O = 1.02; \)

```
STEADY-STATE EQUATIONS
```

```
I first note the equations for \( c \) and \( w \) in the imitation model. The long derivations are in the 'Unequal Imitation Steady-State Solutions Mathematica file';
```
In[288] := "This is the equation for c";

In[289] := \[CON = \frac{c + \lambda (-a_0 k_0 \lambda (dr + iN + iO + \mu O) \Sigma O \lambda + aN kN (dr + iN + iO + \mu N) (\lambda - \mu O \Sigma O))}{mN (\lambda - \mu O \Sigma O)} = 0; \]

In[290] := "This is the equation for North-South relative wage";

In[291] := \[WG = w - \frac{a_0 k_0 \lambda (dr + iN + iO + \mu O) \Sigma O \lambda + aN kN (dr + iN + iO + \mu N) (-\lambda + \mu O \Sigma O)}{a_0 k_0 mN (dr + iN + iO + \mu O) \Sigma O} = 0; \]

In[292] := "I enter the industry equilibrium shares";

In[293] := \[nO = \frac{iO}{iN + iO + \mu N}; nN = \frac{iN}{iN + iO + \mu N}; nS = \frac{iO + \mu N}{iN + iO + \mu N} - \frac{iO}{iN + iO + \mu N}; \]

In[294] := "Profit flow of a Northern firm adjusted for world population LN+LS, ie., divided by LN+LS";

In[295] := \[\pi N = \frac{\Sigma O \Sigma O}{c} \sum \frac{c*mN}{\lambda} - \frac{1}{1+\eta S} \];

In[296] := "Profit flow of an outsourcing firm adjusted for world population";

In[297] := \[\pi O = \frac{\Sigma O \Sigma O}{c} \sum \frac{c*mO}{\lambda} - \frac{\eta S}{1+\eta S} \];

In[298] := "I now rewrite the Northern and Southern labor market equations";

In[299] := \[LN = (iN * aN + kN) + (iO * aO + kO) + \left(\frac{nN * mN}{\lambda}\right) - \frac{1}{(1+\eta S)} = 0 // FullSimplify; \]

In[300] := \[LS = \frac{(nO) * c + mO}{\lambda} + (nS) * c - \frac{\eta S}{1+\eta S} = 0; \]

In[301] := "NUMERICAL SOLUTIONS FOR STEADY-STATE EQUILIBRIUM";

In[302] := Needs["Miscellaneous`RealOnly`"];
NSolve[{LN, LS, CON, WG, {iN, iO, w, c}}]

Out[302] = \{[iN -> -0.606639, iO -> 0.528835, w -> 1.13544, c -> -0.145671],
{iN -> -0.00140532, iO -> -0.00421162, w -> 1.2821, c -> 0.934554},
iN -> 0.005561, iO -> 0.00724858, w -> 1.27703, c -> 1.21029] \]

In[303] := FindRoot[{LN, LS, CON, WG, {iN, 0.01}, {iO, 0.01}, {c, 1}, {w, 1}, MaxIterations -> 10000}
Out[303] = \{[iN -> 0.005561, iO -> 0.00724858, c -> 1.21029, w -> 1.27703] \]

In[304] := Output[1] = %;

In[305] = \{"iA" -> iN + iO, "nN" -> nN, "nO" -> nO, "nS" -> nS, "\pi N" -> \pi N, "\pi O" -> \pi O,
"w*mN" -> w * mN, "mO*\Sigma O" -> mO * \Sigma O, "REPO" -> iO + iN + \mu O, "REPN" -> iO + iN + \mu N]\ / . \%

Out[306] = \{[iA -> 0.0128096, nN -> 0.351749, nO -> 0.304444, nS -> 0.343812, \pi N -> 0.128384,
\pi O -> 0.387294, w*mN -> 1.34088, mO*\Sigma O -> 1.02, REPO -> 0.0238096, REPN -> 0.0158096] \]

In[306] := "The rate of growth in utility is"; 0.012809578328133838` * Log[1.5]

Out[306] = 0.00519384
**Observations for Numerical Outcomes**

1. **IA is around 1.2%**, which renders \( g = IA \times \log(1.5) = 0.005 \). This is the growth rate attributable to technology improvements according to Denison (1985) and also used in Segerstrom papers.

2. The mark-up \( \lambda \cdot mnw \) is around 25%.

3. The industry configuration looks somewhat equally distributed and reasonable.

4. North-South wage gap is around 25%, which sounds low but indeed reasonable given the upper bound dictated by \( \lambda \).

5. Profit flow per world population are positive and thus the model is correctly solved.

**A Decline in AO**

\[
\begin{align*}
\Delta CON &= c - \frac{\lambda \cdot AO \cdot \Delta AO \cdot (dr + iN + iO + \mu O) \cdot \Sigma O + aN \cdot kN \cdot (dr + iN + iO + \mu N) \cdot (\lambda - \mu O \cdot \Sigma O)}{\mu N (\lambda - \mu O \cdot \Sigma O)} = 0; \\
\Delta WG &= w - \frac{\lambda \cdot AO \cdot kO \cdot (dr + iN + iO + \mu O) \cdot \Sigma O + aN \cdot kN \cdot (dr + iN + iO + \mu N) \cdot (-\lambda + \mu O \cdot \Sigma O)}{\mu N (\lambda - \mu O \cdot \Sigma O)} = 0; \\
\Delta nN &= \frac{iO + \mu N}{iN + iO + \mu N} - \frac{iO}{iO + \mu N}; \Delta iO = \frac{c \cdot (\lambda - \mu N \cdot w)}{\lambda}; \Delta nO = \frac{c \cdot (\lambda - (\mu O \cdot \Sigma O))}{\lambda}; \\
\Delta LN &= (iN \cdot aN \cdot kN) + (iO \cdot aO \cdot kO) + \left( \frac{nN \cdot c \cdot mN}{\lambda} \right) - \frac{1}{(1 + \eta S)} = 0 // \text{FullSimplify}; \\
\Delta LS &= \frac{(nO) \cdot c \cdot mO}{\lambda} + (nS) \cdot c - \frac{\eta S}{(1 + \eta S)} = 0;
\end{align*}
\]

**Needs**

\[
\begin{align*}
\text{NSolve} &\{[\text{LN}, \text{LS}, \text{CON}, \text{WG}], \{\text{iN}, \text{iO}, \text{w}, \text{c}\} \}
\end{align*}
\]

Out[322]= {{iN \to -0.69836, iO \to 0.620821, w \to 1.09939, c \to -0.123839}, \\
{iN \to -0.00295836, iO \to -0.00474188, w \to 1.26668, c \to 0.802781}, \\
{iN \to 0.00846547, iO \to 0.0155753, w \to 1.25788, c \to 1.22392}}
\textbf{In[323]} := \texttt{FindRoot[\{LN, LS, CON, WG\}, \{iN, 0.02\}, \{iO, 0.01\}, \{c, 1\}, \{w, 1\}, MaxIterations \rightarrow 10000]}

\textbf{Out[323]} = \{iN \rightarrow 0.00846547, iO \rightarrow 0.0155753, c \rightarrow 1.22392, w \rightarrow 1.25788\}

\textbf{In[324]} := \texttt{dw = (w / . Output[1]) - (w / . Output[1]); dc = (c / . Output[1]) - (c / . Output[1]);
  diN = (iN / . Output[1]) - (iN / . Output[1]); dio = (iO / . Output[1]) - (iO / . Output[1]);
  dnO = (nO / . Output[1]) - (nO / . Output[1]);
  dc = (c / . Output[1]) - (c / . Output[1]);
  dnS = (nS / . Output[1]) - (nS / . Output[1]);
}

\textbf{In[325]} := \texttt{\{"nN" \rightarrow nN, "nO" \rightarrow nO, "nS" \rightarrow nS, 
  "\pi N" \rightarrow \pi N, "\pi O" \rightarrow \pi O,
  "w*mN" \rightarrow w*nN, "mO*\Sigma O" \rightarrow mO*\Sigma O, "REPO" \rightarrow iO + iN + \mu O, "REPN" \rightarrow iO + iN + \mu N\} / . \%
}

\textbf{Out[325]} = \{nN \rightarrow 0.313063, nO \rightarrow 0.444491, nS \rightarrow 0.242446, \pi N \rightarrow 0.146238,
  \pi O \rightarrow 0.391653, w*nN \rightarrow 1.32077, mO*\Sigma O \rightarrow 1.02, REPO \rightarrow 0.0350408, REPN \rightarrow 0.0270408\}

\textbf{In[326]} := \texttt{y = \{iN, iO, iN + iO, w, c, nN, nO, nS, w * mN, mO * \Sigma O, \pi N, \pi O\} / . \%
}

\textbf{Out[326]} = \{0.00846547, 0.0155753, 0.0240408, 1.25788, 1.22392,
  0.313063, 0.444491, 0.242446, 1.32077, 1.02, 0.146238, 0.391653\}

\textbf{In[327]} := \texttt{\{"dw" \rightarrow dw, "{d\texttt{io}}" \rightarrow \texttt{dio}, "{d\texttt{nN}}" \rightarrow \texttt{dinN}, "{d\texttt{ia}}" \rightarrow \texttt{dia},
  "{dnN}" \rightarrow \texttt{dnN}, "{dc}" \rightarrow \texttt{dc}, "{dnS}" \rightarrow \texttt{dnS}\}

\textbf{Out[327]} = \{\{\texttt{dw} \rightarrow -0.0191517\}, \{\texttt{dio} \rightarrow 0.00832672\}, \{\texttt{dinN} \rightarrow 0.00290447\}, \{\texttt{dia} \rightarrow 0.0112312\},
  \{\texttt{dnN} \rightarrow 0.140051\}, \{\texttt{dc} \rightarrow 0.0136226\}, \{\texttt{dnS} \rightarrow -0.0386854\}, \{\texttt{dnS} \rightarrow -0.101366\}\}

\textbf{In[328]} := \texttt{"Expected outcome based on no imitation case:
  \texttt{dw<0, d\texttt{io}>0, d\texttt{nN}>0 iff iO \rightarrow (1-\sigma IO) \phi N(1+\phi O)\},
  \texttt{d\texttt{ia}>0, d\texttt{nO}>0, d\texttt{nN}<0";}

\[ \text{Out[32]} = \text{CON} \]
\[ \text{In[32]} = \text{Check}; \{ \frac{iO}{\text{in}} < \frac{\lambda \Sigma \text{iO} (\alpha \text{o} \text{kO} \lambda \Sigma \text{iO} - \text{aN} \text{kn} (\lambda - \text{mO} \Sigma \text{o}))}{\text{aN} \text{kn} (\lambda - \text{mO} \Sigma \text{o})^2} \}/. \text{Output[1], } \{ \frac{iO}{\text{in}}, \frac{iO}{\text{iN}}, \frac{\lambda \Sigma \text{iO} (\alpha \text{o} \text{kO} \lambda \Sigma \text{iO} - \text{aN} \text{kn} (\lambda - \text{mO} \Sigma \text{o}))}{\text{aN} \text{kn} (\lambda - \text{mO} \Sigma \text{o})^2} \}/. \text{Output[1]} \} \]
\[ \text{Out[32]} = \{ \text{True}, \{1.30347, 20.731\} \}
\]

\[ \text{In[330]} = \text{"BOTTOM LINE"; } \]
\[ \text{In[331]} = \text{"The expected results based on the zero imitation model continue to hold. The analytical condition I identified for an increase in iN also holds under this simulation"; } \]
\[ \text{In[332]} = \text{"2. A DECLINE IN EO (HIGHER SUBSIDY TO MANUFACTURING)"; } \]
\[ \text{In[333]} = \text{Clear[}\text{FEIN, FEIO}]; \text{Clear[}\lambda, \text{aN, aO, kn, kO, } \lambda, \Sigma \text{iO, } \Sigma \text{O, mN, mO, } \eta \text{S, } \text{dr, } \mu]; \text{Clear[}\text{LN, LS}]; \text{Clear[}c, w, \text{iO, nN, nO, } nS]; \]
\[ \text{In[334]} = \text{dr} = 0.07 - 0.014; \mu \text{N} = 0.003; \mu \text{O} = 0.011; \lambda = 1.5; \]
\[ \eta \text{S} = 2; \]
\[\text{kN} = 1; \text{kO} = 1; \]
\[\text{aN} = 1.4; \text{ao} = 3.8; \]
\[\Sigma \text{O} = 1; \Sigma \text{iO} = 1; \]
\[\text{mN} = 1.05; \text{mO} = 1.02; \]
\[\Sigma \text{O} = 1 \ast 0.9; \]
\[\lambda = \Sigma \text{O} = 1 \ast 0.9; \]
\[\text{CON} = c + \frac{\text{mN} (\lambda - \text{mo} \Sigma \text{o})}{\text{mN} (\lambda - \text{mO} \Sigma \text{o})} = 0; \]
\[\text{WG} = w - \frac{\text{mo} \Sigma \text{n} \text{kn} (\lambda - \text{mO} \Sigma \text{o})}{\text{mo} \Sigma \text{n} \text{kn} (\lambda - \text{mO} \Sigma \text{o})} = 0; \]
\[\text{nN} = \frac{\text{nN}}{\text{nN}}; \text{nS} = \frac{\text{nS}}{\text{nS}}; \]
\[\text{LN} = \frac{\text{LN}}{\text{LN}} + \frac{\text{LS}}{\text{LS}} - \frac{\text{LS}}{\text{LS}}; \text{LS} = \frac{\text{LS}}{\text{LS}} + \frac{\text{LS}}{\text{LS}} - \frac{\text{LS}}{\text{LS}}; \]
\[\text{LN} = \text{LN}; \text{LS} = \text{LS}; \]
\[\text{Needs[}\text{Miscellaneous`RealOnly"}; \]
\[\text{NSolve[}\{\text{LN, LS, CON, WG}, \{\text{iN, iO, w, c}\}]; \]
\[\text{Out[343]} = \{ \{ \text{iN} \rightarrow -0.612184, \text{iO} \rightarrow 0.53129, \text{w} \rightarrow 1.10677, \text{c} \rightarrow -0.150598\}, \{ \text{iN} \rightarrow -0.00405946, \text{iO} \rightarrow -0.00463834, \text{w} \rightarrow 1.25238, \text{c} \rightarrow 0.715111\}, \{ \text{iN} \rightarrow 0.00977062, \text{iO} \rightarrow 0.0220986, \text{w} \rightarrow 1.24088, \text{c} \rightarrow 1.20156\} \}
\]
\[\text{In[344]} = \text{FindRoot[}\{\text{LN, LS, CON, WG}, \{\text{iN}, 0.02\}, \{\text{iO}, 0.01\}, \{\text{c}, 1\}, \{\text{w}, 1\}], \text{MaxIterations} \rightarrow 10000 \]
\[\text{Out[344]} = \{ \{ \text{iN} \rightarrow 0.00977062, \text{iO} \rightarrow 0.0220986, \text{c} \rightarrow 1.20156, \text{w} \rightarrow 1.24088\} \]
\[\text{In[345]} = \text{dw} = (\text{w} / . \% \rightarrow (\text{w} / . \text{Output[1]})); \text{dc} = (\text{c} / . \% \rightarrow (\text{c} / . \text{Output[1]})); \]
\[\text{diN} = (\text{iN} / . \% \rightarrow (\text{iN} / . \text{Output[1]})); \text{diO} = (\text{iO} / . \% \rightarrow (\text{iO} / . \text{Output[1]})); \]
\[\text{dia} = ((\text{iO} + \text{iN}) / . \% \rightarrow ((\text{iO} + \text{iN}) / . \text{Output[1]})); \text{dnN} = (\text{nN} / . \% \rightarrow (\text{nN} / . \text{Output[1]})); \]
\[\text{dnO} = (\text{nO} / . \% \rightarrow (\text{nO} / . \text{Output[1]})); \text{dsn} = (\text{nS} / . \% \rightarrow (\text{nS} / . \text{Output[1]})); \]
"w*mN" \[Rule] w*mN, 
%. Out[346]= {nN \[Rule] 0.280208, nO \[Rule] 0.515488, nS \[Rule] 0.204304, 
\[Pi]N \[Rule] 0.157861, 
\[Pi]O \[Rule] 0.466204, w*mN \[Rule] 1.30293, mO*\[Sigma]O \[Rule] 0.918, REPO \[Rule] 0.0428692, REPN \[Rule] 0.0348692} 

In[347]:= y = {iN, iO, iN + iO, w, c, nN, nO, nS, w*mN, mO*\[Sigma]O, \[Pi]N, \[Pi]O} /. 
%. Out[347]= {0.00977062, 0.0220986, 0.0318692, 1.24088, 1.20156, 
0.280208, 0.515488, 0.204304, 1.30293, 0.918, 0.0428692, 0.0348692} 

Out[348]= {{dw \[Rule] -0.0361481}, {diO \[Rule] 0.01485}, {diN \[Rule] 0.00420962}, {diA \[Rule] 0.0190596},
{dnO \[Rule] 0.211049}, {dc \[Rule] -0.00873678}, {dnN \[Rule] -0.0715408}, {dnS \[Rule] -0.139508}} 

In[349]:= "Expected outcome based on no imitation case:

dw<0, diO>0, diN>0 iff

\[ \begin{align*}
\frac{iO \ aO \ kO \ \lambda \ S_iO + aN \ kN \ (-\lambda + mO \ \Sigma O)}{\ aO \ kO \ (\lambda - mO \ \Sigma O)} & \leq iN,
\end{align*} \]

\[ \begin{align*}
diA>0, \ dnO>0, \ dnN < 0";
\end{align*} \]
In[350]:=
\[
\text{"Check":}\left\{\frac{iO}{iN} aO kO} \lambda \Sigma iO + aN kN (-\lambda + mO} \Sigma O) \quad / \text{Output[1]},
\frac{iO}{iN} aO kO} \lambda \Sigma iO + aN kN (-\lambda + mO} \Sigma O) \quad / \text{Output[1]}\right\}\]

Out[350]= \{True, \{1.30347, 2.2089\}\}

In[351]:=
"The if and only if condition holds, the results are in the expected direction";

In[352]:=
"BOTTOM LINE";

In[353]:=
"The expected results based on the equal imitation model continue to hold. The analytical condition I identified for an increase in iN also holds under this simulation";

In[354]:=
"3. A DECLINE IN \Sigma iO (HIGHER SUBSIDY TO OUTSOURCING TECHNOLOGY)";

In[355]:=
Clear[FEIN, FEIO]; Clear[\[Lambda], aN, aO, kN, kO, \[Lambda], \[Sigma]iO, \[Sigma]O, mN, mO, \[Eta]S, dr, \[Mu]]; Clear[\[Lambda]N, \[Lambda]S]; Clear[c, w, iN, iO, nN, nO, nS];

In[356]:=
"THE BENCHMARK PARAMETERS";

In[357]:=
dr = 0.07 - 0.014; \[Mu]N = 0.003; \[Mu]O = 0.011; \[Lambda] = 1.5;
\[Eta]S = 2;
kN = 1; kO = 1;
aN = 1.4; aO = 3.8;
\[Sigma]iO = 1; \[Sigma]O = 1;
mN = 1.05; mO = 1.02;

In[358]:=
\[Sigma]iO = 1 * 0.9;

In[359]:=
CON = c + \frac{\lambda (-aO kO \lambda (dr + iN + iO + \muO) \Sigma iO + aN kN (dr + iN + iO + \muN) \lambda \muO) \lambda \Sigma O)}{mN \lambda \Sigma O} = 0;

WG = w - \frac{\lambda (-aO kO \lambda (dr + iN + iO + \muO) \Sigma iO + aN kN (dr + iN + iO + \muN) (-\lambda + mO \Sigma O))}{aO kO mN (dr + iN + iO + \muO) \Sigma iO} = 0; nO = \frac{iO}{iN + iO + \muO};
nN = \frac{iO}{iN + iO + \muO}; nS = \frac{iO}{iN + iO + \muO}; \[Pi]N = \frac{c (\lambda - mN \muO w)}{\lambda}; \[Pi]O = \frac{c (\lambda - (mO + (\Sigma O)) \lambda)}{\lambda};

\[\text{LN} = (iN * aN * kN) + (iO * aO * kO) + \left(\frac{c * mN}{\lambda} - \frac{1}{(1 + \[Eta]S)}\right) = 0 / \text{FullSimplify};

\[\text{LS} = \left(\frac{nO * c * mO}{\lambda} + (nS) * c - \frac{\[Eta]S}{(1 + \[Eta]S)}\right) = 0;\]

In[360]:=
Needs["Miscellaneous`RealOnly`"];

NSolve[{\[Lambda]N, \[Lambda]S, CON, WG}, \{iN, iO, w, c\}]

Out[360]= \{\{iN -> 0.60945, iO -> 0.530076, w -> 1.12046, c -> 0.148183\},
\{iN -> 0.00299573, iO -> 0.0047368, w -> 1.2667, c -> 0.802352\},
\{iN -> 0.00819856, iO -> 0.0152389, w -> 1.25799, c -> 1.21591\}\}
\textbf{In[367]:=}
FindRoot\{\text{LN, LS, CON, WG}, \{iN, 0.02\}, \{iO, 0.01\}, \{c, 1\}, \{w, 1\}, \text{MaxIterations} \rightarrow 10000\}
\textbf{Out[367]=} \{iN \rightarrow 0.00819856, iO \rightarrow 0.0152389, c \rightarrow 1.21591, w \rightarrow 1.25799\}

\textbf{In[368]=}
dw = \left(\frac{w}{w} - \text{. Output[1]}\right); dc = \left(\frac{c}{c} - \text{. Output[1]}\right);
diN = \left(\frac{iN}{iN} - \text{. Output[1]}\right); diO = \left(\frac{iO}{iO} - \text{. Output[1]}\right);
diA = \left(\frac{(iO + iN)}{(iO + iN)} - \text{. Output[1]}\right); dnN = \left(\frac{nN}{nN} - \text{. Output[1]}\right); dnO = \left(\frac{nO}{nO} - \text{. Output[1]}\right); dnS = \left(\frac{nS}{nS} - \text{. Output[1]}\right);

\textbf{In[369]=}
\{"nN" \rightarrow nN, "nO" \rightarrow nO, "nS" \rightarrow nS, "\pi N" \rightarrow \pi N, "\pi O" \rightarrow \pi O, "w*mN" \rightarrow w*mN, "mO*\Sigma O" \rightarrow mO*\Sigma O, "$\text{REPO}" \rightarrow iO + iN + \mu O, "$\text{REPN}" \rightarrow iO + iN + \mu N\} / . \%
\textbf{Out[369]=} \{nN \rightarrow 0.310111, nO \rightarrow 0.44251, nS \rightarrow 0.247379, \pi N \rightarrow 0.145188, \pi O \rightarrow 0.389092, w*mN \rightarrow 1.32089, mO*\Sigma O \rightarrow 1.02, \text{REPO} \rightarrow 0.0344375, \text{REPN} \rightarrow 0.0264375\}

\textbf{In[370]=}
y = \{iN, iO, iN + iO, w, c, nN, nO, nS, w*mN, mO*\Sigma O, \pi N, \pi O\} / . \%
\textbf{Out[370]=} \{0.00819856, 0.0152389, 0.0234375, 1.25799, 1.21591, 0.310111, 0.44251, 0.247379, 0.32089, 1.02, 0.145188, 0.389092\}

\textbf{In[371]=}
\{"dw" \rightarrow dw, "diO" \rightarrow diO, "diN" \rightarrow diN, "diA" \rightarrow diA, "dnO" \rightarrow dnO, "dc" \rightarrow dc, "dnN" \rightarrow dnN, "dnS" \rightarrow dnS\}
\textbf{Out[371]=} \{dw \rightarrow -0.019042, diO \rightarrow 0.00799032, diN \rightarrow 0.00263756, diA \rightarrow 0.0106279, dnO \rightarrow 0.13807, dc \rightarrow 0.00561796, dnN \rightarrow -0.0416373, dnS \rightarrow -0.0964325\}

\textbf{In[372]=}
"Expected outcome based on no imitation case:
\begin{align*}
dw < 0, \quad & \text{diO} > 0, \quad \text{diN} > 0 \quad \text{iff} \\
& \frac{iO}{iN} < \frac{aO\lambda\Sigma O + aN\lambda (-\lambda + mO\Sigma O)}{aO\lambda\Sigma O}, \quad \text{diA} > 0, \quad \text{dnN} > 0, \quad \text{dnN} < 0";
\end{align*}
In[373]:= "Check"; \{ \frac{iO \ aO \ kO \ \lambda \ \Sigma iO + aN \ kN \ (-\lambda + mO \ \Sigma O)}{\ iN, aO \ kO \ (\lambda - mO \ \Sigma O)}/. \text{Output}[1],
\[ \{ \frac{iO \ aO \ kO \ \lambda \ \Sigma iO + aN \ kN \ (-\lambda + mO \ \Sigma O)}{\ iN, aO \ kO \ (\lambda - mO \ \Sigma O)}/. \text{Output}[1]\} \}

Out[373]= \{\text{True}, \{1.30347, 2.44408\}\}

In[374]= "BOTTOM LINE";

In[375]= "The expected results based on the equal imitation model continue to hold. The analytical condition I identified for an increase in \(iN\) also holds under this simulation";

In[376]= "4. A DECREASE IN \(\mu_O\) and \(\mu_N\) (LOWER IMITATION RATES DUE TO TRIPS)";

In[377]= Clear[FEIN, FEIO]; Clear[\(\lambda\), \(aN\), \(aO\), \(kN\), \(kO\), \(\lambda\), \(\Sigma iO\), \(\Sigma O\), \(mN\), \(mO\), \(\eta_S\), \(\eta_S\), \(\mu_D\), \(\eta_S\), \(\eta_S\));

In[378]= "THE BENCHMARK PARAMETERS";

In[379]= \(dr\) = 0.07 - 0.014; \(\mu_N\) = 0.003; \(\mu_O\) = 0.011; \(\lambda\) = 1.5;
\(\eta_S\) = 2;
\(kN\) = 1; \(kO\) = 1;
\(aN\) = 1.4; \(aO\) = 3.8;
\(\Sigma iO\) = 1; \(\Sigma O\) = 1;
\(mN\) = 1.05; \(mO\) = 1.02;

In[385]= \(\mu_N\) = 0.003 * 0.9; \(\mu_O\) = 0.011 * 0.9;

In[386]= \(\text{CON}\) = \(c + \frac{\lambda \ (-aO \ kO \ \lambda \ (dr + iN + iO + \mu_O) \ \Sigma iO + aN \ kN \ (dr + iN + iO + \mu_N) \ (\lambda - mO \ \Sigma O))}{mN \ (\lambda - mO \ \Sigma O)} \) = 0;
\(\text{WG}\) = \(w - \frac{aO \ kO \ \lambda \ (dr + iN + iO + \mu_O) \ \Sigma iO + aN \ kN \ (dr + iN + iO + \mu_N) \ (-\lambda + mO \ \Sigma O)}{\ iN, aO \ kO \ mN \ (dr + iN + iO + \mu_O) \ \Sigma iO + aN \ kN \ (dr + iN + iO + \mu_N) \ (-\lambda + mO \ \Sigma O)} = 0; \(\text{nN}\) = \(iN + iO + \mu_N\); \(\text{nS}\) = \(iO + \mu_N - \frac{iO}{\ iN + iO + \mu_N}\); \(\text{nN}\) = \(\frac{iO + \mu_N}{\ iN + iO + \mu_N}\); \(\text{nN}\) = \(\frac{c (\lambda - mN \ * \ w)}{\ iO + \mu_N}\); \(\text{nN}\) = \(\frac{\lambda}{\ iO + \mu_N}\); \(\text{nN}\) = \(\frac{c (\lambda - mN * (\Sigma O))}{\ iO + \mu_N}\); \(\text{LN}\) = \((iN + aN + kN) + (iO + aO + kO) + \left(\frac{nN * c * mN}{\lambda}\right) - \frac{1}{\ (1 + \eta_S)}\) = 0 // \text{FullSimplify};
\(\text{LS}\) = \(\frac{(nO) * c * mO \ - \ \eta_S}{\lambda} + (nS) * c - \frac{\eta_S}{(1 + \eta_S)} = 0;\)

In[387]= Needs["Miscellaneous'RealOnly'"];
\(\text{NSolve}[[\text{LN, LS, CON, WG}, \{\text{iN, iO, w, c}\}]

Out[388]= \{{\text{iN} \rightarrow 0.602335, \text{iO} \rightarrow 0.525866, \text{w} \rightarrow 1.14542, \text{c} \rightarrow -0.143763} ,
{\text{iN} \rightarrow -0.00137533, \text{iO} \rightarrow -0.00386175, \text{w} \rightarrow 1.28014, \text{c} \rightarrow 0.922177} ,
{\text{iN} \rightarrow 0.00597715, \text{iO} \rightarrow 0.00890156, \text{w} \rightarrow 1.27516, \text{c} \rightarrow 1.2232}\}
In[389]:= FindRoot[{LN, LS, CON, WG}, {iN, 0.02}, {iO, 0.01}, {c, 1}, {w, 1}, MaxIterations -> 10000]
Out[389]= {iN -> 0.00597715, iO -> 0.00890156, c -> 1.2232, w -> 1.27516}

In[390]:= dw = (w /. %) - (w /. Output[1]); dc = (c /. %) - (c /. Output[1]);
diN = (iN /. %) - (iN /. Output[1]); diO = (iO /. %) - (iO /. Output[1]);
diA = ((iO + iN) /. %) - ((iO + iN) /. Output[1]); dnN = (nN /. %) - (nN /. Output[1]);
dnO = (nO /. %) - (nO /. Output[1]); dnS = (nS /. %) - (nS /. Output[1]);

In[391]:= "nN" -> nN, "nO" -> nO, "nS" -> nS, "πN" -> πN, "πO" -> πO,
"w*mN" -> w*mN, "mO*ΣO" -> mO*ΣO, "REPO" -> iO + iN + μO, "REPN" -> iO + iN + μN] /. %
Out[391]= {nN -> 0.340022, nO -> 0.359242, nS -> 0.300736, πN -> 0.131355,
πO -> 0.391423, w*mN -> 1.33892, mO*ΣO -> 1.02, REPO -> 0.0247787, REPN -> 0.0175787}

In[392]:= y = {iN, iO, iN + iO, w, c, nN, nO, nS, w*mN, mO*ΣO, πN, πO} /. %
Out[392]= {0.00597715, 0.00890156, 0.0148787, 1.27516, 1.2232,
0.340022, 0.359242, 0.300736, 1.33892, 1.02, 0.131355, 0.391423}
"BOTTOM LINE"

"A lower imitation rate for both $\mu_O$ and $\mu_N$ leads to higher aggregate innovation. Both local-sourcing directed R&D and outsourcing-directed R&D intensities attain higher levels.

Recall that the equal imitation rate model predicts that $i_N$ declines and $i_O$ increases. The above result is due to our considering a more reasonable case with $\mu_O > \mu_N$. In the other Mathematica file, we formally show that the impact of lower imitation rates on $i_N$ turns from negative to positive as the $\mu_O - \mu_N > 0$ gap increases.

Note that the outsourcing-directed R&D intensity, $i_O$, responds more. Thus the equilibrium share of outsourcing industries $n_O$ increases at the expense of Southern and Northern industries.

The North-South wage gap declines, because the profitability of local-sourcing directed R&D relative to outsourcing directed R&D goes down once all the change in innovation and imitation are taken into account.

Note that REPN and REPO both increase despite the fall in imitation rates. This is because $i_A$ increases and hence the pace of creative destruction. We can conclude though that the increase in the replacement rate as faced by the Northern producers (the increase in $i_A + \mu_N$) is proportionally larger than the increase in the replacement rate faced by the Outsourcing producers (the increase in $i_A + \mu_O$)."
\[
\lambda \left( -a_0 k_O \lambda (dr + iN + iO + \mu_O) \Sigma iO + a_N k_N (dr + iN + iO + \mu_N) (\lambda - m_O \Sigma) \right) = 0;
\]
\[
\frac{\Sigma iO + a_N k_N}{m_N (\lambda - m_O \Sigma)} = 0;
\]
\[
w = w - \frac{a_0 k_O \lambda (dr + iN + iO + \mu_O) \Sigma iO + a_N k_N (dr + iN + iO + \mu_N) (-\lambda + m_O \Sigma)}{a_0 k_O m_N (dr + iN + iO + \mu_O) \Sigma iO + a_N k_N (dr + iN + iO + \mu_N) (\lambda - m_O \Sigma)} = 0;
\]
\[
n_s = \frac{iO + iN}{iN + iO + \mu_N} - \frac{iO}{iN + iO + \mu_N};
\]
\[
n_N = \frac{c \lambda}{\lambda - \mu_N} - \frac{1}{1 + \eta_N} = 0; \text{ FullSimplify}
\]
\[
\Sigma = \frac{(\eta s) \lambda}{(1 + \eta_N)}.
\]

```
In[408]:= CON = \[ScriptCapitalC] + \[ScriptCapitalD] (\lambda - a_0 \lambda (dr + iN + iO + \mu_O) \Sigma iO + a_N k_N (dr + iN + iO + \mu_N) (\lambda - m_O \Sigma)) / m_N (\lambda - m_O \Sigma);

WG = w - \[Alpha] (\lambda - m_O \Sigma) / a_0 k_O m_N (dr + iN + iO + \mu_O) \Sigma iO + a_N k_N (dr + iN + iO + \mu_N) (\lambda - m_O \Sigma) = 0;

nO = \[ScriptCapitalD] iO/iN + iO + \mu_O;

nN = \[ScriptCapitalD] iO + \mu_N/iN + iO + \mu_N - iO/iN + iO + \mu_N;

\[ScriptCapitalN] = \[ScriptCapitalD] \lambda / \lambda - m_N w - \[ScriptCapitalD] (n_N * c * m_N) / \lambda - \[ScriptCapitalD];

\[ScriptCapitalL] = \[ScriptCapitalD] (n_s * c * m_s) / \lambda + \[ScriptCapitalD] n_s - \[ScriptCapitalD] n_s = 0;
```

```
Needs["Miscellaneous`RealOnly`"];

NSolve[{LN, LS, CON, WG}, {iN, iO, w, b}];

Out[409]= {{iN -> -0.601168, iO -> 0.524801, w -> 1.14913, c -> -0.142836},

{iN -> -0.00112621, iO -> -0.00395518, w -> 1.27926, c -> 0.923907},

{iN -> 0.00607262, iO -> 0.00872964, w -> 1.27455, c -> 1.22145})
```

```
FindRoot[{LN, LS, CON, WG}, {iN, 0.02}, {iO, 0.01}, {c, 1}, {w, 1}, MaxIterations -> 10000]
```

```
Out[409]= {iN -> 0.00607262, iO -> 0.00872964, c -> 1.22145, w -> 1.27455}
```

```
dw = (w / \[Lambda]) - (w / Output[1]);
dc = (c / \[Lambda]) - (c / Output[1]);
diN = (iN / \[Lambda]) - (iN / Output[1]);
diO = (iO / \[Lambda]) - (iO / Output[1]);
diA = ((iO + iN) / \[Lambda]) - ((iO + iN) / Output[1]);
dnN = (nN / \[Lambda]) - (nN / Output[1]);
dnO = (nO / \[Lambda]) - (nO / Output[1]);
dnS = (nS / \[Lambda]) - (nS / Output[1]);
```

```
"nN" = nN, "nO" = nO, "nS" = nS, "\[Pi]N" = \[Pi]N, "\[Pi]O" = \[Pi]O,
"w*mN" = w*mN, "mO*Z" = mO*Z, "REPO" = iO + iN + \mu_O, "REPN" = iO + iN + \mu_N} / .\%
```

```
Out[411]= \{nN -> 0.341115, nO -> 0.353394, nS -> 0.30549, \[Pi]N -> 0.131691,

\[Pi]O -> 0.390865, w*mN -> 1.33828, mO*Z = 1.02, REPO -> 0.0247023, REPN -> 0.0178023\}
```

```
y = {iN, iO, iN + iO, w, c, nN, nO, w + mN, mO*Z, \[Pi]N, \[Pi]O} / .\%\%
```

```
Out[412]= \{0.00607262, 0.00872964, 0.0148023, 1.27455, 1.22145,

0.341115, 0.353394, 0.30549, 1.33828, 1.02, 0.131691, 0.390865\}
```
In[413]:= {{"dw" → dw}, {"diO" → diO}, {"diN" → diN}, {"diA" → diA},
{"dnO" → dnO}, {"dc" → dc}, {"dnN" → dnN}, {"dnS" → dnS}}

Out[413]= {{dw → -0.00248238}, {diO → 0.00148106}, {diN → 0.000511625}, {diA → 0.00199269},
{dnO → 0.0342084}, {dc → 0.0111584}, {dnN → -0.0106335}, {dnS → -0.0235749}}

In[414]= "BOTTOM LINE";

In[415]= "A lower imitation rate targeting the Outsourcing industries
μO leads to higher aggregate innovation. Both local-sourcing directed
R&D and outsourcing-directed R&D intensities attain higher levels.

Note that the outsourcing-directed R&D intensity, iO,
responds more. Thus the equilibrium share of outsourcing industries
nO increases at the expense of Southern and Northern industries.

The North-South wage gap declines, because the profitability of
local-sourcing directed R&D relative to outsourcing directed R&D goes down
once all the change in innovation and imitation are taken into account.

Note again that REP and REPO increase. However, the increase in
the replacement rate as faced by the Northern producers (the increase
in iA+μN) is proportionally larger than the increase in the replacement
rate faced by the Outsourcing producers (the increase in iA+μO).";